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Proceedings of the American Society of Civil Engineers

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VEHICLE LOADS AND HIGHWAY BRIDGE DESIGN*

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(Proc. Paper 1302)

SYNOPSIS

Certain inconsistencies which are evident when existing highway bridges have to be rated for overload capacity justify a study of the causes and their rectification wherever practicable. The study involves consideration of a realistic loading and the logical safety factors to be applied to all loads and forces acting on the structure. The paper aims to present facts and ideas relating to the choice of a reasonably correct live loading and its application to highway bridge design. Typical bridge spans designed in accordance with current specifications have been studied with particular reference to their overload capacities.

INTRODUCTION

Why should we consider the advisability of changing present highway bridge design practice? Apparently, it produces structures of pleasing appearance which are carrying the ever-increasing highway traffic loads. No ordinary well-designed highway bridge, based on the H-15 or heavier standard loadings, has failed or shown signs of collapse because of the loads that have passed over it. But, when we have to rate an existing bridge for maximum safe load we find that this may vary considerably with the type of structure and length of span. This alone has raised a question as to the logic of the present design practice and at least it justifies a study of the causes of the discrepancy and practicable means of rectifying it.

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It is seen immediately that in determining the overload capacity of an existing bridge, only the live load moment or shear stress is being raised above its designed value, whereas in the design, an equal margin of safety was provided against an increase in dead load. To assume, however, that the overload capacity of a bridge span is solely a function of the dead-to-live load ratio is an over-simplification. There are other provisions or requirements of the design specifications that have a bearing on the ultimate load capacity and must be given consideration. And in considering them it must be kept in mind that bridge design is subject to conditions which make it impractical to determine the exact value for a safety factor against the possibility of failure. Many indeterminate or unpredictable conditions, including the possible combinations of vehicle loads together with the other forces acting on the structure, make the problem of safety factor one that requires experience aided by the theory of probability for its final solution.

A factor that has to be considered when setting up or modifying our design specifications is the need for comparatively simple and clearly defined procedures which will permit the ordinary designer to prepare his plans expeditiously with, of course, due regard for structural economy. Also, very exact theoretical computations to determine minimum sections and quantities of material are seldom necessary since it is often more economical to design structural members on the basis of standard sections or a size and shape easy to construct by present methods of fabrication and erection. Other factors of importance that govern the design are limitations on deflection or vibration, provisions for deterioration, and cost of maintenance.

It is not the purpose of this paper to cover all these factors or to present a set of revised specifications for the design of highway bridges. Its aim is to present facts and ideas that will help in determining a practicable type and a reasonable magnitude for a bridge design live loading giving due consideration to its practical application. Factual data are presented on the size, weight and frequency of occurrance of presently operated highway vehicles used for regular hauling and also those used for special purposes. Also presented is information on the status of existing bridges with a discussion of the economics of replacing them in connection with the adoption of a proper design loading. The overload capacity of various types of bridge spans of moderate length, designed under the current specifications, has been computed in order to study the cause of any inconsistency that exists. It is hoped this information will be helpful in solving a problem the answer to which must be based chiefly on engineering judgement fortified by experience.

Vehicle Data

a. Actual Vehicle Loads

The State vehicle codes which regulate the weight of highway vehicles and vehicle combinations have been approaching uniformity in recent years but major differences still exist. The AASHO Policy of Limiting Sizes and Weights of Motor Vehicles, issued in 1946, did tend to establish a measure of uniformity and was adopted as it stands by several states. However, a large number of the State codes permit heavier vehicles than would be permitted under the AASHO recommendations. State codes that permit the

heavier vehicles fall into two general classifications: (1) those of the New England and Mid-Atlantic States which allow single-axle loads up to 22,400 lb., but place the greater restrictions on length and gross vehicle weights, and, (2) those of the Rocky Mountain and Pacific Coast States which restrict single-axle loads to 18,000 lb., and dual-axle loads to 32,000 lb., (as does the AASHO policy) but permit longer and heavier vehicle combinations. It is the predominant practice in these Eastern States to limit gross vehicle weights to specified amounts for the various types of vehicle and vehicle combination, whereas the Western States use a table that limits the weight on axle groups or vehicles according to the length of wheel base of the group or vehicle. Axle group weights are related to their spacing by a formula in the AASHO policy. However, since previous state laws had permitted a large investment in motor vehicles with closer axle spacings than those conforming to the AASHO policy, some increase in the axle group weights was necessary to avoid what was considered an undue economic hardship.

Figures taken from studies made at Texas A & M College (1) illustrate the practical effect of the two forms of weight restriction. They are based on the average of the 25 heaviest vehicles of each of six major vehicle types weighed in each state in the 1951 heavy-vehicle loadometer survey. The axle weights for three representative vehicle types are shown in Table I.

These figures, while they indicate that the different codes have not resulted in any startling difference in vehicle loads, do show the prevalence of heavier single- and tandem-axle loads in the Eastern States. These axle loads are of primary importance in the service life of a pavement and the shorter length bridge spans that make up the majority of highway bridges.

Loads operating under the provisions of properly enforced vehicle codes should be based on the ability of pavements and bridges to withstand the stresses caused by repeated applications. In addition, it is necessary to consider the ability to withstand very infrequent loads of much greater magnitude which, in cases of either military necessity or the general welfare, are permitted to travel the highways under special permit. Useful information from which to decide what should be done to provide against these occasional loads is the design of special-purpose vehicles now in use for transporting heavy machinery and equipment and those in use for hauling ore or logs over private roadways. The latter, which are not allowed to operate over public highways, would very likely cause serious damage to existing bridges and pavements. Examples of such special purpose vehicles together with a maximum weight 3S2 vehicle operated under the California Vehicle Code, and the H20-S16 design truck loading, are shown in Fig. 1.

b. Stresses from Vehicle Live Loads

The relative stress produced by legal and special-purpose vehicles and vehicle combinations as compared to that resulting from the design loadings and legal loadings, is illustrated in Fig. 2. For this purpose, comparative stress is based on the moment per lane produced by a single line of vehicles in a simply-supported span of the length shown. A similar pattern results if vertical shear is used as the index of stress. Stresses in continuous spans, girders of various spacing, multiple lane structures, etc., are more complex and require special investigation. Other factors that complicate the study of comparative load stresses are fatigue effects, impact and the combination of live load stresses with those produced by dead load and other concurrently acting forces.

c. Frequency of Actual Heavy Vehicle Loading

Repeated stressing can cause a structural member to fail under a much lower stress than the ultimate strength of the material as determined by laboratory tests. Hence, in order to decide upon a future design loading, it is necessary to estimate the probable frequency of occurrence of heavy loads. Statistics obtained from present-day vehicle operation are again the main source of information upon which to base such an estimate.

Periodical weighings of the heavier vehicles and vehicle combinations at loadometer stations are a good indication of the probable frequency of occurrence of these heavy vehicle loads. It will be noted that the cost and the need to avoid undue interference with traffic, render such statistics somewhat incomplete. But they are a sufficiently accurate representation of the conditions resulting from present-day heavy vehicle operation and enforcement of the weight limits to serve as a basis for determining the proper loading for future bridge design.

The data on which Fig. 3 and 4 are based are loadometer station records taken in California in 1953 and 1954. The practical criterion for actual vehicle loads producing maximum stresses in the shorter spans—up to about 35 ft.—is the tandem-axle load. Fig. 3 shows the number of tandem-axle loads, in groups of 1,000 lb., that were weighed at 10 loadometer stations in 1953. It is seen that one-third of all the vehicles weighed, loaded or empty, fall between 27,000 lb. and the maximum legal weight of 32,000 lb. Only 2.5% of these tandem-axle loads exceeded the legal limit.

For longer spans, up to the point where more than one vehicle combination must be considered (about 100 ft.), the 3S2 tractor and semitrailer is a satisfactory criterion.

Fig. 4 shows the relation between the number of 3S2 vehicle combinations and the moment they produce in one lane of a simply supported 60-ft. span. It will be noted that a large percentage of the combinations produce a moment in a 60-ft. span that exceeds the 418.5 kip-ft. moment of the H15-44 design loading but all are below the 800 kip-ft. moment of the H20-S16 loading. The moment produced by the 3S2 combination (shown in Fig. 1) is the greatest that will be produced by a practicable combination of axle loads and spacings for this type of vehicle combination conforming to all provisions of the California Vehicle Code. The moment of 672 kip-ft. produced by this 38-ft-long 3S2 combination is greater than the moment produced by all but one or two of the combinations weighed at the ten stations.

For very short spans (less than 10 ft.) and for pavements, the maximum legal single axle load of 18,000 lb. or 32,000 lb. dual-axle load is the critical loading. They produce less stress on the short spans, however, than does the single 24,000 lb. axle loading of the design specifications. (Two 24,000 lb. axles, 4 ft. C-C are used for Interstate Highway System bridges.)

Highway Bridge Data

a. Data on Existing Bridges

We still have to cross highway bridges that were designed to carry a road roller or some similar live load fancied by the designer. Uniformity, and a set of definite design rules, can be said to have started with the AASHO design specifications and their "H" loadings. We may also say that the first

modern highway bridges were designed for an H-15 loading which still constitute a large portion of the highway bridges of a permanent type. Although the proportion of H-15 bridges has been decreasing, this is largely due to the building of new highways with bridges of heavier design rather than replacement of older bridges.

Some idea as to the situation on our principal highways can be had from an October, 1956, tabulation of capacity ratings of 4,827 bridges on the California State Highway System. (See Table II) It should be noted, however, that the percentage of bridges of less than H-15 capacity rating is much greater on county roads.

It is apparent that the investment in bridges of H-15 or less capacity is very large, and the investment in the newer bridges of H20-S16 design is becoming enormous. And since most of these are on densely travelled highways, replacement would be extremely difficult as well as costly—although some strengthening may be practicable. Furthermore, since it is impractical to restrict heavy hauling to particular highways or sections of highways, we often find that the bridges on roads in sparsely settled areas get a serious pounding from heavy vehicles such as logging trucks. Hence, as a matter of economy, it does not seem logical to design future bridges for greater overload capacity than that of the stronger bridges now in use. To replace the existing bridges on the California State Highway System alone (some 30,000,000 square feet) would cost on the order of half a billion dollars.

An October 1956 tabulation of span lengths in California highway bridges is shown in Table III. Spans less than 10 ft. are not included, and about one-half the spans in the 10- to 19-ft. bracket are 19-ft. span timber trestle. One can see that about 96% of all of the spans are less than 100 ft. in length. And the fact that up to 87% of the spans are less than 60 ft. in length accounts for so much emphasis in this paper on these short spans. Even in long bridges, the data on short-spans applies to the floor system.

b. Dead Load Moments and Shears in Highway Bridges

In order to present data which involves the dead load of bridge spans, typical bridge designs have been chosen. The H20-S16 vehicle of the AASHO and the bridge design standards prepared by the Bureau of Public Roads "Standard Plans for Highway Bridge Superstructures, 1953 Edition" which are generally representative of design practice throughout the nation are used in the following illustrations.

Table IV shows dead load moments at the center of the span and the end shears, per lane, for various types of bridges designed for H20-S16 loading.

Examination of the table shows that for comparable spans, the moments and shears from dead load in reinforced concrete "T" beam bridges are almost twice those in Standard "I" beam bridges with non-composite decks. The reinforced concrete box girder which is also used extensively has about the same dead load moments and shears as T-beam bridges. There is evidently some reduction in dead load when composite construction is used for I-beam bridges.

c. Relation Between Dead Load and Live Load
in Highway Bridges

The use of lane loadings as a basis for determining stress relationships in individual members of bridges can lead to erroneous answers. For a

correct comparison it is necessary to determine the dead and live load shears and moments per beam or per foot-width-of-slab rather than on a per lane basis.

In distributing dead-load to the beams the usual assumptions on dead weight of slab have been used. Distribution of live load to the beams is a more uncertain procedure. Tests performed at the University of Illinois over a period of several years (2) show that the distribution of loads to longitudinal beams and slabs as set forth in the AASHO Specification, 1953 Edition, is fairly accurate for practical designing.

Fig. 5 illustrates the variation of the dead-to-live load ratio "R" with span length, for an interior and an exterior beam of standard non-composite I-beam bridges, and for reinforced concrete T-beam bridges. The live loads used are the "maximum legal vehicle" shown in Figure 1. with no allowance for impact and the H20-S16 design load plus impact for the interior beam only. Dead-load moments are computed from the B.P.R. standard plans and the live loads are distributed according to the AASHO design specifications.

Overload Capacity in Current Bridge Design

In theory, other things being equal, the overload capacity of a bridge designed by present criteria should increase proportionately with "R." Assume that we are computing the overload capacity of an existing bridge based on an increase in the moment stress up to a maximum value beyond which the structure is likely to be permanently damaged. Assuming the stress from forces other than dead and live load to be negligible, we can represent the design condition by the equation:

$$MLL + MDL = f.Z. \tag{1}$$

Wherein: MLL = the live-load moment stress caused by the design loading.

MDL = the calculated dead-load moment stress.

f. = the allowable design, or "working" stress.

Z. = the section modulus or "form factor."

If we agree that the stress in the critical members can be raised to a maximum value f' under an infrequent load (the AASHO rating specifications permit a maximum increase in the allowable design stress of 50% for this condition), then the design live-load can be increased N times and we have the following equation for ultimate moment value MU:

$$MU = N.MLL + MDL = f'.Z. \tag{2}$$

Dividing equation (2) by (1) and, for convenience, letting MDL/MLL = R and f'/f = C, we have:

$$N = C + R(C-1) \tag{3}$$

Hence, for a given value of C, N should be a straight line function of R starting with the minimum value of C. This relationship assumes that the resisting section Z does not change as the stress approaches its ultimate value which is the case in non-composite, "I" beam spans but not in the case of reinforced concrete beam spans. In the illustrative examples and computations which follow, the minimum specified value for the elastic limit of steel

will be taken as the ultimate stress value. Then MU will be the moment value when the stress in the bottom fibers of an I beam reaches 33,000 psi or the tensile reinforcement in reinforced concrete beams reaches a stress of 40,000 psi (for intermediate grade steel). Ultimate moment in the case of R.C. slab and T-beam spans will be computed by the method of the ACI-ASCE Joint Committee Report on Ultimate Design. In general, R.C. beam sections are under-reinforced. Composite beam sections are computed on the assumption that the dead loan is carried by the I-beam alone and the live load by the combined beam and deck slab. A modular ratio of 10 has been assumed. Then, from equations (2) and (3),

$$N = (MU - MDL)/MLL = C + R(C-1) \qquad (4)$$

wherein, for structural carbon steel, C = 33,000/18,000 = 1.83, and for intermediate grade steel C = 40,000/20,000 = 2.00, assuming the design stresses to be the maximum allowable stress of the AASHO design specifications — which is seldom the case.

It is evident that the value of MU so determined is not strictly correct for all conditions, particularly since it neglects the effect of other forces and factors pertinent to the safety and service life of the structure. Nevertheless, these simplifying assumptions do not invalidate the use of the equation to show the inconsistencies that may result from present design practices. It is to be noted, however, that in applying equation (4) to actual designs, the relationship between N and R is affected by many practical factors of design and use of fabricated materials. These include: the designer's choice of beam depths and spacings to meet conditions of economy or required clearance; the use of standard-size rolled or otherwise fabricated sections; empirical rules for distributing the load to members of the structure; and, maximum design stresses somewhat lower than the specified allowable stress which is frequently the case in practice. In other words, the value of C is not a constant for all design. Figures 6 and 7 show the values of N determined from equation (4) applied to the B.P.R. standard bridge designs previously referred to. Two live loadings, with no allowance for impact, have been used for computing the design or "service load" moment to be compared with the ultimate resisting moment. These are the standard H20-S16 design loading, and, the practical maximum 3S2 vehicle combination legal in California.

Figure 6 shows the variation of N with the length of span for non-composite I-beam spans. It is shown for both the loadings and for both exterior and interior beams. Figure 7 shows the same information for reinforced concrete slab and T-beam spans. It will be seen that the maximum overload capacity, N, of reinforced concrete spans is definitely higher than that of the steel beam spans, due to the considerably greater values of R. In the case of R.C. spans the value of N increases with the length of span although not proportionately as would be expected from the theoretical assumption of a constant value of C. Variation in the value of C (i.e. the variation of the "working" stress from the maximum allowable value) together with an increasing value of R with span length causes the value of N for the steel beam spans to remain fairly constant although with some irregular fluctuation.

The general observations to be drawn from Figures 6 and 7 are:

(1) Present design practice seems to produce a fairly uniform overload capacity in the case of steel I-beam spans and probably in any type in which the range of R is relatively small. In this particular case, however, the overload capacity of the exterior beam is greater than that of the interior beam.

(2) The overload capacity of reinforced concrete spans is relatively greater than that of steel spans and increases considerably with the length of span and its relatively large increase in the value of R.

The following computed values of N, based on extreme values of C and R taken from the foregoing data for standard bridge spans and the H20-S16 loading, give a general picture of the relation between these factors in ordinary designs. Referring to equation (3) and applying it to the interior girder of a non-composite steel beam span:

(1) For an approximate average value of $C = 1.95$ and extreme values for R of 0.28 and 0.71,
$N = 1.95 + 0.95 \times 0.28 = 2.22.$ - 100%
$N = 1.95 + 0.95 \times 0.71 = 2.62.$ - 114%

(2) For an approximate average value of $R = 0.5$ and extreme values for C of 1.88 and 2.12,
$N = 1.88 + 0.88 \times 0.5 = 2.32.$ - 100%
$N = 2.12 + 1.12 \times 0.5 = 2.68.$ - 115%

Thus it can be seen that ordinary variations in the actual design working stress which are reflected in the value of C can affect the value of N as much as the normal change in the value of R with increase in span length.

Comparing these values with those of a reinforced concrete span we find that for a 20-foot slab span, N has a value of about 2.0 and for a 60-foot T-beam span its value is about 5.0. For these values of N the values of R are about 0.45 and 1.55 respectively. The value of C would thus be about 1.7 and 2.6 respectively showing that in the case of reinforced concrete spans the ratio of ultimate (elastic limit) stress to the design stress is not an accurate criterion.

Discussion of the Safety Factor

There are two things to be settled in connection with the amount to be provided as a factor of safety against future eventualities and unknown conditions. First, is it large enough to provide reasonable safety without being unnecessarily costly? Second, is it as near the proper magnitude for all parts of structures as is practicable under the normal procedures used in design and construction?

In answer to the first question, we may be uncertain as to what the future may bring in the way of highway vehicle loads; but we are faced with the necessity of deciding upon the magnitude of a maximum loading to be provided for (both infrequent and repetitive). We cannot avoid the decision. The safe stress that should be permitted under this maximum loading is another matter. Here we have to leave a margin for the uncertain factors that are often referred to as "factors of ignorance." They include such matters as the use of approximations and empirical rules in practical design

procedures, and the variable strength of materials as normally manufactured and fabricated. Another uncertain factor is impact. What is its magnitude under all conditions affected by type of structure, length of span, type, weight and speed of vehicle, vibration, multiple vehicle loads, etc? In certain cases deterioration in use is a factor that cannot be accurately predicted and may depend on how well the structure is maintained.

It may be noted that even though the magnitude of stresses caused by impact were known, there is a question as to its application in the case of an infrequent heavy load. These overloads are permitted to travel only under a special permit which can require them to cross a bridge at slow speed when no other heavy vehicle is on it. Adherence to this would justify omitting impact stress in computing the overload capacity. Likewise the calculation of combined stress from loads and other forces under these conditions is a moot question.

It would seem, therefore, that if the maximum load capacity of the average span designed for an H20-S16 loading should be chosen as a criterion for future designs, the difficult problem will be the determination of the maximum permissable stress under this maximum loading. In other words, what should be the magnitude of the factors of ignorance? Except for such enlightenment as can be obtained from applying the theory of probability to available statistics (3) this must be based on broad experience and judgement backed by careful thinking.

Following tradition, a design loading has been used that is of the same order of magnitude as the maximum legal vehicle loads. This "service load" (the exact meaning of which has never been clearly defined) in the more recent specifications is multiplied by a "load factor" to obtain the "ultimate" load. This assumes that the ultimate load, insofar as its effects are concerned, is a replica of the design or service load. In view of the many uncertainties enumerated this may be a reasonable, though not a necessary assumption. Why not specify an ultimate live load only? And as for the dead load, if maximum weights of material are specified and future surfacing computed as additional uniform load, there seems no reason to use a load factor (1.2 to 1.5) for dead load except to take care of approximations and carelessness in design computations.

Let us now consider the second question which relates to the need for proper relative values of the safety factors. From the data presented it is seen that the effect of using the same safety factor for both live and dead load does not always result in spans of materially different overload capacities. It is evident, however, that it does make a considerable difference in some cases such as when steel beam spans are compared to R.C. girder spans. It cannot be denied that, at least in theory, designing for a maximum load and a maximum safe stress for each and every necessary condition of loading is the more correct and logical procedure. It will bring about more nearly equal overload capacities in all cases. As more is learned about the behavior of materials, and progress in their manufacture permits more rigid requirements, the matter of correct procedure becomes increasingly important. At the present time many agencies are testing the properties of structural materials in various forms and under different kinds of loadings. The idea of "ultimate design," "plastic design," or whatever it may be called as a method of computing the structural section is gaining headway. More attention needs to be given to an "ultimate loading" to go with the maximum

permissable stresses so determined. The designer of airplanes and of machines has always had to face this problem and has met it quite successfully. It is probably because he has not been forced to face it that the highway bridge engineer has been content to leave well enough alone.

Summary and Recommendations

It is pointed out in this paper that there is an apparent lack of consistency in the overload capacity of existing bridges designed for the same loading under the current specifications. But, since several provisions of the specifications affect the design and resultant overload capacity, no change, however logical it may seem, should be made without consideration of all the factors involved. At this stage the following conclusions seem reasonable:

1. The first matter of importance is whether the stresses produced in the bridge members conform to the design stresses based on one of the presently used design loadings. Using moment in a simple span as a criterion, it appears that the H20-S16 design loading simulates fairly well the effect of actual vehicle loads operating under the state vehicle codes.
2. In order to arrive at the safe value for maximum stresses in the bridge members, it is necessary to look into the possibility of having to reduce maximum stress under a single loading in order to take care of stress repetitions. It would appear from the limited data presented herein that the possible occurrence of loads heavy enough to cause a reduction in strength due to their repetition is not a matter of concern under present conditions.
3. From the data presented, it is evident that there is a large investment in the older bridges of a "permanent" type of construction which are carrying the current highway loads without any signs of distress. It would be very difficult to finance their replacement particularly if the later spans of H20-S16 design are included. And it is very doubtful if it could be justified by the advantages it might offer to heavy hauling over the highway. This is a good reason for not adopting a higher strength standard for future bridge design than is now obtained by using the current specifications and the H20-S16 loading.
4. Overload capacity, or the maximum load the bridge should be designed to carry, should be the same for all bridges—as nearly as practical matters will permit. It is of interest, therefore, to see how consistent our present bridges are with respect to the maximum load they can carry and to see what factors influence the results. It is found that although, from a theoretical standpoint, the overload capacity of existing bridges is a direct function of the dead-to-live load ratio, there are other factors which affect the result. It would be expected that the overload capacity would increase with length of span since the dead-to-live load ratio normally increases. However, it is found by checking the overload capacities of standard bridges designed by the Bureau of Public Roads that this rule does not hold in many cases. In general, the overload capacity as a rule does not increase consistently with the length of span in the case of steel beam spans but does so in the case of reinforced concrete bridges. And concrete bridges as a whole, because of their relatively greater dead load, have greater overload capacity than the lighter steel beam bridges.

5. It is the opinion of the authors that bridges designed by the current specifications and using the H20-S16 loading are of as high a standard of load carrying capacity as the economy can justify. While the variation in load capacities may not be considered a very serious matter, it is felt that the inconsistencies that do exist can be smoothed out by revision of the design specifications on the basis of ultimate strength design. This will result in a more nearly correct relation between the dead and live load and it is in keeping with the present trend evidenced by recent design specifications for prestressed concrete and the alternate specifications for conventional reinforced concrete. All these base the ultimate load on present design loadings multiplied by an arbitrary "load factor." The load factor differs in the specifications referred to and it is the opinion of the authors that there is need for agreement on a realistic and practical type of ultimate loading to be applied to any length of span.

REFERENCES

1. Bulletins and other publications of the Engineering Experiment Station and the Transportation Institute of the A. & M. College of Texas covering highway vehicle weights and frequency studies.
2. University of Illinois, Eng. Experiment Station, Bulletins Nos. 304, 1938; 336, 1942; 363, 1946; 375, 1948; 396, 1952. "Design of Slab and Stringer Highway Bridges," Newmark and Siess, Public Roads Vol. 23, No. 7, Jan. Feb. Mar. 1943. Also, Report 14-B, Highway Research Board, 1952, "Distribution of Load Stresses in Highway Bridges."
3. "Safety and the Probability of Failure," Freudenthal, Paper No. 2843, Trans. A.S.C.E. Vol. 121, 1956.

TABLE I

ACTUAL HEAVY VEHICLES IN THE UNITED STATES

From The

1951 Heavy Vehicle Loadometer Survey By Texas A. &M.

TYPE OF VEHICLE	AXLE NO.	WEIGHT IN POUNDS	
		EASTERN STATES	WESTERN STATES
TWO AXLE VEHICLE	1ST	8,008	6,714
	2ND	23,089	21,021
	G.V.W.	31,097	27,735
THREE AXLE VEHICLES	1ST	8,268	8,372
	2ND	18,278	17,400
	3RD	17,310	15,941
	G.V.W.	44,216	41,713
FIVE AXLE COMBINATIONS	1ST	8,329	9,079
	2ND	13,943	16,389
	3RD	14,143	15,181
	4TH	13,385	15,879
	5TH	14,543	15,980
	G.V.W.	64,343	72,508

TABLE II

CAPACITY RATING OF 4,827 BRIDGES IN THE CALIFORNIA STATE HIGHWAY SYSTEM

CAPACITY RATING AND TYPE OF CONSTRUCTION	% OF TOTAL NO. OF BRIDGES	
H20, H20-S16 OR STRONGER OF STEEL OR CONC. CONST.	35.7%	
H15 OR EQUIVALENT OF STEEL OR CONCRETE CONSTR.	37.7	54.5
H15 OR EQUIVALENT OF TIMBER CONSTRUCTION	16.8	
LESS THAN H15 OF MISC. CONST.	9.8%	

TABLE III

FREQUENCY DISTRIBUTION OF BRIDGE SPAN LENGTHS IN THE CALIFORNIA STATE HIGHWAY SYSTEM

SPAN LENGTH FEET	NUMBER OF SPANS	PERCENT OF TOTAL NO. OF SPANS	CUMULATIVE PERCENT
10 TO 19	7,655	35.32	35.32
20 TO 29	4,078	18.85	54.17
30 TO 39	3,749	17.30	71.47
40 TO 49	1,936	8.97	80.44
50 TO 59	1,414	6.54	86.98
60 TO 69	1,116	5.15	92.13
70 TO 79	416	1.93	94.06
80 TO 89	355	1.64	95.70
90 TO 99	142	.70	96.40
100 TO 109	214	.99	97.39
110 TO 199	416	1.93	99.32
200 TO 399	121	.56	99.88
400 TO 999	14	.06	99.94
1,000 TO 4,200	13	.06	100.00

TABLE IV

SIMPLE SPAN DEAD LOADS AND SHEARS PER LANE IN HIGHWAY BRIDGES

Bridges designed by the Bureau of Public Roads for H20-S16 loadings using AASHO Spec.

SPAN FEET	CENTERLINE DEAD LOAD MOMENT - KIP FEET PER LANE / DEAD LOAD END SHEAR - KIPS PER LANE			
	STD. I BEAM, NON-COMPOSITE CONCRETE DECK BRIDGE	STD. I BEAM, COMPOSITE, CONCRETE DECK BRIDGE	REINFORCED CONCRETE SLAB BRIDGE	REINFORCED CONCRETE "T" BEAM BRIDGE
20	113 / 22.7		142 / 28.5	
25	180 / 28.7		250 / 40.0	
30	251 / 33.5		402 / 53.5	
35	361 / 41.3		672 / 76.8	
40	475 / 47.5			748 / 74.8
45	611 / 54.3			
50	769 / 61.5	744 / 59.5		1,325 / 106
60	1,200 / 80.0	1,105 / 73.7		2,265 / 151
70	1,683 / 96.2	1,577 / 90.2		
78	2,104 / 113.0			
80		2,372 / 118.6		
90		2,968 / 131.7		
100		3,888 / 155.5		

VEHICLE LIVE LOADS

FIG. 1

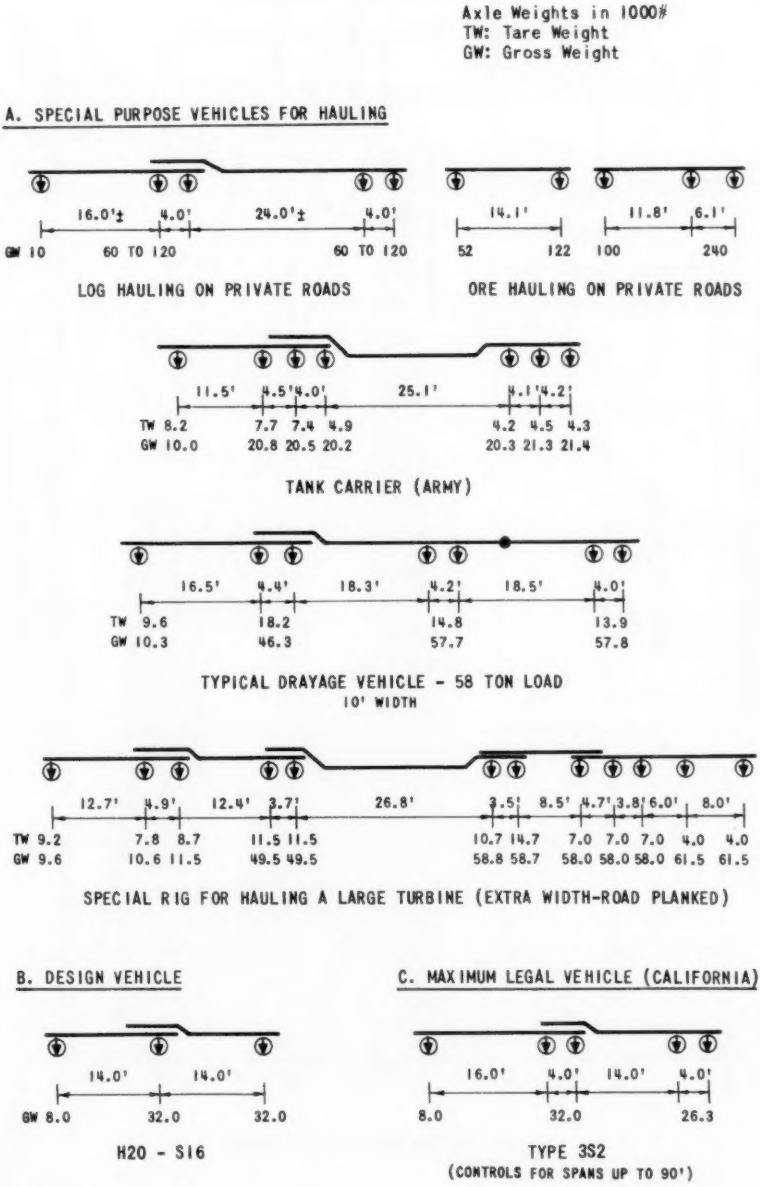


FIG. 2

LIVE LOAD MOMENTS PER LANE IN SIMPLE SPAN BRIDGES
PRODUCED BY VARIOUS VEHICLES

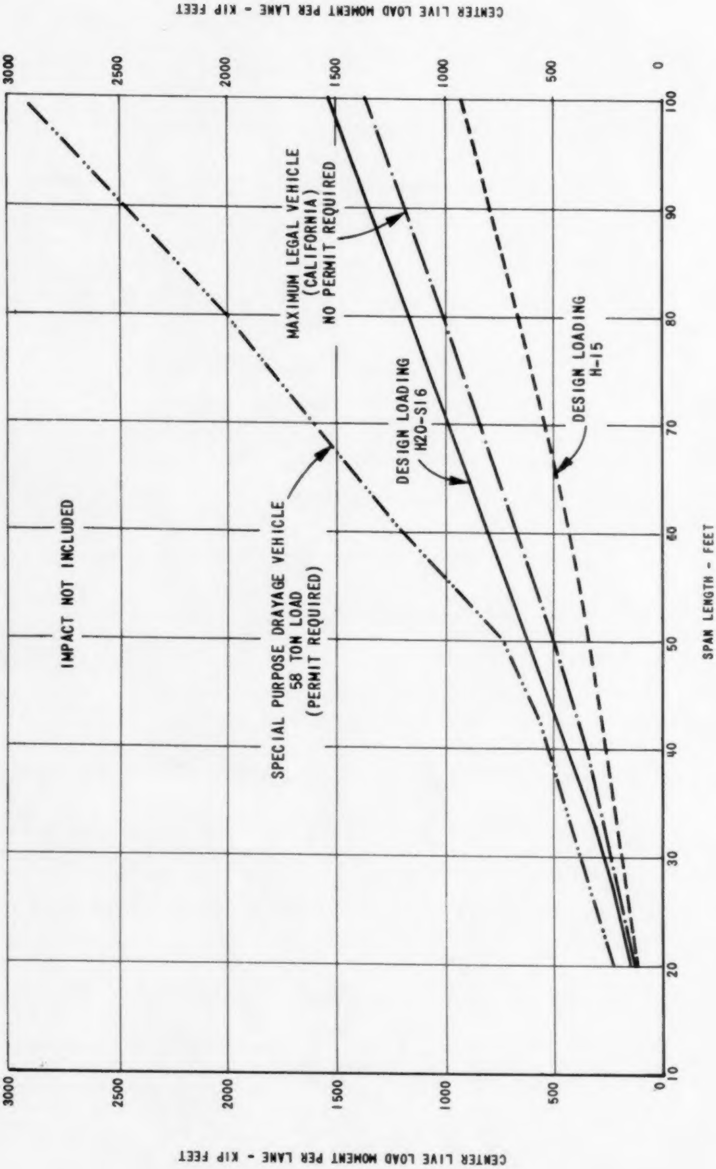


FIG. 3

FREQUENCY DISTRIBUTION CHART
OF
LARGE TRUCK TANDEM AXLES

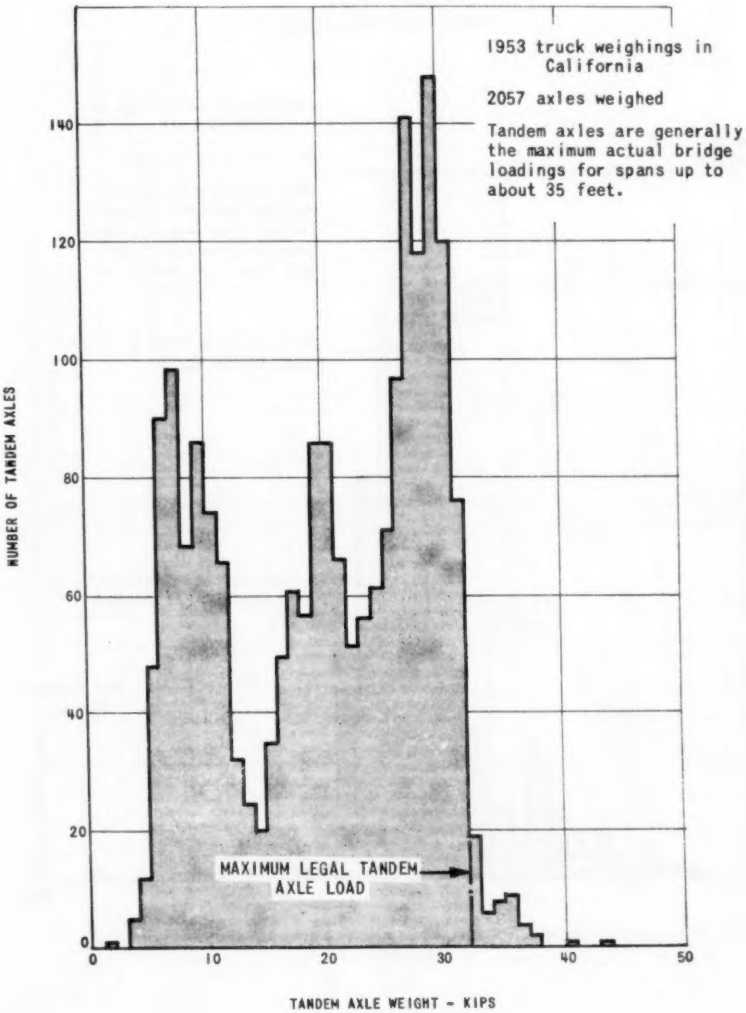


FIG. 4

FREQUENCY DISTRIBUTION CHART
OF
MOMENT AT THE CENTERLINE OF 60 FOOT SIMPLE SPAN BRIDGES
PRODUCED BY
LARGE 3S2 VEHICLES

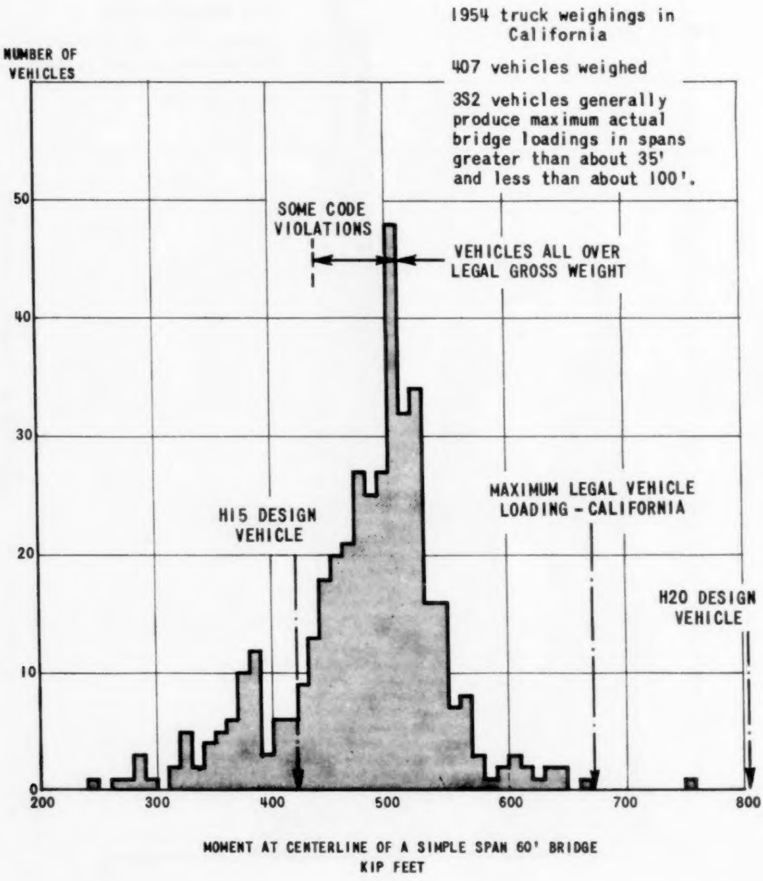


FIG. 5

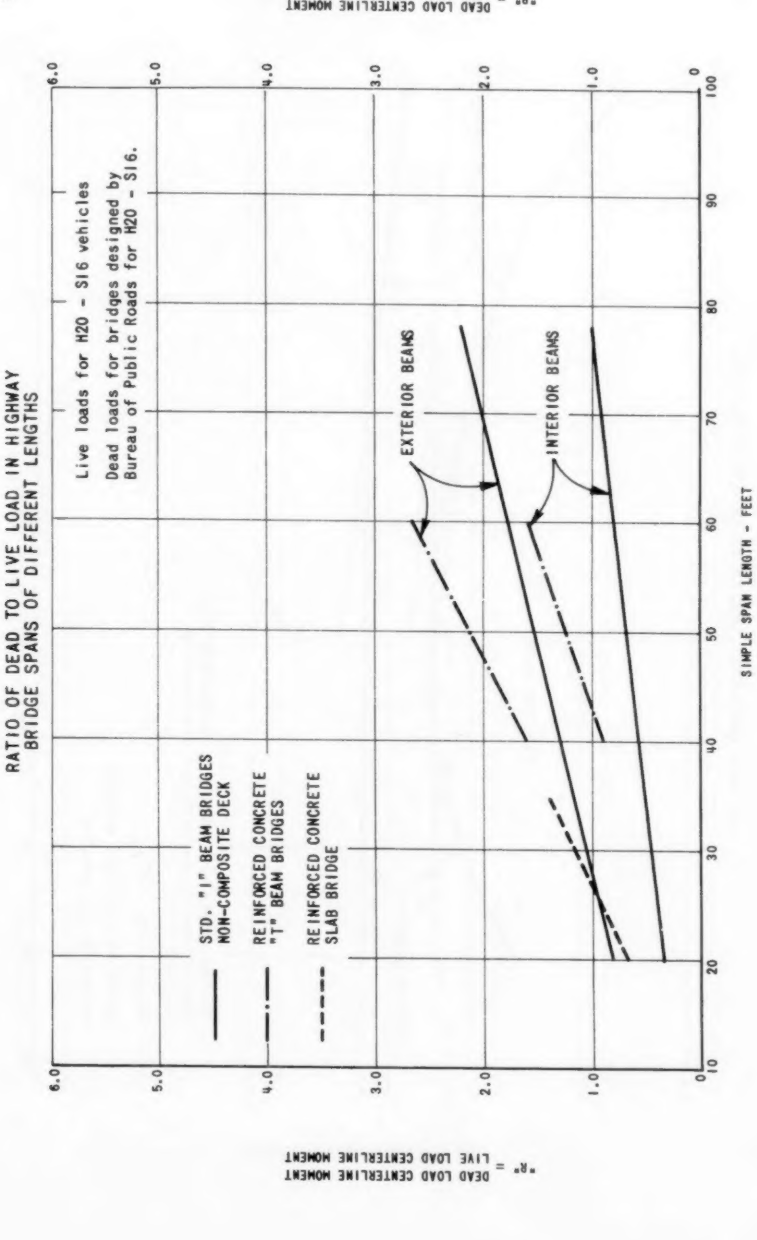

RATIO OF DEAD TO LIVE LOAD IN HIGHWAY BRIDGE SPANS OF DIFFERENT LENGTHS
Live loads for H20 - S16 vehicles
Dead loads for bridges designed by Bureau of Public Roads for H20 - S16.
EXTERIOR BEAMS
INTERIOR BEAMS
STD. "I" BEAM BRIDGES NON-COMPOSITE DECK
REINFORCED CONCRETE "T" BEAM BRIDGES
REINFORCED CONCRETE SLAB BRIDGE
"R" = DEAD LOAD CENTERLINE MOMENT / LIVE LOAD CENTERLINE MOMENT
6.0
5.0
4.0
3.0
2.0
1.0
0
10
20
30
40
50
60
70
80
90
100
SIMPLE SPAN LENGTH - FEET

FIG. 6

MAXIMUM OVERLOAD CAPACITIES OF STEEL BRIDGES

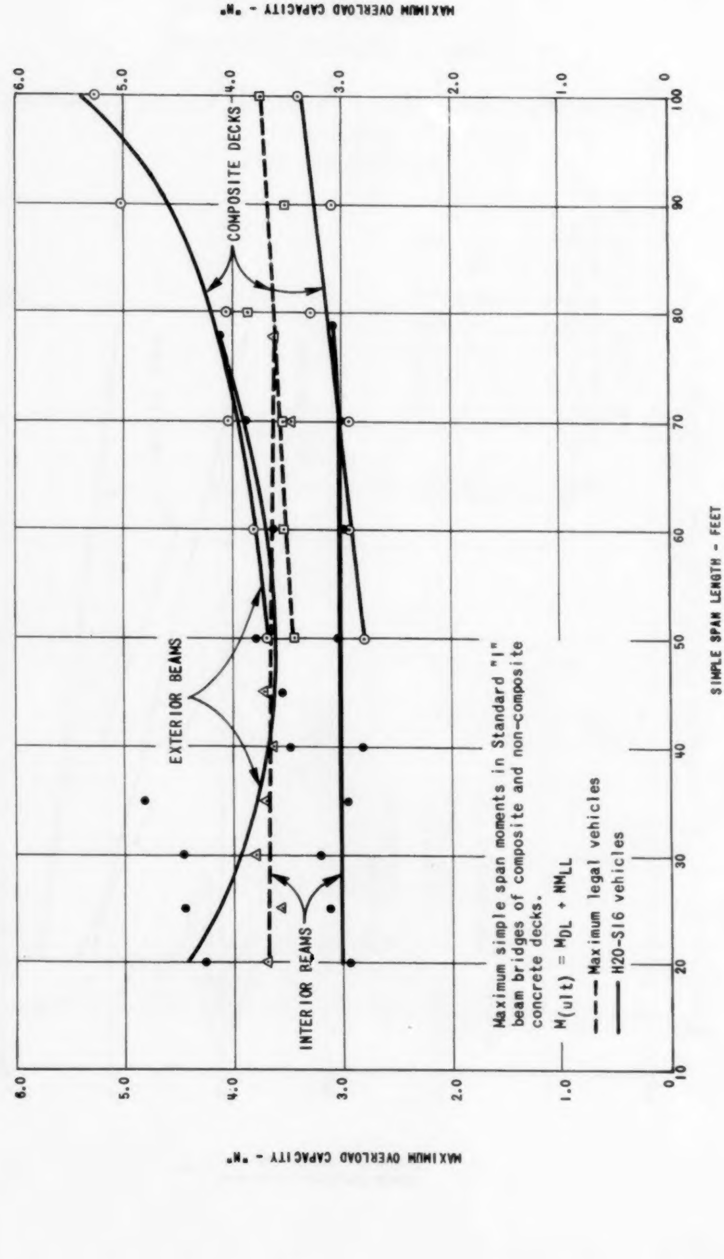
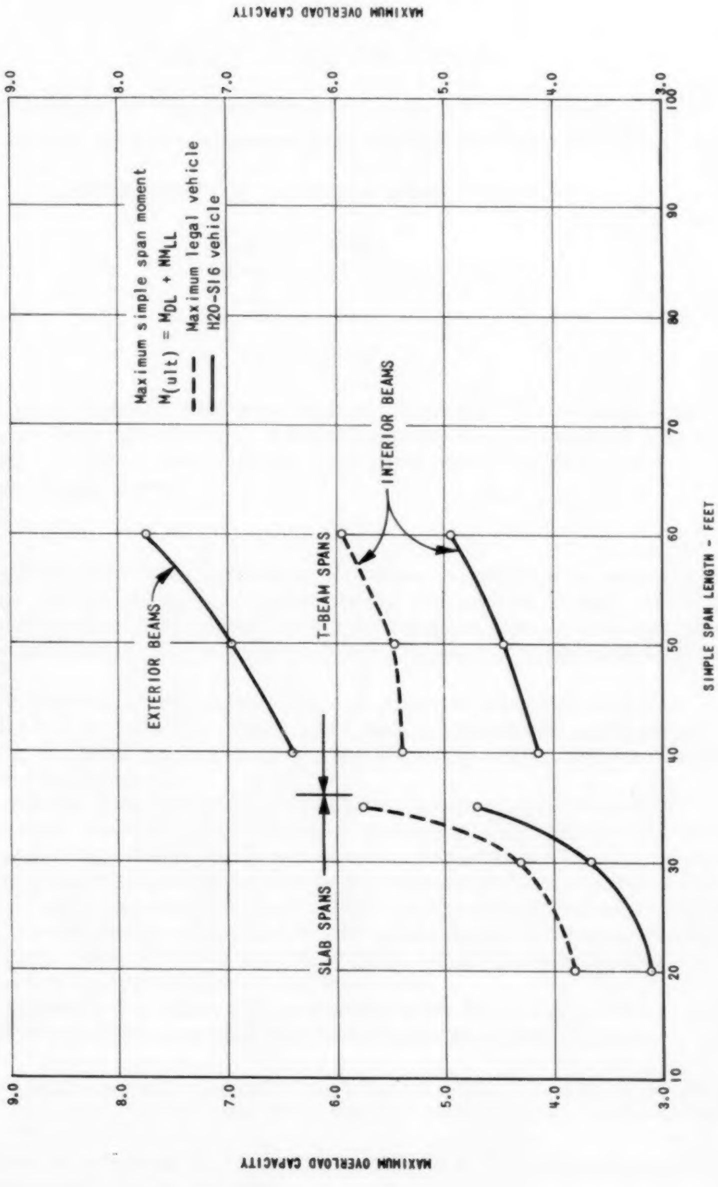


FIG. 7

MAXIMUM REINFORCED CAPACITIES OF REINFORCED CONCRETE BRIDGES



Journal of the
STRUCTURAL DIVISION
Proceedings of the American Society of Civil Engineers

DISTRIBUTION OF LOADS ON BRIDGE DECKS[a]

A. M. Lount[1]
(Proc. Paper 1303)

ABSTRACT

This paper examines recent developments in computation techniques as applied to the determination of load distribution between longitudinal members of bridges. The action of diaphragms, their beneficial effects and economy are emphasized.

The determination of load distribution between longitudinal members of bridges has been the subject of much study and research in recent years. The purpose of this paper is to examine some recent developments in computation techniques and the resulting reappraisal of the problem thereby made possible.

There are two basic approaches to the problem of computation of load distribution in a bridge deck. The first of these places major emphasis on the action of the deck slab, whereas the second places major emphasis on the action of the diaphragms.

In general, the first has been the traditional approach, particularly in North America. However, recent work on both sides of the Atlantic has been placing greater importance on the action of the diaphragms.

In either case the exact calculation of the interaction forces involved has been tedious and sometimes complex. It has been made particularly difficult by want of the necessary tools which would enable the job to be done quickly and efficiently. The result has been very great emphasis on both experimental work and on analogous methods of calculation.

The remarks in this paper will be directed towards a more detailed study of the second approach, and in particular towards the methods of exact

Note: Discussion open until December 1, 1957. Paper 1303 is part of the copyrighted Journal of the Structural Division of the American Society of Civil Engineers, Vol. 83, No. ST 4, July, 1957.

a. Presented at a meeting of the American Society of Civil Engineers, Jackson, Miss., February, 1957.

1. Partner, T. O. Lazarides, Lount and Partners, Cons. Engrs., Toronto, Canada.

analysis now possible with modern computational aids. The results will be compared with those obtained by methods using simplifying assumptions and approximations as well as with experimental work and the current AASHO empirical distribution formulae.

Fundamental Difference Between Bridge Structures with and Without Transverse Diaphragms

In order to appreciate the importance of a correct evaluation of these studies, we should first consider briefly the difference in behaviour between bridges with effective transverse diaphragms and those without such diaphragms.

The flexibility of deck slabs is usually such that their effectiveness cannot be expected to be more than purely local. As a result the load distribution between girders is largely a function of the stiffness of the slab in the immediate vicinity of the load. In bridges without diaphragms, the longitudinal girders are affected far more by a load in the immediate vicinity of the girders themselves than they would be by loads in adjacent lanes. In fact it makes relatively little difference to the maximum loading whether or not all the lanes on a particular bridge are loaded or not. However, when a bridge is built with transverse diaphragms of sufficient stiffness it acts as a single unit and the effect of a load on any one lane on the girders under that lane is substantially less than in the previous case. In essence, this means that the bridge is being treated as a single unit rather than as a number of individual units with some local distributing effect occurring through the slab. It may be seen, therefore, that quite apart from any effect on the actual maximum moment to be carried by an individual girder, the nature of the loading itself is inherently different. This is best seen in considering the factors of safety of the two structures.

The component parts of a factor of safety can be briefly stated as follows:

(1) A factor to be applied to the load, allowing for possible overloads, and
(2) A factor which allows for a possible weakness in the material itself.

In the case of bridges without transverse diaphragms, a large proportion of the load carried by any one girder may be attributed to loads in the lane over the girder in question. Reaching the elastic limit in any one girder will probably result in the collapse of the structure since little or no assistance can be given that member from adjacent members. These considerations govern the current empirical load distribution formulae and working stresses specified in most highway bridge codes.

Allowance is usually made for multiple lane live load reductions in structures such as trusses where more than two lanes contribute to the loads acting on one element of the structure. It is therefore entirely reasonable to consider similar live load reductions in the design of homogeneous grid bridges since it would require simultaneous overloads in several lanes to overload any one member.

Further, in truss bridges where multiple lane live load reductions are usually made, the reaching of the elastic limit in any one member will probably also result in the failure of the structure, whereas in homogeneous grid bridges the reaching of the elastic limit merely means the creation of a plastic hinge permitting other elements of the structure to assist the overstressed element.

We see therefore that in the case of a bridge with effective transverse diaphragms the dangers of both overstressing and of defects in the material are considerably reduced.

This brings us to the question of what is the most likely overload to which a bridge structure may be subjected. Most probably this will be a single heavy load such as a heavily loaded diesel truck, a crane or shovel, a transformer or other heavy equipment on a float or perhaps even an army tank. It is interesting to note that few bridges in North America could support army class 100 loading without severe overstressing or even failure. As will be shown later in this paper, grid structures can usually carry such single loads centrally without appreciable overstressing of any longitudinal members or even of transverse members if these have been well proportioned.

In addition to this basic increase in the real factor of safety, there is the added advantage that during the life of the structure, the number of times that the maximum design stresses will be reached or exceeded are considerably reduced. This will result in a considerable lessening of possible fatigue loading on structures, resulting in another argument for increased working stresses allowable in a homogeneous structure.

From the point of view of cost, in our experience rarely if ever does a grid cost more than a corresponding structure without diaphragms. Usually the cost is somewhat less and in cases where dead load is small, such as in steel bridges with steel open decking, the savings can be very substantial. Possible savings, however, are in reality secondary to the basic advantages of greater safety and higher permissible single emergency loading. Savings may be even higher if full advantage is taken of ultimate design concept. It is based on these latter points that a number of countries have now enacted design codes requiring certain minimum transverse strength in bridges.

Grid Analysis

The elastic analysis of a grid presents no fundamental problems other than those caused by the number of unknowns involved in the computation. Solution for single point loads or uniform loads may be undertaken by means of the method of relaxation. Such a solution was developed by Lazarides in 1952. However, the elastic analysis for moving loads usually necessitates the solution of basic simultaneous elastic equations for each of the individual members in terms of the interaction forces between the longitudinal members and the transverse diaphragms. Since these interaction forces are unknown at the time of making up the equations, a series of simultaneous equations is produced in terms of these unknowns. These equations may be extremely sensitive. This means that a successful solution requires that the work be carried out to a large number of decimal points. We have found in our experience that a total of twelve decimal places is essential for work of this nature.

In addition, the possibility of human error is very great and a system of checks and counterchecks must be established to prevent the introduction or accumulation of errors. This necessity has to date discouraged the use of elastic analysis as a solution and among a large number of investigators are Guyon and Massonnet who have considered the grid to be an anisotropic slab, that is to say, a slab with different bending properties in the longitudinal and transverse directions. The main difference between the Guyon and the

Massonnet approaches is that the Guyon approach considers the bending rigidity of the beams and diaphragms and neglects the torsional rigidity, whereas the Massonnet method combines both bending and torsional rigidity.

The validity of this approach has been investigated by Morice & Little in Great Britain and experimental work carried out by them indicates that the Guyon-Massonet approach provides a safe design basis for certain cases investigated.

There are, however, objections to the use of such analogous methods. First of all, precise determination of the stresses in the transverse diaphragms is difficult. The extension of the theory to unequal beam spacing and to variable moments of inertia on continuous spans is somewhat difficult and questionable. The methods would at present appear to be limited to right spans with relatively few identical girders and diaphragms and with constant spacing and moment of inertia. As against this, methods of analysis of the grid, based on elastic calculations, can be applied to any circumstance.

As mentioned above, the main difficulty involved in making an elastic analysis in the computation of a grid is the number of unknowns and the sensitivity of the resulting equations. The first of these difficulties can be reduced by means of load transformation. The principle of load transformation is shown in Figure 1. A load at an intersection point is transformed into four quarter loads at corresponding intersection points at each of the four quadrants of the grid. Thus by means of symmetry and antimetry the number of unknowns is immediately reduced by four. The use of electronic computers reduces the amount of time required for the solution of the equations to a matter of minutes. The method has therefore become practical and there is no necessity for any uncertain approximation. Operating to twelve decimal places with a computer also reduces the problem of sensitivity of equations.

One of the most important problems in this type of analysis is the elimination of errors which must be considered as an ever present possibility. As a result, continuous cross checks both mathematical and elastic must be evolved for computer techniques to ensure the precision of the calculations. Apart from the routine mathematical checks required during the setting up and the solution of the equations, a series of checks by Maxwell's theorem should be conducted after the unknowns have been established. It, of course, goes without saying that the introduction of the solved unknowns must satisfy the basic equations initially set up and that each diaphragm must be under equilibrium when the interaction forces are applied. These points are mentioned because the elimination of error is fundamental to any such analysis. Any computer programme established must have these structural checks built in to the programme. When this is done the designer may have confidence that he has in fact a true elastic representation of the forces existing in the structure depending only upon the validity of his initial assumptions.

We will now investigate the distribution of load in an actual structure. Figure 2 shows the diagrammatic layout for a grid system for a bridge for the Department of Highways of Ontario over the Big East River, north of Huntsville, Ontario. The girder spacing is 9'6" on the three interior panels and 8'6" on the outer panels. There are four diaphragms spaced at 24 ft. The overall span of the structure is 120 ft. The transverse distribution was investigated in terms of the effect of the point loads at each of the intersection points between the beams and the diaphragms. Figure 3 shows the transverse distribution computed by elastic analysis on both of the diaphragms.

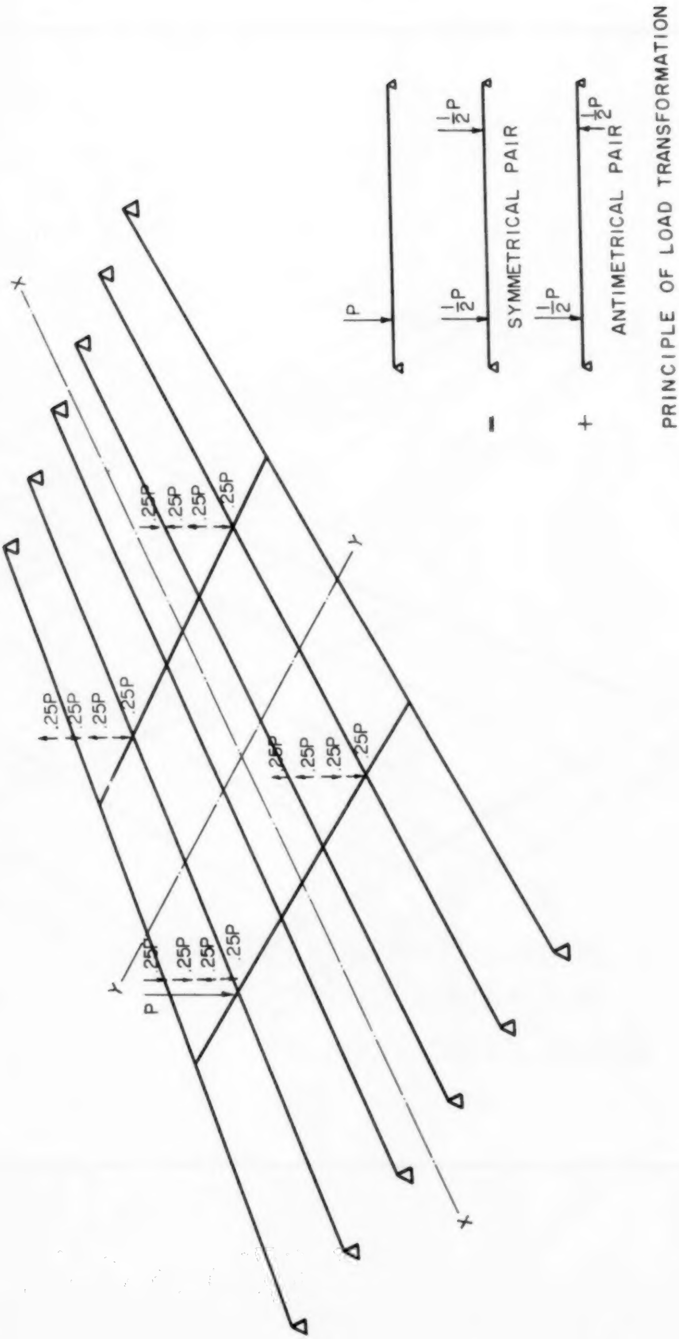
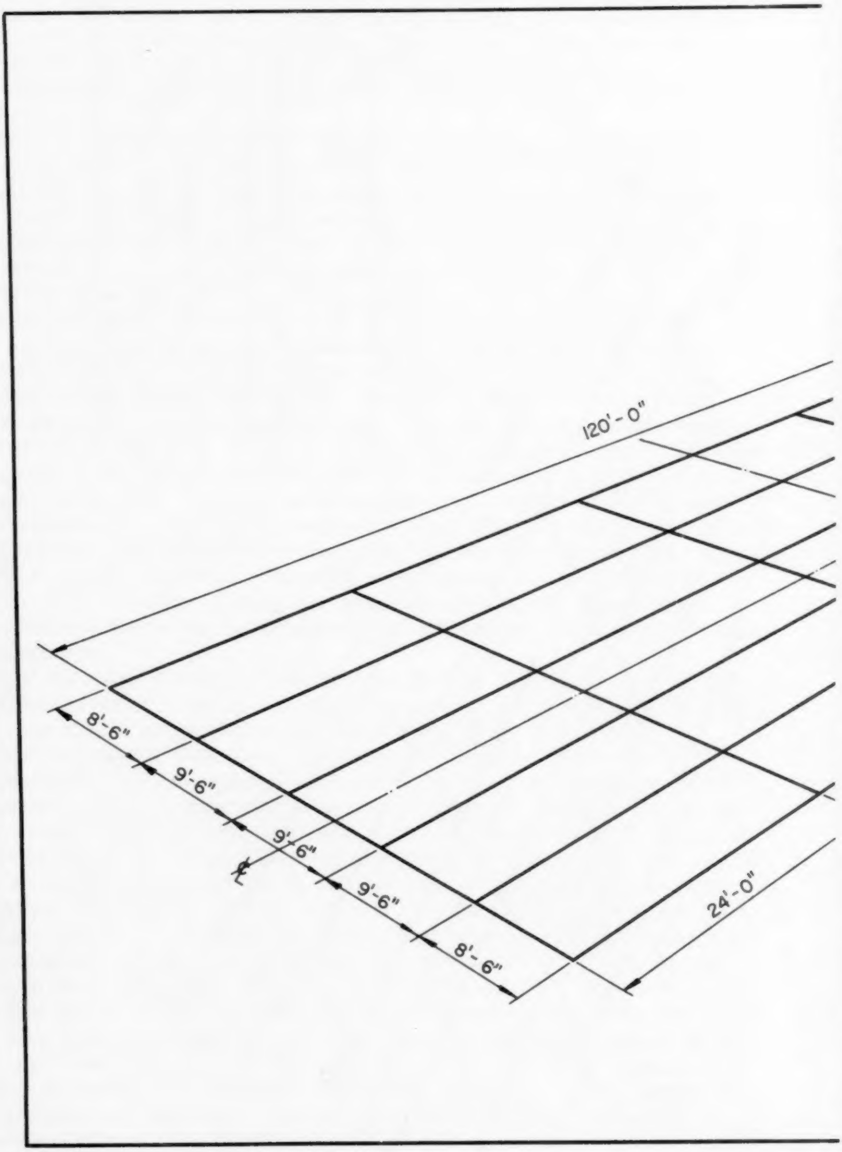


FIG. 1. LOAD TRANSFORMATION APPLIED TO A SIMPLE GRID


120'-0"
8'-6"
9'-6"
9'-6"
℄
9'-6"
8'-6"
24'-0"

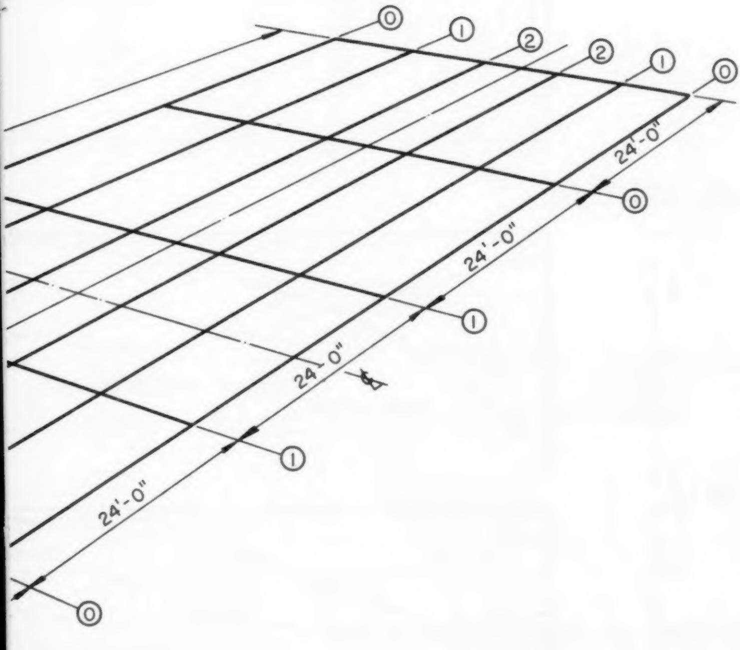


FIG. 2 DIAGRAMMATIC LAYOUT OF GRID SYSTEM FOR BIG EAST RIVER BRIDGE

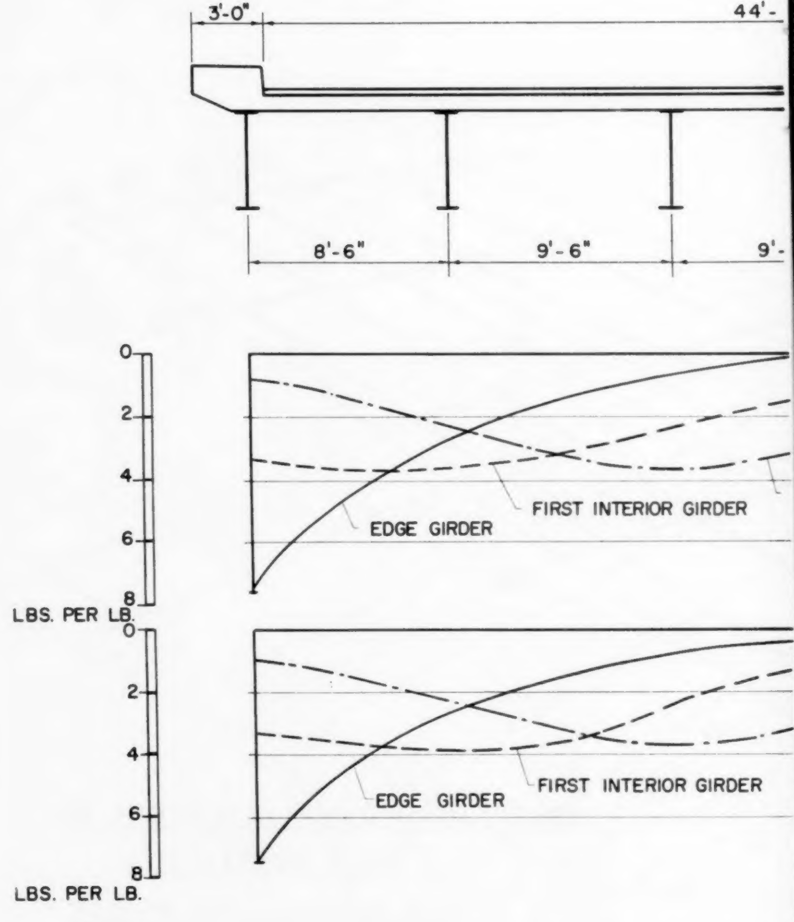
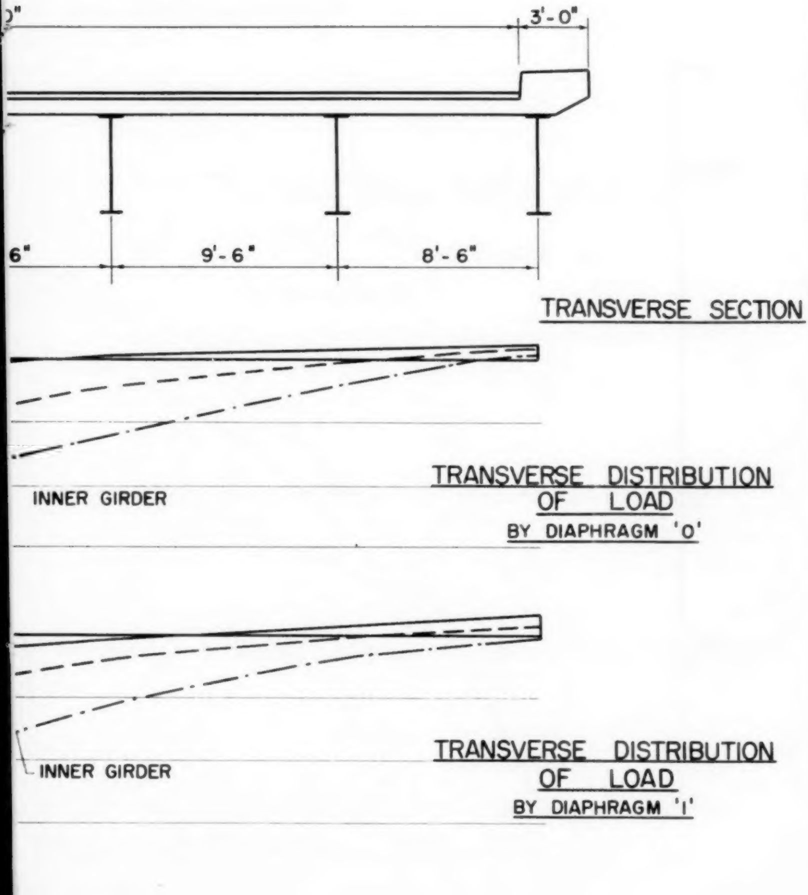


FIG. 3 TRANSVERSE DISTRI



RI BUTION OF POINT LOAD

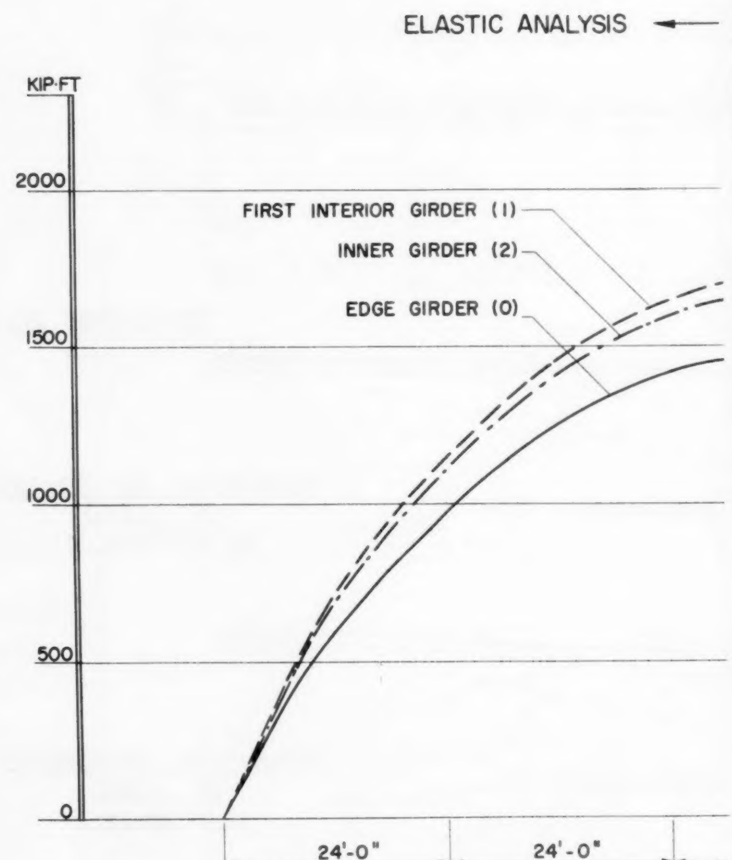
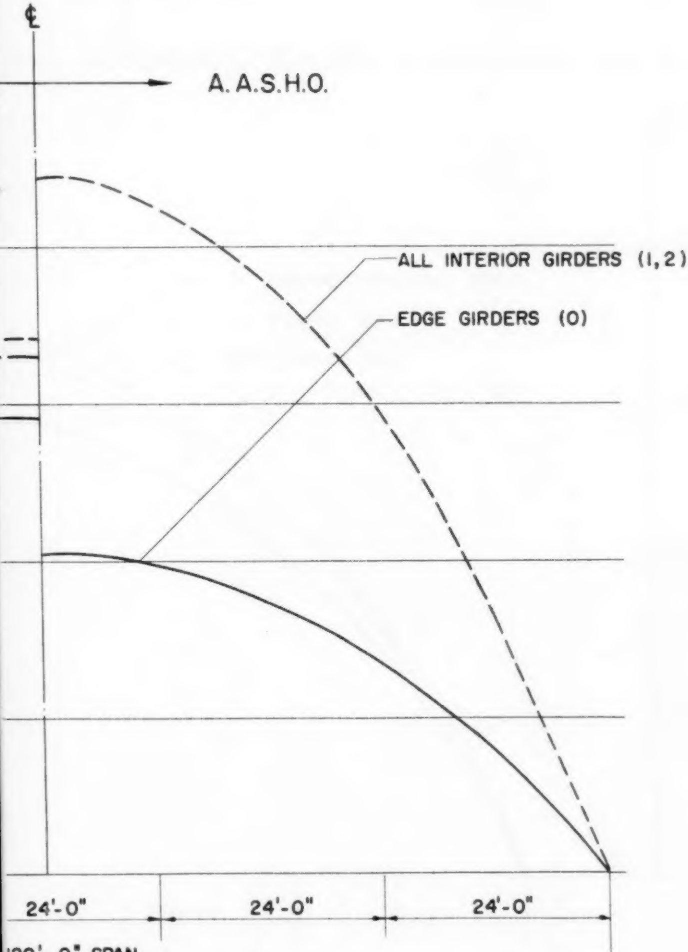


FIG. 4 MAXIMUM BENDING MO



MENT ENVELOPE DUE TO LIVE LOAD

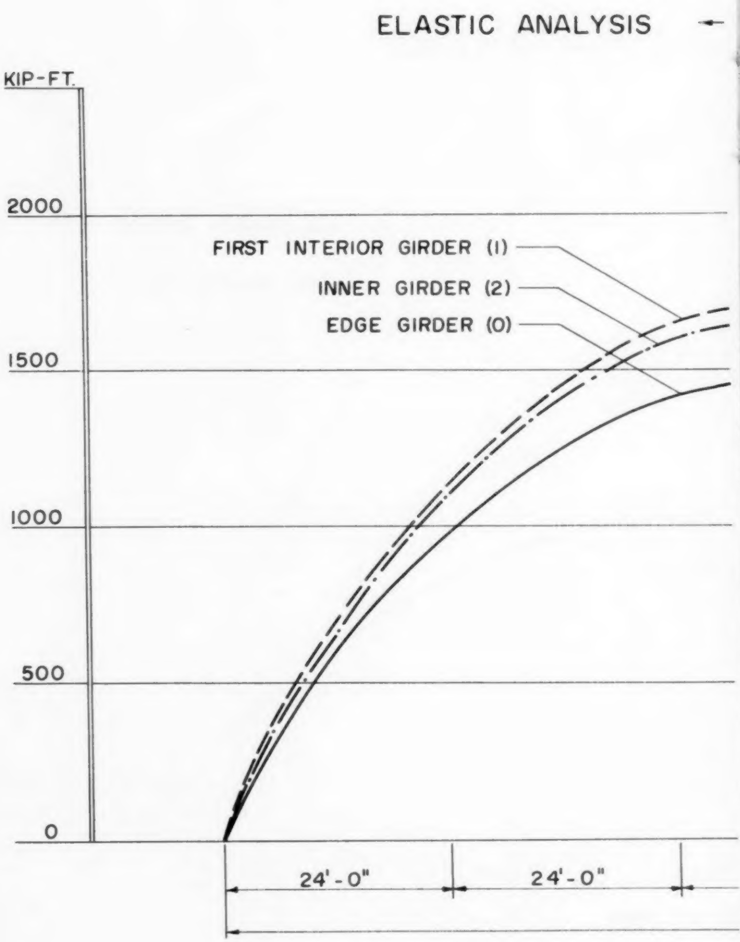
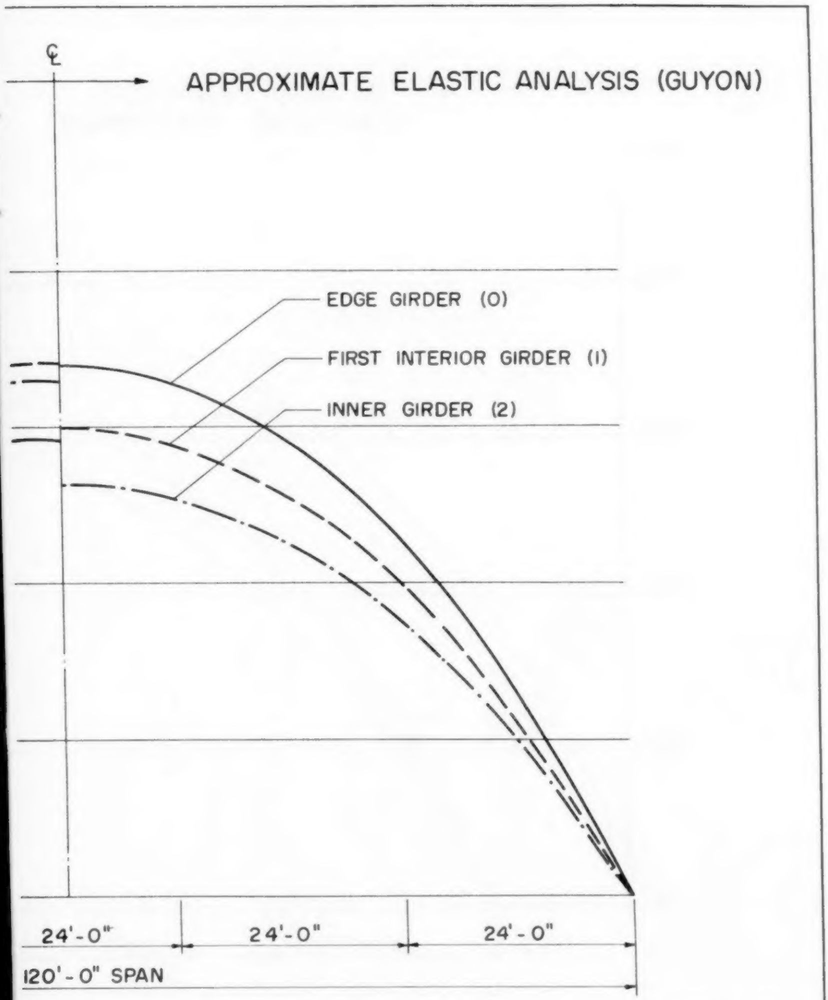


FIG. 5 MAXIMUM BENDING MOMENT EI



IVELOPE DUE TO LIVE LOAD

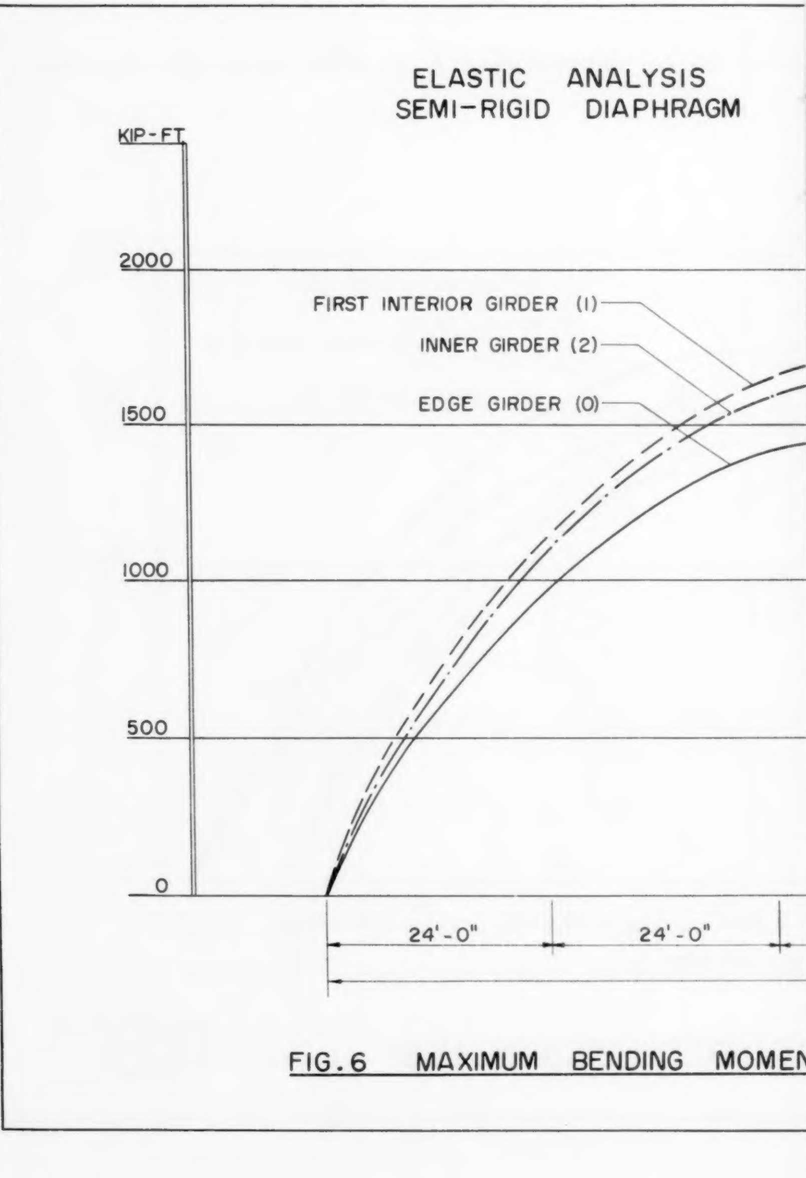
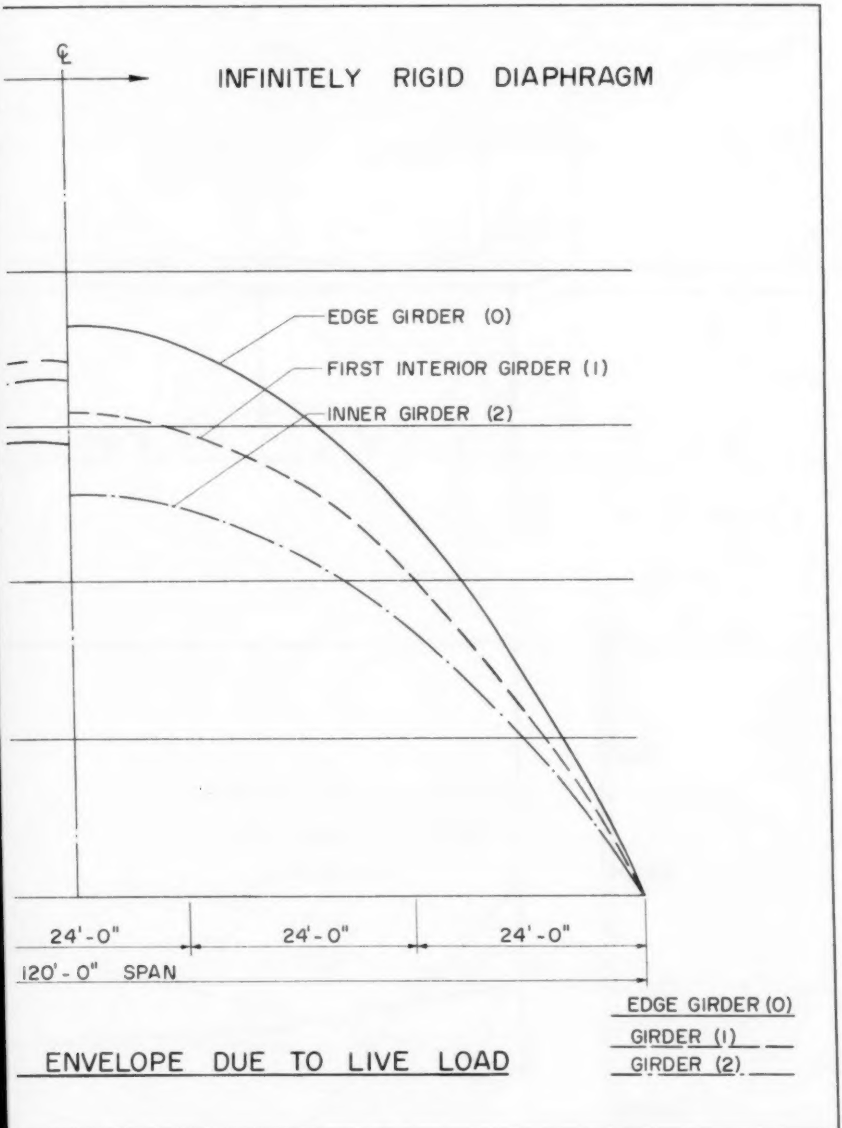
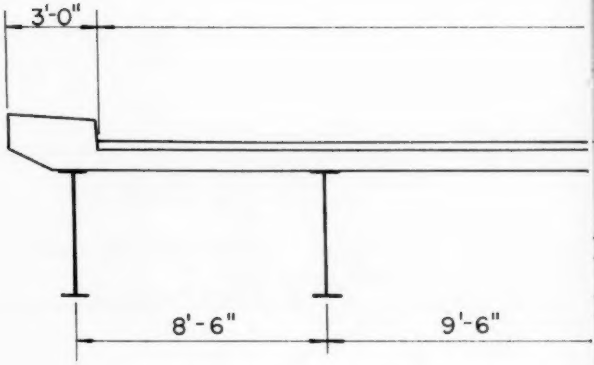


FIG. 6 MAXIMUM BENDING MOMEN


INFINITELY RIGID DIAPHRAGM
EDGE GIRDER (0)
FIRST INTERIOR GIRDER (1)
INNER GIRDER (2)
24'-0"
24'-0"
24'-0"
120'-0" SPAN
EDGE GIRDER (0)
GIRDER (1)
GIRDER (2)
ENVELOPE DUE TO LIVE LOAD



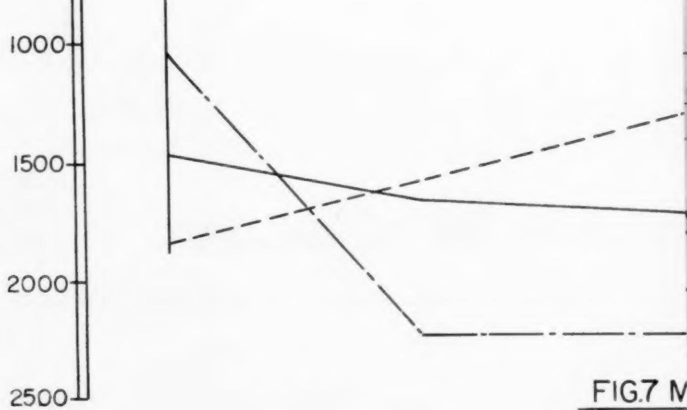
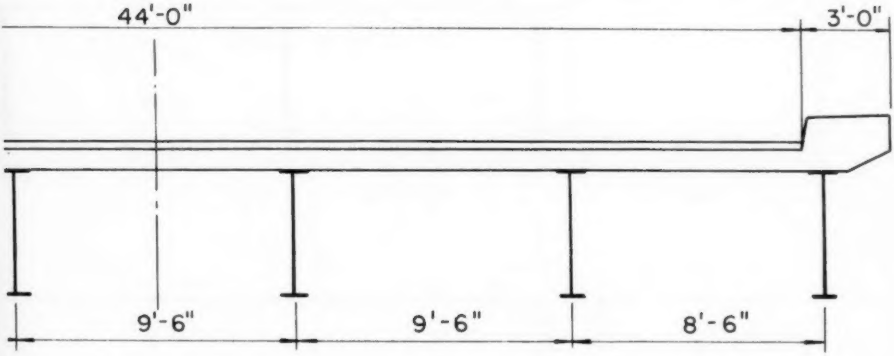
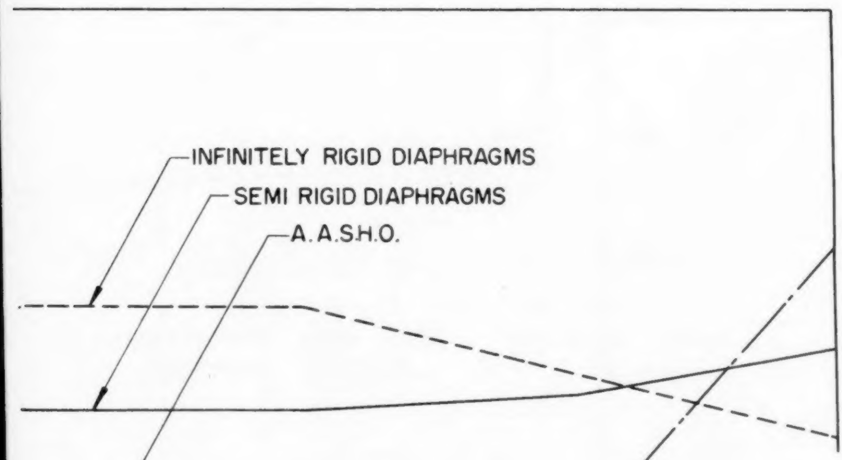


FIG.7 M
ALONG





AXIMUM LIVE LOAD MOMENT
TRANSVERSE ℄ OF BRIDGE

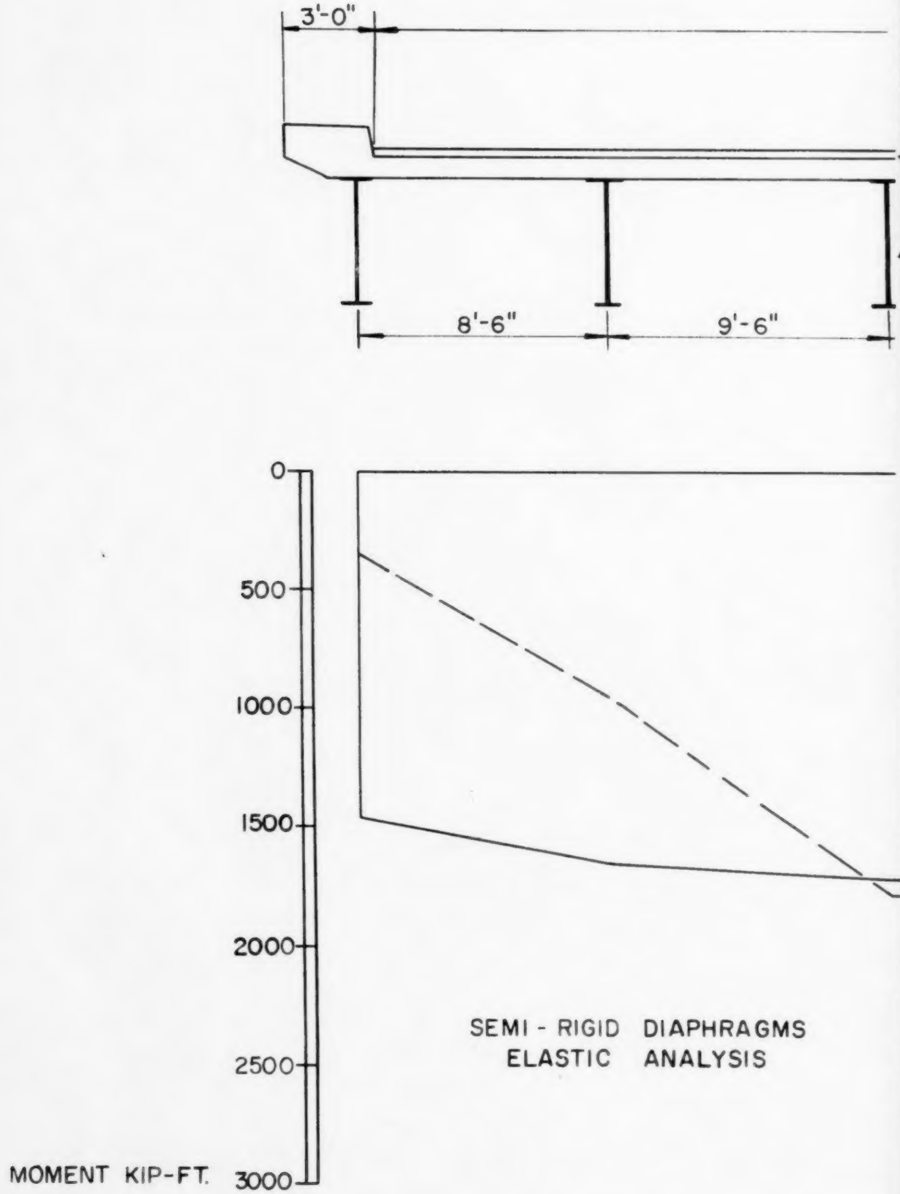
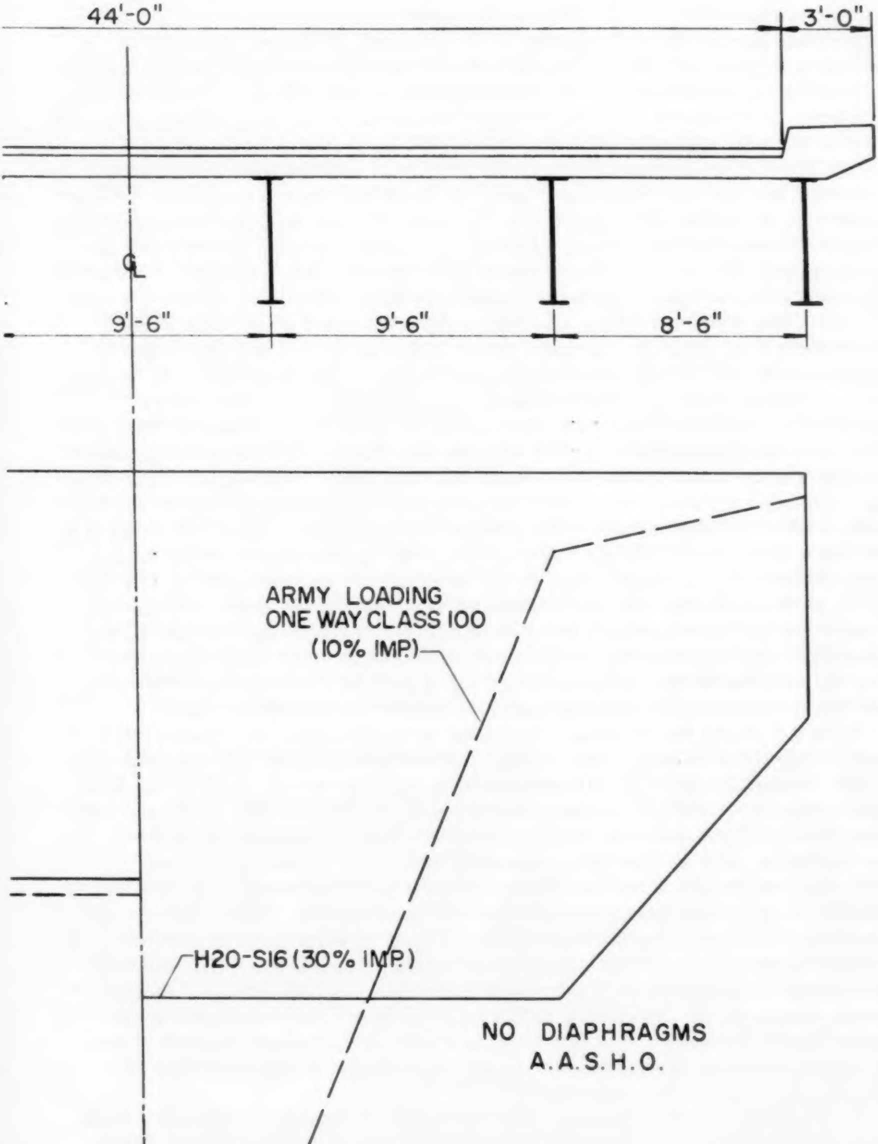


FIG.8 MA
ALONG T



(IMUM LIVE LOAD MOMENT
RANSVERSE ℄ OF BRIDGE

The computations were carried out by electronic computer. The distribution is expressed in lbs. per lb. The diaphragms in this structure consist of structural steel built up members with a moment of inertia of 50,800 in^4 which is compared with the longitudinal section 60 inches deep with a moment of inertia of 131,800 in^4 for the interior girders and 165,600 in^4 for the edge girders. The ratio of the total moment of inertia of the longitudinal members to that of the transverse members is about 4.2.

Figure 4 gives a comparison in the maximum live load bending moment envelope for a bridge with diaphragms using elastic analysis and the AASHO empirical formulae which applies to bridges both with and without diaphragms. It will be noted that the values obtained by AASHO for edge girders are approximately 40% less than those determined by the elastic analysis. It should be noted that the elastic analysis neglects torsion. The effect of torsion will be discussed later in this paper. These curves show that the empirical formulae established for bridges without diaphragms are not applicable to bridges with substantial transverse rigidity, and emphasize that it is necessary to examine not only the desirability of providing such transverse rigidity, as has been previously discussed, but also the methods by which the effects of this transverse rigidity may be calculated. The method suggested by Guyon is in general safe for simple span structures with equal beam spacing. Figure 5 gives a comparison between an elastic analysis and an approximate elastic analysis on the method suggested by Guyon. It will be noted that the main difference is an inversion in the order of the forces acting on the edge girders and the interior girders. In effect the variation in the spacing of the girders distorts the basic assumptions made in the Guyon analysis. Figure 6 gives a comparison between an elastic analysis for a semi rigid diaphragm and the load distribution for an infinitely rigid diaphragm. As may be seen the Guyon analysis results in a picture somewhere between that for the semi rigid diaphragm and the infinitely rigid diaphragm.

Figure 7 shows the maximum live load moments along the transverse centre line of the bridge. The AASHO distribution is shown for the purposes of this discussion and may be considered as approximately equal to the load distribution for a bridge without transverse diaphragms. The maximum live load moment anticipated in a structure with infinitely rigid diaphragms is also shown as well as that for a structure with semi rigid diaphragms.

It must be remembered that these curves are for the maximum live load moment to which any particular girder will be subjected. These are not the distribution curves for individual loads. It may be seen from this curve that a judicious selection of diaphragm strength is of importance. The selection of the diaphragm stiffness is a critical factor in economic design. As previously shown by Dr. Lazarides in his paper in the "Civil Engineering and Public Works Review" there is a critical diaphragm stiffness beyond which additional stiffness does not result in any appreciable decrease in the required total longitudinal strength.

To illustrate the effectiveness of diaphragms in handling emergency loads, Figure 8 shows the maximum live load moment due to a single Army Class 100 load crossing the bridge along the centre line. As may be seen the overstress in a bridge with diaphragms is negligible and the deflections to be expected are moderate and show that the deck slab should not be subjected to much distortion. In a bridge without diaphragms there would be considerable overstressing and the transverse deflection pattern in the deck would be

severe, probably leading to cracking. Extending this point further it may be readily seen that the ratio of failure loading due to single overloads to normal design loading is far higher in a bridge with diaphragms than in a structure without such diaphragms. The reason is that in the case of the single overload causing failure the important factor is the distributing capacity of the deck system.

This illustration brings out the very great difference that exists in reality in the behaviour with respect to single loads between a bridge with and without diaphragms. A very careful distinction should be made in the interpretation of distribution coefficients. As illustrated in Figure 8 there is a basic difference between the determination of moments resulting in individual girders due to loading at individual points on a structure and the determination of the maximum design load moment that may exist in any one girder due to the worst combination of design loads. This difference becomes more important as ultimate design concepts gain favour. The question becomes no longer what combination of design live loads are we liable to have normally on a structure, but rather what load or combination of loads is likely to cause failure. The pattern of loading is entirely different at design load and for failure loading. It is my belief that when this latter view is taken, the need for a new approach to transverse strength requirements in bridges will become even more apparent.

In the bridge deck analyzed above, the girders are of structural steel and have little torsional rigidity. The elastic analysis used assumes no torsional stiffness. In the case of another structure, that of a bridge built for the Ontario Hydro and U.S. Corps of Engineers over the Niagara River, the longitudinal beams were spaced much closer and had considerable torsional stiffness in themselves. To ascertain the effect of torsion on the load distribution a series of tests were conducted on the structure by the Hydro-Electric Power Commission of Ontario. Figure 9 shows the results of these tests.

As may be seen this bridge had very high transverse stiffness approximating infinitely stiff diaphragms. The Guyon approximate method and the elastic analysis both neglect the effects of torsion. The Massonnet approximate method, which is an elaboration of the Guyon method considers torsion. Comparing the estimated with the actual, it may be seen that the effect of torsion on this structure is considerable. It also may be seen that it has the effect of reducing the maximum moment both positive and negative for which the structure had to be designed. If an estimate of torsional rigidity is made by comparing the Guyon and Massonnet results and is then applied to the elastic analysis previously made, then it may be seen that the calculated results are remarkably close to the actual load distribution.

CONCLUSIONS

From the foregoing, we may conclude that the addition of diaphragms to bridge structures provides better transverse distribution, stiffens the bridge, reduces vibrational and deflection effects, increases the real safety of the structure, reduces hazards from fatigue loading and permits the crossing of very heavy individual loadings in emergencies. Our experience has shown that these benefits may be obtained without additional cost and often an appreciable saving may be achieved, especially in structures with high live load to dead load ratios.

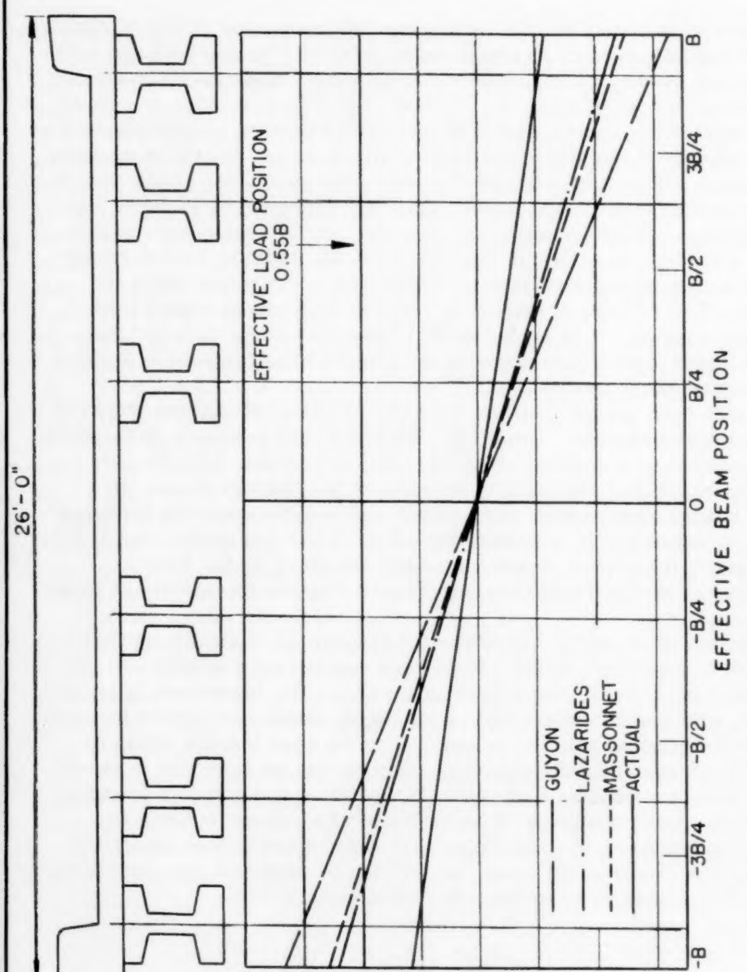


FIG. 9 DISTRIBUTION COEFFICIENTS FOR LOAD 2
NIAGARA REMEDIAL WORKS

REPRODUCED BY COURTESY OF
THE HYDRO-ELECTRIC POWER COMMISSION OF ONTARIO.

We may also conclude that AASHO distribution is not applicable to grid structures. The advent of the electronic computer has made it possible to calculate from first principles the load distribution pattern in grid structures. This tool can be extended for use in consideration of torsional effects subject only to the economics of such refinement. The method is absolutely general and applies equally to continuous structures, differential beam spacing, unequal girder stiffness and variable moments of inertia. The addition of diaphragms at right angles to girders on skew bridges is a field which is now open for study and it is anticipated that very substantial savings can thereby result. This is a field in which the electronic computer is not called upon to streamline an existing design method, but rather makes possible an entirely new approach to a design problem.

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FATIGUE RESISTANCE OF PRESTRESSED CONCRETE BEAMS IN BENDING[a]

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(Proc. Paper 1304)

SYNOPSIS

A method is presented to predict the fatigue strength of prestressed concrete based on the failure envelope of the materials involved. A discussion of factors influencing the fatigue strength, such as percentage of steel, level of prestress, and ratio of dead to live load is also included. Recommendations are given on the notion of safety and on the safety factors for prestressed concrete under fatigue loading.

INTRODUCTION

A survey of the present status of research in fatigue of prestressed concrete members leaves a rather perplexing impression. Various authors have conducted and reported numerous tests and drawn conclusions which are difficult to compare, because of the complexity of the problem.

A wide-spread opinion that prestressed concrete is safe against fatigue failure, a belief probably stemming from the early tests of Freyssinet,(1) is confronted by a reluctance on the part of many engineers to use prestressed concrete where fatigue loading exists. The existence of these two notions side by side is not surprising when one considers the large scatter of dynamic safety factors from as low as 1.15 reported by Leonhardt(2) to as high as 2.60 reported by Lin.(3) Other authors mention ratios of dynamic ultimate strength to static ultimate strength of 0.40 (Inomata)(4) to 1.00 (Abeles).(5)

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On the basis of the above facts, it seems imperative that the causes for such wide variation should be investigated theoretically. This gives rise to the question of whether or not it is possible to predict the dynamic ultimate strength of a member with about the same degree of accuracy which can be achieved in calculating the static ultimate strength.

The literature offers little help in answering these questions, and therefore it is the primary purpose of this paper to present a method for the calculation of the dynamic ultimate loads which permits an adequate consideration of the safety of a member in bending. The method which is presented here has been used successfully in checking the results of several investigations, including those of Lehigh University.

Prediction of the Behavior of Beams under Dynamic Loads

The Stress-Moment Diagram

The first information required is the stress-moment relationship for a critical cross-section of the beam in question. In order to make this relationship as general as possible, the ordinate and the abscissa are nondimensionalized by expressing the actual stresses and moments as a percent of the corresponding ultimate stresses and moments (Fig. 1).

The uppermost curve gives the theoretical relationship between steel stress and total applied moment, from release of prestress, through cracking, and up to the static ultimate moment. In the elastic range, i.e., before cracking, the steel stresses increase only very little and at cracking load there is theoretically a sudden break. In reality there is no such discontinuity, since the cracks develop gradually. For theoretical considerations, however, it is convenient to assign the total stress increase due to cracking to one moment—the cracking moment. The curves in this example are drawn for a beam having a percentage of steel required for balanced design. A design is called "balanced" if the stress in the steel at the ultimate moment corresponds to the stress causing 0.2% permanent set in the steel. The stress-moment relationship for the concrete top fiber starts out as a straight line, and gradually becomes concave downward as static ultimate load is approached. The bottom fiber stresses, on the other hand, increase more or less linearly to their terminal point at the initial cracking moment. Such a diagram is characteristic for a given cross-section and a given effective prestressing force, but it does not depend on the location of the cross-section, because the dead load is included in the external bending moment. Hence for a beam with straight tendons and constant moment of inertia the above diagram is valid for any cross-section. It will be noted that the requirements of the Bureau of Public Roads Criteria(6) were observed in laying out these curves, in that, the steel stress under working load is at 60% of ultimate for steel, and the maximum tensile and compressive concrete stresses are respectively 8% and 40% of static ultimate for concrete.

An accurate determination of the stress-moment diagram after cracking is, admittedly, not an easy task for two main reasons. First, the formulation of a strain compatability condition for a cracked cross-section leads to some difficulty because of the initial state of strain of the uncracked cross-section imposed by the prestressing. Second, the stress-strain relationship of steel and concrete cannot any longer be taken as a straight line. The difficulty

with the usual methods of equilibrium and compatibility condition can be overcome by following a method first proposed by Colonetti:[7]

Any external moment M (including the dead load moment) can always be split into a couple, say

$$M = hT = hC$$

$$T = C$$

For example, one could arbitrarily choose T to be the initial prestressing force minus the losses due to creep and shrinkage.

$$T = T_{initial} - \Delta T_{creep} - \Delta T_{shrinkage}$$

(Note that the elastic losses are not subtracted.)

If this T is imagined to act alone on the centroid of the prestressing tendons, it would just eliminate all the concrete stresses. The actual concrete stresses can thus be calculated under the action of the eccentric compressive force C alone by means of the usual methods of equilibrium and compatibility. (The force C can very well lie outside the cross-section; if M is large enough, the lever arm of the external forces

$$h = \frac{M}{T} = \frac{M}{C}$$

can become larger than the effective depth of the beam.) The steel stresses, in turn, can be found by superimposing the action of T and C. However, since this involves several steps by trial and error, and furthermore, requires a definite knowledge of the stress-strain relationships of concrete and steel, the method remains somewhat lengthy.

The Fatigue Failure Envelope for Prestressing Steel and Concrete

Thus far only the static properties of the critical cross-section have been dealt with. In order to find a connection to the dynamical behavior, two fundamental diagrams which show the dynamic properties of steel and plain concrete are introduced. It is customary in strength of materials to present this information as a failure envelope for a given fixed number of cycles. It is beyond the scope of this paper to extensively deal with the question of how large to choose this number of cycles—a question which involves statistical considerations of the expected load frequency as well as consideration of the dynamic characteristics of the material itself. We only mention that most of these materials show a more-or-less distinct fatigue limit, i.e., above a certain number of cycles, which can vary from one to several millions, the ultimate capacity levels asymptotically to a fixed value. For our combination of concrete and high tensile steel it seems to be justified, although not beyond critical consideration, to restrict the discussion to a fatigue limit assumed at one million cycles.

The fatigue failure envelope for prestressing steel, shown in Fig. 2, is a modification of the well-known Goodman-Johnson diagram.[8] This envelope indicates how much we can increase the stress from a given lower level to obtain a failure at about one million load-cycles. (Again the stresses are expressed as a percentage of static ultimate strength.) The higher this lower stress limit is chosen, the smaller becomes the possible stress amplitude. Thus, the steel may resist a repetitive range of stress amounting

to 27% of static tensile strength if the lower stress limit is zero, but only an 18% range of stress if the lower stress limit is increased to 40%. The trend of possible stress ranges may be easily observed by noting the vertical displacement and width of the shaded area. At the point where the abscissa is 40%, for example, it will be seen from the vertical axis that the shaded area extends from 40% to 58%. Furthermore, it may be seen that if the stress were increased so that the minimum stress limit was 80%, the shaded area extends to only 85%, thus giving a possible stress range of only 5%.

The failure envelope in Fig. 2 readily provides a definition of the dynamic yield point for materials with no distinct flow point. It can be shown by tests that there exists a stress level smaller than the static ultimate strength above which even an extremely small stress alternation will cause dynamic fracture. This stress level, at about 95% of the ultimate, shall henceforth be defined as the dynamic yield stress.

A failure envelope analogous to the one for steel can be found for concrete (Fig. 3). This curve is more complicated because it must be drawn for a range covering both tensile and compressive stresses.[9] For example, if a point is considered where a tensile stress range occurs, the portion of the envelope in the upper right quadrant adjacent point A will apply. If a point, such as the bottom fiber of a prestressed beam is being considered, it might readily be visualized that the fiber may be stressed between, say, 20% compression and 10% tension, for one million cycles before cracking took place. The point on the envelope representing a possible range of stress between 20% compression and 10% tension is denoted by the point B. The third quadrant contains the portion of the failure envelope representing compressive stress ranges. For example, a compressive stress could be applied between the limits of 40 and 80% (as denoted by point C) for one million cycles before failure. The possible stress amplitudes are again accentuated by the shaded area in the same way as for the steel failure envelope. Apparently there exists also a dynamic yield stress in concrete approximately at 90% of the static ultimate strength.

The Combined Diagram

Returning now to the problem of predicting the dynamic ultimate strength in bending of a critical cross-section, we simply combine the three previous diagrams, with the stress-moment relation as the central part, and the steel and concrete failure envelopes plotted for reasons of clearness, to the left and to the right. Figure 4 shows the resulting combined diagram, which makes possible the determination of the dynamic cracking load, and the dynamic ultimate moment as limited by steel or concrete.

Determination of the Dynamic Cracking Moment

Several simple steps are necessary in the determination of the dynamic cracking load. In Fig. 4, point A on the stress-moment diagram represents the bottom fiber stress at dead load moment and is the starting point from which a horizontal projection is made over to the failure envelope of concrete. Having a lower stress limit at point B on the failure envelope, it is only necessary to project vertically to point C and establish the upper limit of the stress range which causes cracks after 1 million cycles. A horizontal projection from point C back to an intersection with the bottom fiber curve

on the central diagram at point D results in the establishment of the dynamic cracking moment at 40% of static ultimate moment. In other words, in this particular example, 1 million load cycles from the dead load moment to 40% of the static ultimate moment will induce cracks in the bottom fiber of concrete.

Determination of the Dynamic Ultimate Moment

The ultimate moment under dynamic loading is found in the same manner as the cracking load, i.e., by projecting in turn from the moment-stress diagram to the failure envelope and back. In order to obtain the minimum failure moment, however, we have to observe two failure conditions—one for steel and one for concrete. The dynamic ultimate moment based on the steel is 56% of static ultimate as found by projecting from E to F to G and finally to H. The dynamic ultimate based on the top fiber of concrete is determined by projecting from I to J to K to L and is 83% of the static ultimate moment. Thus, the dynamic ultimate moment of the beam for which Fig. 4 was drawn is limited by the steel, and is 56% of static ultimate moment.

DISCUSSION

The method described above provides an adequate means for determining the dynamic capacity of a prestressed concrete member in bending. It is therefore possible to discuss the effect of the level of prestress, the effect of over- and under-reinforcing, and the cracking characteristics.

Level of Prestress

In Figure 5 the stress moment relationships are plotted for different levels of prestressing. Imagine three beams which are identical except for the fact that the steel in the first beam has no initial prestress, the steel in the second beam has a prestress which is 30% of its static ultimate strength, and the third beam has steel with 60% initial prestress. The moment-stress diagrams for the beam with zero initial prestress is represented by the solid lines, for 30% initial prestress by the dashed lines, and for 60% initial prestress by the broken lines. For zero prestress the maximum possible stress range in the steel is seen to be from zero to 30%, thus giving a dynamic ultimate moment of only about 27% of static ultimate moment by projecting from A to B to C to D. Correspondingly, it can be found by projecting from E to F to G to H that the beam with 30% initial prestress has a dynamic ultimate moment based on the steel of 42% of static ultimate moment. The highest dynamic ultimate moment results from the beam with the highest initial prestress of 60% of its static tensile strength (projecting from I to J to K to L) and this value is 52% of static ultimate moment.

The values of dynamic ultimate bending moment obtained above for various levels of prestress indicate that under otherwise identical circumstances these values increase with the increasing level of prestress. This statement has been confirmed by many tests, the first having been performed by Freyssinet(1) in 1934. It should be kept in mind, however, that the concrete stresses are assumed to be not critical.

Over- and Under-reinforcing

Fig. 6 shows the stress-moment relation for three beams which are identical except for the amount and arrangement of the prestressing steel. The first beam, for example, has a small percentage of steel and is under-reinforced. The second beam is assumed to be a balanced static design on the basis of the definition previously stated in this paper. The third beam has more steel than is needed for a balanced design and is therefore over-reinforced.

The dynamic ultimate moments for all three beams based on the steel may be found quite easily. It will be observed that the possible stress range, denoted by the line B - C on the failure envelope for the steel is practically common to all three beams because of the close correlation of the stress-moment curves over the elastic range. The dynamic ultimate moments based on the steel are observed to be the abscissas of the points D, E, and F for the under-reinforced, the balanced, and the over-reinforced designs, respectively.

It is evident from an examination of the relationship between the stress-moment curve G - J and the failure envelope for the concrete, that the top fiber of the concrete does not become critical unless there is a percentage of steel which is larger than the balanced design percentage for static loads. In other words, there exists a balanced design percentage of steel for dynamic loading which results in an optimum dynamic ultimate moment. In Fig. 6, this optimum dynamic ultimate moment is approximately 85% of static ultimate moment.

It has been observed that the variation of the percentage of steel, within the range of practicable values, can result in a wide range of variation of the ratio of dynamic ultimate moment to static ultimate moment. It follows that probably the higher the percentage of steel in any given cross-section, the higher will be the resistance to dynamic loading, but the arrangement of steel must also be considered. In fact, it appears to be advantageous from the point of view of fatigue loading to use a percentage of steel higher than the percentage for static balanced design, that is, over-reinforce the section. It is known that in most cases the steel, not the concrete, is the critical material in fatigue, even though statically the section might fail by crushing of the concrete in the top flange. The amount of increase in steel stress at cracking of the bottom fiber is inversely proportional to the percentage of steel in the section. Furthermore, the slope of the curve becomes flatter with increased steel percentages, and hence leads to an advantage in over-reinforcing for fatigue conditions.

There has always been an aversion against using over-reinforced concrete members of any kind. The principal reason for this seems to stem from the fact that over-reinforced members usually fail without warning, and before any large deflections are observed. However, in the case of failures caused by dynamic loading, failure can occur suddenly, whether the beam is over- or under-reinforced, and before deflections become large, moreover, the chance that the strands or bars do not fail simultaneously increases with the number of tendons, thus again favoring over-reinforcing. Also, the failure can be sudden, no matter whether it occurs in concrete or steel.

Cracks

One fact, which is quite apparent from a study of Fig. 4 is that the dynamic cracking load will always be less than the static cracking load. However, this should not be regarded with too much concern by the engineer.

A recent investigation conducted at the Fritz Engineering Laboratory, consisting of static and dynamic testing of two full scale pretensioned bridge members, tends to substantiate the statements in the preceding paragraph.(10) It was found that the stiffness and general behavior of the dynamically tested beam compared quite favorably with the second beam tested by a gradually applied load. Observations revealed that whereas cracks in the dynamically loaded beam occurred below static cracking moment, their number and width at higher moments compared very favorably with values for the statically tested beam.

Further study of Fig. 4 reveals that the ratio of dead load moment to live load moment has very little effect on the dynamic cracking moment (bottom fiber). For example, a ratio of dead to live load of zero will result in a theoretical cracking moment of about 40% of static ultimate moment, whereas a ratio of five increases the cracking moment to only about 43% of static ultimate moment.

Although repetitive loading necessarily reduces the cracking load, the advantage of prestressing remains the same in that it delays the occurrence of cracks very substantially.

Notion of Safety

Now that a means of predicting the dynamic strength of prestressed members is established, it seems justified to discuss the notion of safety in order to arrive at a suitable means of providing sufficient safety against the probability of failure.

Since the actual stresses due to prestressing do not increase linearly with the applied moment, it was felt that merely specifying allowable stresses is insufficient. Thus an additional restriction is imposed on the external moment by all the current codes.

Formulas for limiting the maximum allowable bending moments are as follows:

$$M_{ult} \geq K_1 (D + L + I)$$

$$M_{ult} \geq D + K_2 (L + I)$$

When: D = dead load moment
L = live load moment
I = impact moment
K_1, K_2 = load factors[4]

The nature of these formulas often leads to the idea that they provide a margin of safety against overloading. Such an idea is both erroneous and dangerous. The necessity for safety factors has quite a different reason, namely, it stems from the fact that the prediction of the ultimate moment always

4. As discussed later, K_1 (but not K_2) can be considered as a safety factor.

includes a number of uncertainties which can be summarized, according to Rüsch,[10] as follows:

1) Uncertainties in the calculation of the internal forces and stress.
2) Uncertainties in the properties of the materials (concrete and steel).
3) Local weaknesses of the materials (secregations in concrete and inclusions in steel).

The real purpose of the factor of safety should be to guarantee a sufficiently small probability of failure under working load assuming that an unlucky coincidence of malpredictions due to the above uncertainties might lead to a failure under working load. Two points are significant, first, that a possible failure is not excluded but only made highly improbable, and second, that this probability refers only to a given load—usually taken to be the maximum working load. The probability itself can be obtained by a theoretical evaluation of statistical surveys of simple tests.

Since these uncertainties are fully independent of the nature of the load, it follows that dead and live loads have to be dealt with in the same manner. A formula of the kind

$$M_{ult} \geqq D + K_2 (L + I)$$

is therefore notionally unsatisfactory although it is clear in its intentions. It might be mentioned, incidentally, that the notion of allowable stresses is clear in this respect, because nobody ever suggested that one should distinguish between allowable stresses due to live and due to dead loads.

If there are believed to be some uncertainties in the live and impact load assumptions, then it is necessary to increase these load assumptions, but the safety factor should not be used to take care of overloading since this would simply mean that the required safety margin would no longer be guaranteed. It seems especially desirable for short bridges to assume occasional overloading, thus eliminating the need for any restrictions of the above type.

From all the foregoing it follows that the factor of safety should clearly be separated from the overloading capacity. The former is connected with the uncertainties of our calculation and assumptions while the latter is a definite value and can only be determined by the actual destruction of the structure. Only in two cases does the safety factor coincide with the overloading capacity; if the deviations from our assumptions are all zero (which is never the case), or if the underestimations are accidentally equalized by overestimations.

If a given bridge should, during its life, sustain permanently larger loads than those for which it was designed, then a careful consideration of the actual failure probability can always decide whether or not such overloading can be tolerated.

All these remarks apply to dynamic as well as static loading. Since the uncertainties under the latter are not smaller for one than for the other, the design moment should be subjected to a restriction of the form

$$M_{dyn\text{-}ult} \geq K (D + L + I).$$

As to the actual value of the safety factor K, this might very well be a matter for further discussion, but it is believed that it should be somewhere around two.

Many engineers will argue that in an actual bridge 1 million cycles of the

maximum load will be very unlikely; this being true under certain conditions, we can only see one way of taking into account such possibilities—that is, by a statistical proof that the expected maximum load frequency minimizes the danger of fatigue failure. Such proof would certainly be possible for many highway bridges, but probably not for railway bridges.

Another widespread argument against considerations of dynamic failures is based on the fact that the steel-stress variation is definitely small and that the corresponding failure envelope leaves a margin of say three times as large a stress variation. From all the foregoing, it follows that this factor of three cannot be considered as a factor of safety. Certainly no one would pretend that the safety factor of any structure is two if only the allowable stresses are taken to half of the ultimate stresses. The above argument, however, would amount to an analogous, misleading contention.

The foregoing discussion of the desirable requirements on prestressed structures under repetitive loading may seem rather pessimistic. To leave such an impression, however, is quite opposite to the aim of this paper. An attempt was made to show that prestressed concrete is very well suited to structures subjected to dynamic loads. The method described in this paper is intended to give more certainty as to what is to be expected under such conditions. Since it is believed that the estimated dynamic capacity can be predicted, it seems only fair to connect this also with sufficient safety requirements.

Although a dynamic factor of, say two, may seem very severe, attention is drawn to the fact that fatigue loading, especially if combined with severe exposure conditions, involves more uncertainties than those mentioned previously for static conditions.

Now investigate the possibility that the present design requirements may provide sufficient safety against fatigue failure. If one considers the two formulas given by the Bureau of Public Roads Criteria(9) for limiting the design moment based on the ultimate strength of the member, it will be seen readily by considering typical ratios of live load to dead load that the ratio of dynamic ultimate strength to static ultimate strength must be greater than about 0.8 if a factor of safety of two against dynamic ultimate moment is achieved. A beam must be carefully designed based on the method of analysis given in this paper to have so high a ratio of dynamic ultimate to static ultimate strength. Members designed on the basis of only static considerations normally have ratios of dynamic ultimate to static ultimate of 0.4 to 0.8.

The required dynamic factor of safety cannot be achieved economically by making the required static ultimate moment more conservative because designing for a higher static ultimate moment would result in a larger cross-section of member when all that may be required is a similar member with a larger percentage of steel or a better arrangement of steel.

The majority of prestressed bridges will never be in danger of fatigue failure. But if a structure is subjected to severe repetitive loading, the problem of failure deserves adequate consideration. For any specific problem, the dynamic ultimate moment should be calculated by the procedures outlined in this paper.

CONCLUDING REMARKS

Based on the knowledge of the fatigue properties of concrete and prestressing steel and the determination of the stress-moment relationships of

prestressed beams, it has been shown that it is possible to predict the fatigue properties of members in bending revealing the following facts:

1) The dynamic ultimate moment is always less than the static ultimate moment and can vary over a very large range.

2) The dynamic cracking moment is always smaller than the static cracking moment. Above static cracking moment, however, the number and width of cracks due to dynamic loading correspond to those due to static loading.

3) The ratio of the dynamic ultimate moment to the static ultimate moment is increased by:

   a) Increasing the level of prestress.
   b) Increasing the percentage of steel in a beam.

The design of prestressed members under severe fatigue loading should be based on the determination of the ultimate dynamic moment. Considering the uncertainties always present in the analysis and in the properties of materials, adequate safety should be provided by a restriction of the following kind:

$$M_{\text{dynamic ultimate}} \geq K\,(D + L + I)$$

The desirable value of the safety factor K (about 2) deserves a careful consideration and should be specified according to the particular type of structure and site conditions. In the case of fatigue loading, it is quite important that possible overloading be considered separately by increasing the design loads and using the above equation with the accepted value of K.

It should be emphasized that this paper is restricted to pure bending of bonded prestressed beams. If the future will, as is hoped, provide more information about the stress distribution under the combined action of moment and shear or if more becomes known about bond stresses, then a further discussion of such failures under repetitive loading will certainly be possible and follow the same pattern as suggested in this paper.

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REFERENCES

1. Rowe, R. E.: "An Appreciation of the Work Carried Out on Fatigue in Prestressed Concrete Structures," Technical Report. Cement and Concrete Association, London.

2. Leonhardt, F.: Spannbeton für die Praxis. Ernst and Son, Berlin, 1955.

3. Lin, T. Y.: "Strength of Continuous Prestressed Concrete Beams under Static and Repeated Loads." Proceedings of the American Concrete Institute, Vol. 51, p. 1037, 1955.

4. Inomata, S.: "Fatigue Tests in Bending of Prestressed Concrete Beams." Proceedings of the American Concrete Institute, Vol. 49, p. 766, 1952-1953.

5. Abeles, P. W.: "Static and Fatigue Tests on Partially Prestressed Concrete Constructions." Proceedings of the American Concrete Institute, Vol. 51, pp. 361-376, 1955.

6. "Criteria for Prestressed Concrete Bridges." Department of Commerce, Bureau of Public Roads, 1954.

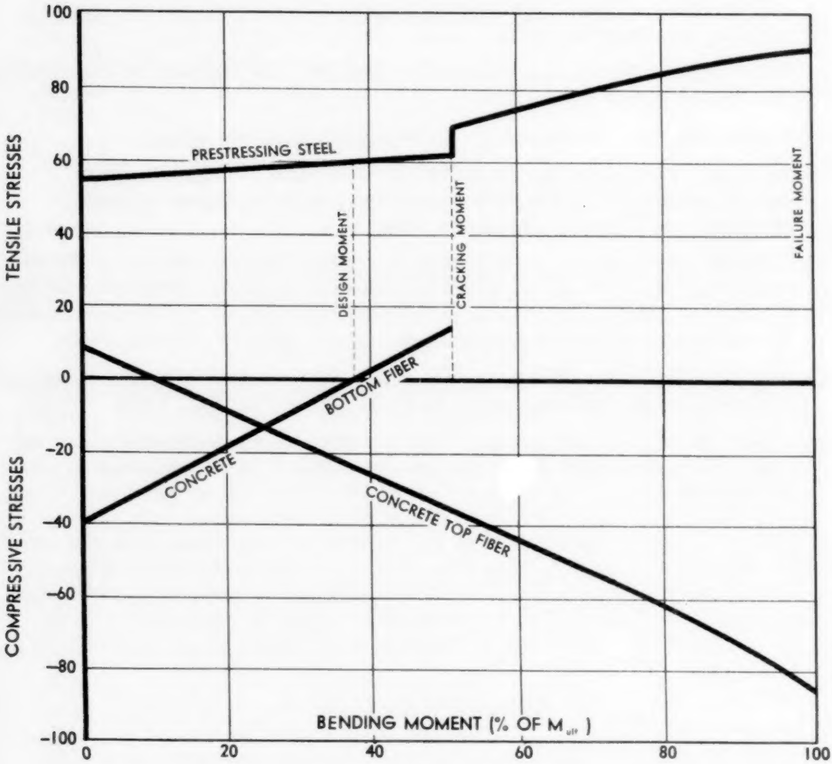
7. Colonnetti, G.: "Scienza delle Costruioni." Einaudi, 1941.

8. Roš, M.: Vorgespannter Beton (Bericht Nr. 155). Eidgenössische Matialprüfungs- und Versuchsanstalt für Industrie, Bauwesen und Gewerbe. S. 79, Abb. 78, Zürich, 1946.

9. Roš, M.: Die materialtechnischen Grundlagen und Probleme des Eisenbetons im Hinblick auf die zukünftige Gestaltung der Stalhbeton-Bauweise (Bericht Nr. 162). Eidgenössische Materialprüfungs- und Versuchsanstalt für Industrie, Bauwesen und Gewerbe. S. 36, Abb. 55, Zürich, 1950.

10. Ekberg, Jr., C. E.: "Dynamic Tests of a 55 Foot Amdek Bridge Member." Concrete Products Company of America, Pottstown, Pa., 1956.

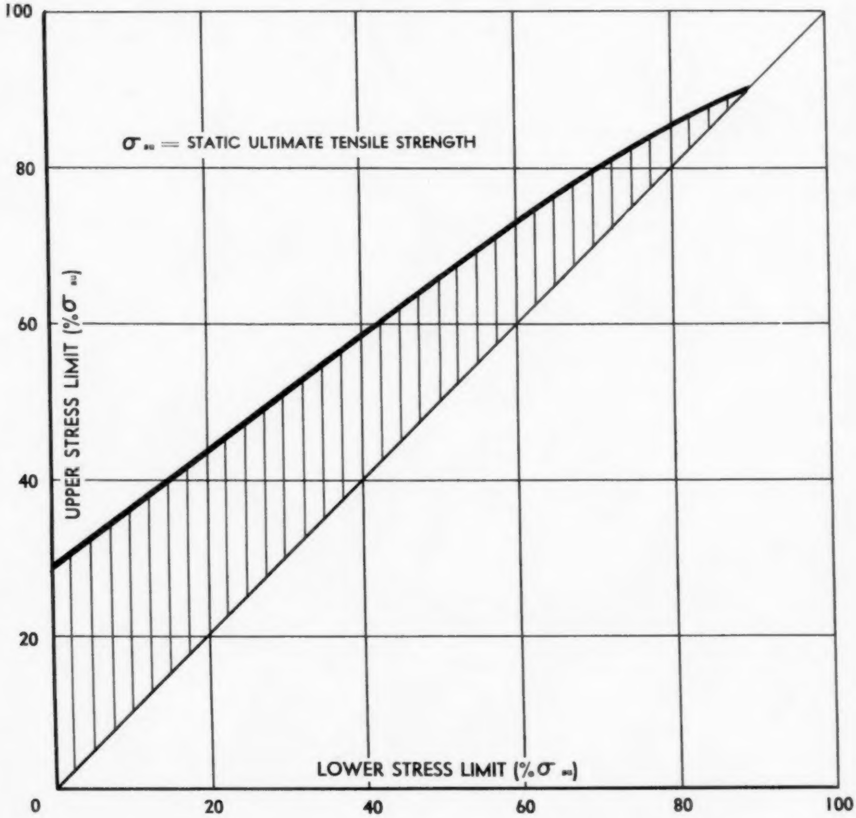
11. Rüsch, H.: Schweizer Archiv, "Der Einfluss des Sicherheitbegriffes auf die technishen Regeln für vorgespannten Beton." pp. 85-93, Heft 3, March, 1954.



NOTE: ALL STRESSES IN PERCENT OF CORRESPONDING STATIC ULTIMATE STRESS

STRESS — MOMENT DIAGRAM

FIGURE 1

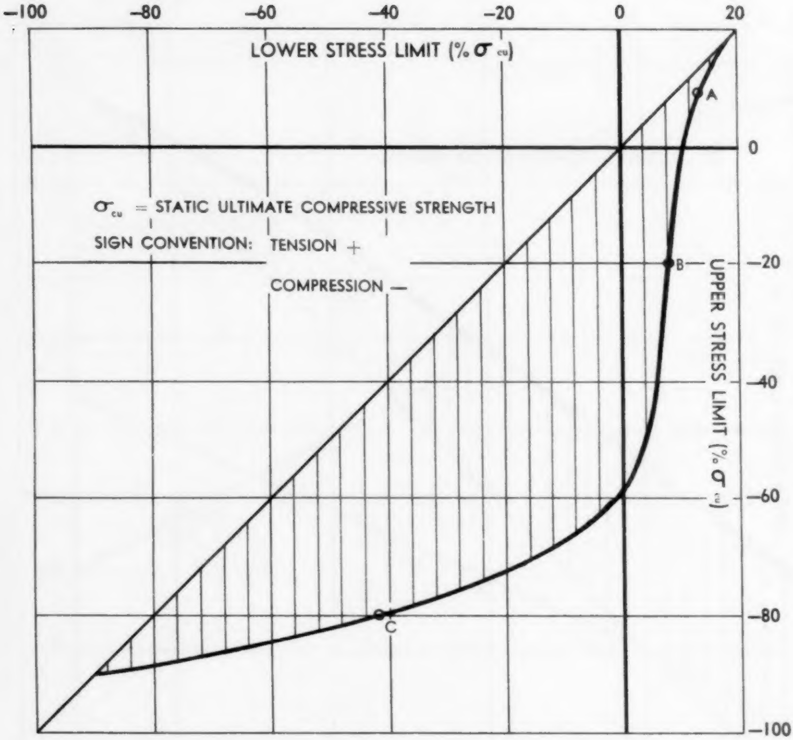


FATIGUE FAILURE ENVELOPE FOR

PRESTRESSING STEEL

(10^6 LOAD CYCLES)

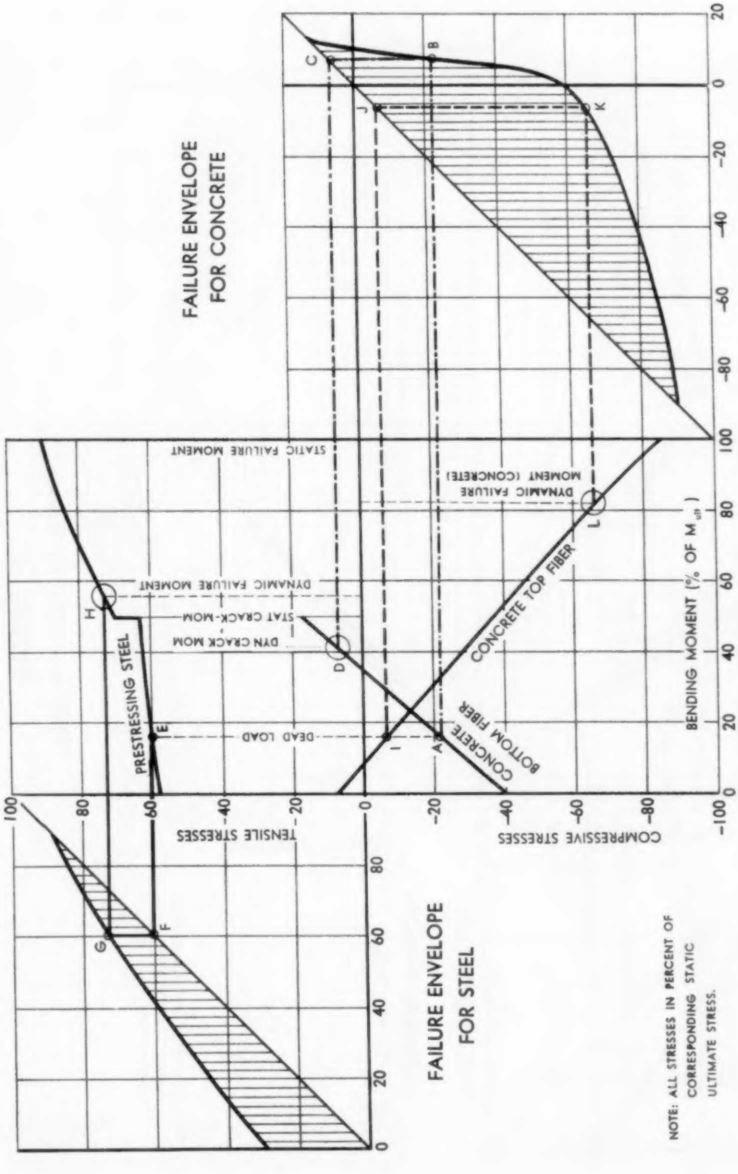
FIGURE 2



FATIGUE FAILURE ENVELOPE FOR CONCRETE

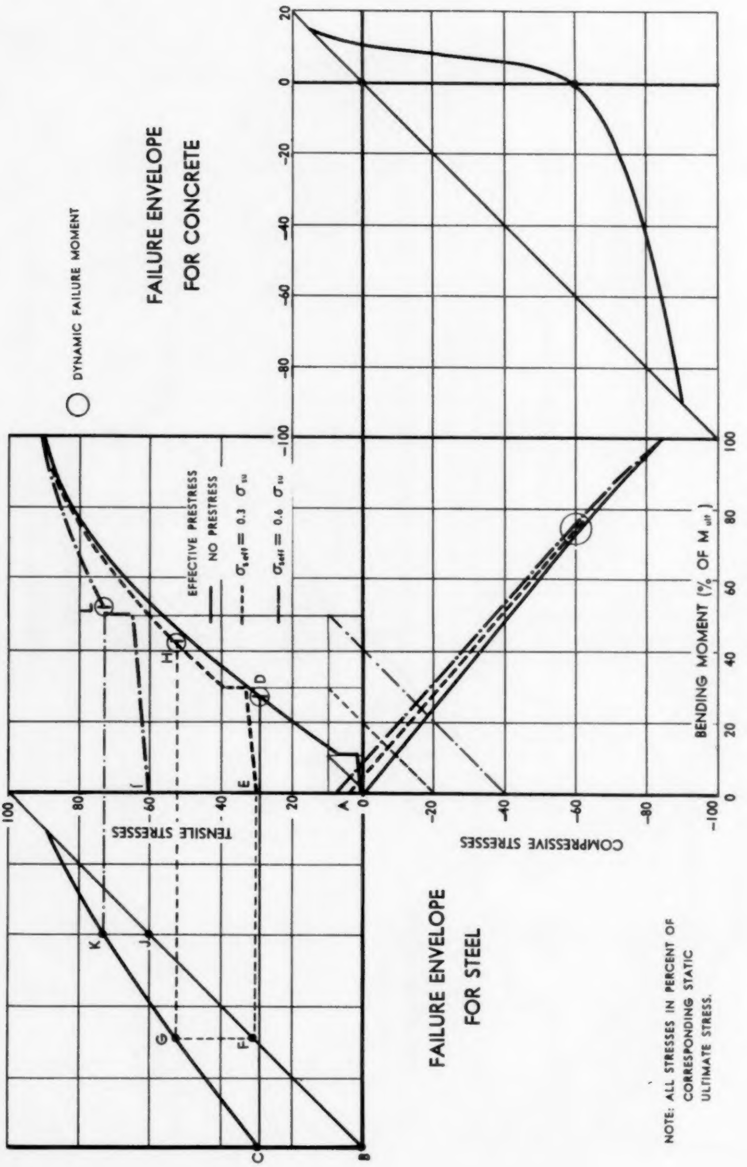
(10^6 LOAD CYCLES)

FIGURE 3



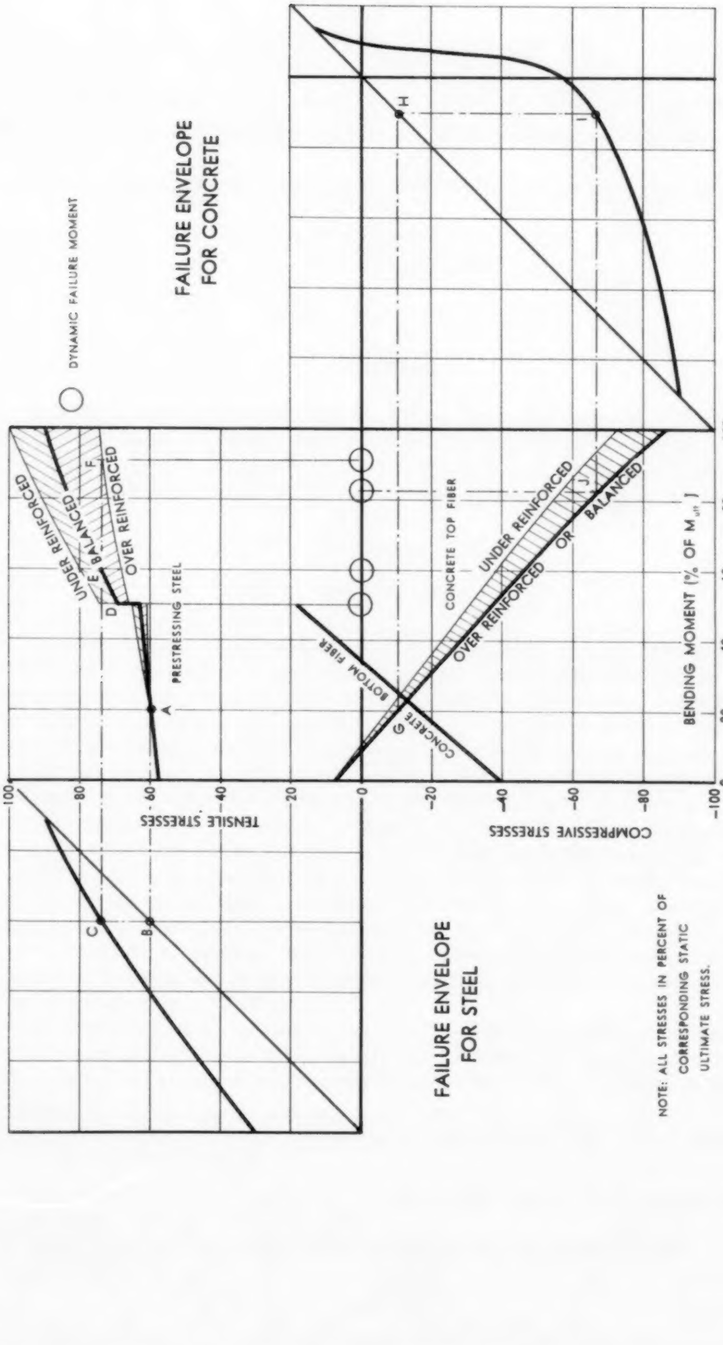
STRESS — MOMENT DIAGRAM

FIGURE 4



STRESS—MOMENT DIAGRAM

FIGURE 5



STRESS – MOMENT DIAGRAM

FIGURE 6

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APPLICATION OF DIGITAL COMPUTERS TO BRIDGE DESIGN*

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(Proc. Paper 1308)

ABSTRACT

This paper presents a discussion of the characteristics of digital computers and the steps necessary to apply them to problems in bridge analysis and design. Recent advances in coding technique and the potentialities of the computer in more refined analysis and optimization of design are indicated.

INTRODUCTION

The availability of the modern digital computer has been viewed by some members of the engineering profession as a mere substitute for the engineering manpower required in today's design and analysis work. In the opinion of the author, this is a very limited viewpoint. Just as the modern machine tools offered new possibilities in dimensional and quality control as well as greater output per worker, so also the digital computer opens new possibilities in the refinement and reliability of analysis and design work. It is the purpose of this presentation to review the present stage of development of the application of such equipment to the special field of bridge analysis and design and to indicate the concepts and techniques which give promise of exploiting the full potentialities of the digital computer in future design work. Specific programs designed for particular computers will not be presented in detail at this time, but the results of some of the work now in progress by the author will be reported in the somewhat more comprehensive paper which will be submitted for publication at a later date.

In order to give definition to the direction of this presentation and to avoid misunderstanding due to difference in terminology, we shall begin by a brief discussion of the various characteristics of presently available digital

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computers. A digital computer, as contrasted to an analog computer, performs arithmetical operations upon quantities represented by discrete digits rather than continuous functions. The power of the digital computer to perform complex mathematical operations such as integrations and differentiations is wholly dependent upon the skill of the programmer in converting these operations to the basic arithmetic elements of addition, subtraction, multiplication and division. The digital computers likely to be available to engineers working in the field of bridge design fall into three broad groups, which for simplicity we shall call merely the small scale, medium scale, and large scale computers. It should be recognized that the boundaries between these groups are somewhat arbitrary. The range of problems which occur in bridge design and analysis is such that each of these general classes of computers is useful and most efficient for some particular problems. All classes of digital computers have five main components: The (1) input mechanism, (2) output mechanism, (3) the "memory" in which data is placed by the input, intermediate results are stored until converted, and output data is held for use by the output mechanism, (4) the arithmetic units in which the actual additions, subtractions, multiplications and divisions are performed, and (5) the control unit, which synchronizes the activities of the other four components.

In the small scale computer (Fig. 1), these functions may all be contained in a single cabinet (Example IBM604A). Typically, these computers operate by reading data from a punched card or group of punched cards, performing a limited number of arithmetic operations and then punching the results into the same card or set of cards. The memory unit usually has a rather small capacity, say six or eight ten digit numbers. However, the thing that is particularly characteristic of this class of computer is the manner in which the control unit receives the instructions to perform the computations. In these machines, the sequence of operations is specified by the wiring of groups of relays and switches by the operator—a so-called "wire program" system. Patch panels are available so that wiring may be prepared in advance and inserted into the computer quickly when a given program is to be used, but a separate panel must be wired for each type of problem to be run.

In the medium scale computer (Fig. 2, Example IBM650, Datatron 304), the input and output are also usually a card reader and a card punch, or an equivalent system employing punched paper tape. The significant advances are in the memory capacity, which is normally able to hold several thousand ten digit decimal numbers or the equivalent in bianary form, and in the manners in which the sequence of operations is specified. The "program" which furnishes the instructions to the control unit is now a part of the information which is stored in the memory. It consists not of a wired panel, but of a series of coded "instructions" written by the programmer for his particular problem and fed into the memory through the input mechanism just as though it were data. Another important aspect of this "stored program" is the fact that it may be modified during the course of the problem by operating upon the code numbers of the instructions with the arithmetic elements of the machine. This permits the same series of instructions to serve for any portion of the problem in which only minor differences of computing steps or changes in the variables to be operated on are involved. These features make it possible to prepare long problems which can be conveniently stored as decks of punched cards or tapes until needed. The efficiency with which large and complex problems can be handled upon a computer is partly dependent upon the capacity of the machine to hold large numbers of intermediate results

during the course of the problems for ready reference at any time. If this capacity is exceeded, the problem must be broken into blocks and each block processed independently. In this respect, the medium scale computer is a great improvement over the ordinary punch card computing punch and greatly expands the scope of problems which can be handled with a minimum of input-output time and card handling.

The size of the block which can be computed without interruption is increased almost without limit in the large scale computer, Fig. 3 (Examples: IBM701, 704, Univac 1103A, Whirlwind, NORC, LARC), in which the working memory of several thousand units is backed up by one or more auxiliary memory components such as magnetic drums, and tapes, the latter of which may contain several million elements of data or instructions. The other chief features of the large scale computer which increase its capabilities over those of the medium scale computer are its much higher speed—with the basic add cycle measured in microseconds—and the increased flexibility of input, output, and control systems. Input capabilities may include a card reader, magnetic tape, punched paper tapes and an analog-digital device, while output may be possible in the form of punched cards, printed list, magnetic tape, or digital-analog display. Special control features include additional arithmetic units for computations which control the program and more flexible manual controls.

The Steps in Solution of a Problem on a Digital Computer

The first step in preparation of any problem for a digital computer is to examine its suitability for automatic computation and the class of computer which it should be coded for. Not all laborious or lengthy computations are profitable to put on a computer just because they are laborious or lengthy. They should possess the element of repetition, either internal or external; that is, the problem should either occur many, many times in the work of an office or it should be one in which a certain pattern of computation is repeated many times in the solution of a single case of the problem. Otherwise, the time spent in "coding" the problem in a form so that the computer can perform the work may not be a wise investment. The type of repetition which is characteristic of the problem determines to some extent the class of computer which is most effectively used in its solution. A relatively simple computation which occurs many thousands of times, such as the computation of area of earthwork cross sections, can be efficiently done on a small scale computer. Such problems are not frequent in the field of bridge design, which we are now considering. The problems of this field occur less frequently and are more complex. Fortunately, however, they do often contain the elements of both external and internal repetition. The computations of the ordinates to an influence line for a continuous truss, for instance, involves repeated applications of a unit load to the same structure or some equivalent of this. These problems are well suited to the capabilities of the medium and large scale computers.

The next thing which must be done is to reduce the problem to a numerical technique which can be performed by the basic arithmetic capabilities of the computer. It is at this stage that the greatest demands are made upon the knowledge and experience of the individual preparing the problem. He must have a clear understanding of the physical basis of the problem as well as a knowledge of the capabilities of the computer in order to select the most

effective system of calculation. In the opinion of the author, the full capabilibilities of computers in structural analysis and design will not be realized until engineers in this field learn to do their own "coding" for the computers. With the mechanical and automatic aids to coding which will be described later in this paper, it is possible for a structural engineer to achieve reasonable facility with the techniques required with a week or more of formal instruction plus a few months' experience in coding problems under the direction of an experienced coder. It can hardly be expected that a professional coder, even with the excellent mathematical background which he usually possesses, can be trained to recognize the many facets peculiar to each problem in structural analysis in an equal amount of time.

The problem formulation is normally accompanied by the construction of a flow chart which diagrams the sequence of calculations which is to be followed so that the relationships between the various steps may be visualized. Such a flow chart (Fig. 4) will indicate clearly those portions of the calculations which are to be performed many times during the processing of a single case. Such repetitive operations are called loops. The sequence of calculations performed during each passage through the loop can be controlled by the same set of instructions in the program if the addresses or locations in memory to which these instructions refer are modified to select the proper data each time the sequence is executed. Such "address modification," as it is called, is accomplished by arithmetic operations on those portions of the instructions in the program which specify the memory location from which an element of data is to be extracted or to which a result is to be assigned.

When the entire calculation has been clearly laid out on a flow chart, the next problem is to connect the mathematical operations indicated symbolically on the flow chart into the rather elementary arithmetic operations which the computer is capable of interpreting. In order that the enormous amount of detail involved in the coding operation may be appreciated by those not familiar with this aspect of computer application, a sample sequence of coding for a rather simple operation is shown in Figure 5. The instructions indicated here are for the IBM 701 computer. It will be noted that each instruction includes a number identifying the instruction, a coded two-digit number which indicates the operation to be performed, and a four-digit number giving the address of the location in memory to be referred to. The particular problem being coded here is the familiar expression for computing the axial deformation in a member; that is,

$$\Delta_n = \frac{P_n L_n}{A_n E_n}$$

It is assumed that the deformations for thirty such members are to be computed. The loads for each of the thirty members have previously been placed in thirty successive full word locations in the memory, the full word addresses being -0102 through -0160. In similar fashion, the thirty values of length and thirty values of area occupy successive addresses as indicated on the figure. The reciprocal of the elastic modulus is assumed to be constant for all members and is stored at location -0050. Certain integers are required in the manipulation of the program and these are stored in the locations indicated at the points where these integers are used. The sequence of calculations begins with a "Reset Add" instruction referring to memory location

-0202. When this particular instruction (No. 1171) is encountered by the control component of the machine, it will clear the arithmetic register and then add the constants of location -0202 to the register; that is, it will place the value of the length of the first member of the accumulator. The next five instructions accomplish the various multiplications, divisions and decimal point shifts to evaluate the expression being coded. Instruction No. 1177 stores the result of the computation for the first member in memory location -0402. The next twelve instructions modify the addresses of the working instructions in the calculation sequence so that upon the next pass through the loop, the data for the second member will be processed. Instructions No. 1178 through 1180 modify the address of instruction No. 1171 by placing this instruction in the accumulator, advancing it by two and storing the advanced address back into instruction No. 1171. The fact that the instruction is advanced by a -2 is a peculiarity of this particular machine, in which all full words are designated by negative even addresses. Instructions No. 1190 through 1192 test the value of a number located in location -0070 to determine whether or not the loop has been executed for all members. Location -0070 initially contains the number 30. Each time the loop is executed, instruction No. 1191 subtracts from this number, and then instruction No. 1192 puts the resulting number back into location -0070. Instruction No. 1193 then examines the value of this number which would still be standing in the accumulator, and if it is positive, causes the machine to return to instruction No. 1171 and repeat the sequence of calculations. When all thirty members have been computed, the number standing in location -0070 will be a zero, and instead of returning to instruction No. 1171, the machine will escape from the loop and continue with the next portion of the program.

It will be noted that three rather laborious and exacting tasks are involved in the process of coding a problem in straight machine language as above. First of all, the entire complex program of computations must be broken into the elementary arithmetic operations of addition, subtraction, multiplication and division. Second, every piece of data manipulated and every result produced must be assigned a location in memory and an extensive bookkeeping procedure is necessary to insure that proper addresses are referred to at all points in the computation. Finally, it will be noted that a considerable fraction of the total number of instructions for even such a simple program are those concerned with address modification. In a program of calculation involving a thousand or more arithmetic operations plus a number of evaluations of special functions such as trigonometric sines or cosines which must be developed by the machine as an infinite series, the task of coding the straight machine language is understandably time consuming. If the program contains several loops and loops within loops, the address modification instructions alone may amount to several hundred steps.

Fortunately for engineers desiring to make use of computers in bridge design work, straight machine language coding is not always necessary to solve a given structural problem. As a matter of fact, there is a large class of structural problems which may be solved effectively with a computer with no coding at all; that is, the individual desiring to solve the problem need not do any coding of his own. This happy state of affairs can be achieved whenever the program of computation can be converted to a standard mathematical computation such as the solution of a large number of simultaneous equations. For such operations, many general purpose routines have been written by professional coders for most of the types of computers which are available. To make use of these general purpose programs, one need only

determine the form in which the data must be prepared for the input equipment to the machine at one's disposal and learn enough of the description of the routine to be able to interpret the results. The availability of such general purpose programs gives renewed importance to all of the classical methods of formulating structural analysis, particularly those methods which pertain to statically indeterminate structures, such as slope-deformation and virtual work. Recent publications by R. K. Livesley(1) and J. Foulkes(2) in England and H. C. Martin(3) in this country have demonstrated the power of formal methods to adopt digital computers to the analysis of highly indeterminate frameworks. These investigators have employed the techniques of modern matrix algebra to manipulate the relationships of equilibrium and computability written for each joint or member in the framework from the basic considerations of axial, torsional and bending deformations.

There is another class of structural analysis problems which can be formulated in a rather standard mathematical form and then solved by the use of general purpose computer programs. This group of problems includes all those structures in which the relationships are governed by formal differential equations which can be converted to finite difference expressions. The long span flexible arch is a problem of this variety. The necessary finite difference equations for this problem have been presented in a recent paper by E. Popov and E. Hirsch.(4) Unfortunately, the amount of study and special programming necessary to use the general purpose program for the solution of such problems is somewhat more extensive than those involving matrix manipulations, particularly if the engineer wishes the machine not only to solve the finite difference equations but also to formulate the terms of these equations when furnished with nothing but the initial geometry of the arch rib and the loading systems to be studied. The writer has a program under development at this time to accomplish this purpose.

Were the employment of digital computers in bridge design to be restricted merely to the manipulation of the laborious and purely mathematical aspects of structural analysis, our profession would indeed be missing a great opportunity to exploit the potentialities of the digital computer. Fortunately, many groups which have access to computers have begun to see the wisdom of training their engineering staff to do their own coding. This has led to the development of various special purpose programs of great value. The American Bridge Company staff, for instance, has reported the development of a special program which enables the computation of stresses in a truss structure of any triangulated configuration. The California State Highway Department has developed a program for computing the properties of composit concrete slabs and steel stringer deck systems. There are, without doubt, many problems encountered frequently in bridge design offices from which automatic computation routines would be welcome. It is the opinion of the writer that while the digital computer does offer the possibility of greatly reducing the number of engineering man hours required in such extensive computations as seconary stress analysis, indeterminate truss analysis, and section property calculations, there is also a fruitful field of application beyond this. It is expected that the availability of a means of carrying out extensive theoretical calculations will, first of all, permit the designer greater freedom from many of the empirical techniques which he is presently forced to use in order to complete a design in a reasonable length of time. It is no surprise to anyone in the bridge design field to learn that many worthwhile, rational approaches to some of the vexing problems of design have not been incorporated into our design

specifications nor widely used because of the sheer labor involved. With the present rapid development of new materials and structural forms, it is doubted that even extensive experimental test programs can keep up with the demand for the extension of some of our present empirical methods. The computer offers a ready means of practical application of existing rational methods to many problems now handled by empirical means, such as the distribution of wheel loads to stringers, the evaluation of impact analysis, stability studies for low cord trusses, and examination of potential weak points in complicated plate girder stiffener arrangements. The computer should enable the bridge engineer not only to complete designs with less engineering man power, but also to do a better job and to move freely into new forms for his structure.

Recent Developments in the Advancement of Coding Methods

In the previous section, we have indicated the desirability of training members of an engineering staff to do their own coding for the digital computer. Unfortunately, when one begins to examine the coding problem, he is often completely disenchanted by the lengthy programs which must be prepared with meticulous care to solve even the simplest of problems worthy of programming. A program which is 99 and 44/100% pure is of no value until the other 56/100% is also made pure. One incorrect digit in any one of the thousands of address references may produce completely meaningless results.

Structural engineers examining the coding problem are not the only individuals who have been concerned about this fact. Many devices and techniques have been developed by the professional coders to reduce the complexity and labor of preparing programs for a computer. These systems fall into several classifications. The first advancement is the use of standard "library" programs. If an engineer wishes to extract a square root or develop an inverse tangent at some point in his program, he need not write his own set of arithmetic operations for generating these functions, but need only refer by a transfer instruction to a location in memory at which he will cause to be stored a standard set of instructions which he can conveniently pick up from a file drawer. Such library routines are available for a large number of common mathematical operations and also for circumventing the annoying problem of writing one's own input and output routines for getting the data and program into the machine and picking up the results. The second aid to coding is the so-called interpretative routine. This type of routine is excellent for programming complex problems which will not be run too frequently. The concept of interpretative program is to furnish the coder with a large number of powerful instructions which enable the problem to be written in abbreviated form. Unfortunately, such interpretative schemes require that a large block of the machine memory be reserved for storing a standard program which enables the machine to interpret the pseudo instructions and convert them to its elementary machine language. This makes the interpretative routine approach expensive with regard to memory space and machine time.

Several routines are available which permit what is known as "regional coding." When using this system, one writes in straight machine language but breaks his problem into small blocks or regions which are largely self-contained; that is, only a few items of data need be obtained from points

outside the region to perform the operations of this region and only a few of these results need be carried into the next region. Within each region, the programmer sets up an independent set of addresses, thus greatly reducing the bookkeeping problem and eliminating many of the mistakes commonly made in address referencing. Before the problem is run on the computer, the instructions written for each of the regions for the program are key punched on to punch cards and assembled together with any standard library routines for special functions and a library routine to provide for input and output arrangements. This deck of cards is placed in the card reader of the computer together with a special program deck which provides the machine with instructions which enable it to arrange the entire program, assign absolute addresses to all data and variables, and punch out the resulting straight machine language program on a new deck of cards. This new deck is then fed into the computer with each set of data for the actual calculations to be performed.

The concept of mechanization of coding which has most potential usefulness in the structural analysis and design field is so-called automatic coding or "compiling." Automatic coding is based upon the realization that many of the operations involved in writing a program are purely routine in nature and involve multiple arithmetic operations particularly in those sections of the program which involve address assignment and modification. No programmer can compare with the digital computer itself in performing such arithmetic operations with speed and accuracy. In automatic coding, the programmer writes his program in symbols rather than by referring to specific locations in the memory. The code in which he writes is, like the case of the interpretative schemes, a pseudo code which contains many powerful instructions, some of which will perform such feats as providing for the complete address modifications necessary to generate a loop. Unlike the interpretative schemes, however, the machine does not depend upon a special set of instructions stored in its memory to execute this pseudo code. Instead, the program prepared in the pseudo code is treated much like the regional program in that the computer itself is used to manipulate the pseudo code and punch out a deck of program cards in straight machine language which is then used to run the problem. This gives the automatic coding technique a great advantage over both the interpretative and regional coding methods, since a powerful system of instructions is obtained without the sacrifice of memory space or machine time in calculation. It will be noted also that since the programmer does not assign any addresses himself, the bookkeeping and the mistakes common in address manipulation are eliminated. In Figure 6, the same sequence of calculations previously coded in straight machine language has been recoded by an automatic coding system. The particular system employed is known as PACT, standing for Procedures for Advancement of Coding Techniques. This system was developed by a group of IBM 701 users(5) in the Los Angeles area and has been employed by the writer in coding a number of extensive calculations in research work. A few words of explanation as to what this program accomplishes are in order. The first instruction SET initializes the subscript of all the variables in the sequence at the value one. The next instruction, which is a blank, places L_i into the accumulator. This quantity is then divided by A_i and multiplied by P_i and the constant 1/E. The first time the loop is executed, the symbols will all refer to the first member. The instruction 6 is encountered, which is a TEST. This instruction compares the value of the subscript existing when the instruction is encountered with the number 30. If

the subscript is still less than 30, the calculation will return to the first step following the SET. Just after the TEST instruction, instruction No. 5.1 has been written which is an EQ instruction meaning "equals." This instruction should, of course, have followed instruction No. 5, but was omitted by the programmer. In the PACT system, such an omission can easily be corrected by the use of decimal steps, thus avoiding the necessity for renumbering all the instructions in the entire program which follow the point of omission. The function of this EQ instruction is to assign the symbol "DEL_i" to the result of the computation for each member, thus insuring that the machine will recognize this quantity as one which is to be used in subsequent parts of the calculation and will assign it a block of addresses so that it will be properly stored in the memory. This little region, when compiled with all the other regions in the PACT program, and processed by the compiling routine, will produce a set of machine instructions, the absolute addresses for each of the variables and provision for modification of instruction addresses like that developed in the previous section, except that if a more efficient way of coding this loop exists the compiling routine will probably discover it and employ it in writing its machine language program.

Automatic coding techniques not only possess the features which make them attractive for any type of coding, but also some features which are particularly useful in the field of structural analysis and design. It is typical of such problems that very seldom do we encounter two problems which are identical. Consider the problem of developing the influence lines for a continuous beam structure. The variables involved include the number of beam elements, the number of spans, the ratios between the lengths of the individual spans, the ratios between the moments of inertia at the centers of each span, and the ratios between the center line and haunch depths at each span. This problem in its general form is frequently encountered, but if the utility program is to be written which will solve a variety of cases, not only the numerical values of certain ratios must be capable of specification but also the integers which define the number of spans and the numbers of points within the spans at which loadings and internal moments and shears are to be considered. Automatic coding offers a convenient means of providing for this flexibility. A master symbolic program can be written in which the loop size controlling the number of items to be considered are left as variables to be specified on a single card of the symbolic program deck. A highly efficient machine language program for a given configuration of structure can then be developed completely automatically, except for key punching this one card to define the integers which control loop sizes. The machine language program which is developed will contain loops of just the proper size for the specific problem being solved and will avoid some of the inefficiencies in general programs written to accommodate the largest of the particular class of problems being solved. Another advantage of the automatic coding technique for structural problems is the fact that the symbolic language employed to identify the variables may very well be precisely those which are used in ordinary structural calculations, thus providing the definite psychological advantage of familiarity to the programmer.

Special Fields of Investigation Opened by Digital Computers

In the preceding sections, we have indicated the fact that the digital computer not only opens the possibility of savings of engineering man power in

performing the analysis and design computations which are now customary, but that it also opens the way to do a more thorough analysis job and a better design job. Some of the areas in which the computer is useful for extending the range of problems in which rational analysis may be substituted for empirical methods have already been indicated. In concluding this paper, the writer would like to share his thoughts upon some studies now under way in the optimization of structures.

Every designer is aware that in the computations to perfect a statically indeterminate structure, he is forced to base the successive refinements of his analysis upon successive corrections to an assumed shape and stiffness distribution for his structure. To some extent this same condition exists in statically determinate structures. The final design should always be a safe structure if careful procedures had been followed, but it is not always the most economical structure. Further variation of the parameters which define the shape and proportions of the structure might well produce a structure of less cost. At the present time, the designer must depend heavily upon his own experience and that of others to select these optimum shape parameters to insure reasonable economy. When the structure is of a conventional form, these empirical methods, no doubt, produce configurations close enough to the theoretical optimum that further refinement is not necessary. However, with the rapid changes in structural form and material now taking place, it is extremely desirable to develop rational procedures for optimizing new structural systems. It is the opinion of the writer that the digital computer will not only be a prominent tool in the development of such methods, but that it also is likely to become a working tool for optimization procedures.

The concept of optimizing continuous beams and frames is not new. In 1939, L. E. Grinter published Automatic Design of Continuous Frames(6) a treatize on the application of successive approximation methods quite similar to the Hardy Cross procedure for the selecting of suitable sizes of beams and columns in continuous frames in order to produce a well-balanced set of maximum stresses. Extensions of his method to members of variable moments of inertia and to frames subjected to moving loads were discussed. More recently, J. Foulkes in England has applied the digital computer to the optimization of portal frames based on plastic concepts of failure. The writer now has several programs under consideration for extending the optimization concepts to certain problems in bridge design. Consider a program for the computation of influence lines for a continuous beam of variable moment of inertia. Suppose that we add to this program an additional sequence of computation which will locate the proper positions for the application of loads to these influence lines, compute the maximum life load effects produced, and finally tabulate the moment and sheer envelopes for the beam. If we have employed in the coding of this problem one of the physical successive approximation methods, such as moment distribution, it is now a fairly simple matter to make local alterations in the stiffness of the beam so that to obtain better balance between the resulting stresses. If this feature is added to our program and the computer is instructed to recycle the entire process after making each set of alterations, we have the beginning of an optimization routine.

When the other parameters which govern the over-all economy of the structure such as the span ratios are allowed to vary, a further degree of complexity is introduced. The writer is currently exploring the possibility of employing the concepts of linear programming as developed by the economists to this problem. Whether the more effective optimization routine will result

from such a formal and abstract procedure or from educating the computer by special routines to the quantitative value judgments possessed by the structural engineer is still an open question.

Several other optimization problems are under consideration at the present time. One of the most interesting concerns the geometry of a flexible arch rib designed for a system of moving loads. The problem is to find the initial geometry for the rib and the distribution of stiffnesses along the rib axis which will result in the minimum amount of material for a given load system. Another problem under study is the continuous truss in which the variation of depth along the spans is one of the controlling parameters. None of these programs has as yet been concluded, but it is expected that the experience being gained by their formulation may open some fruitful possibilities in this field.

CONCLUSIONS

In summarizing the writer would like to emphasize the following points:

1. The digital computer can and should play an important part in reducing engineering man power and requirements in performing customary design calculations.
2. To fully exploit the computer facilities available to a design office, the engineers themselves should invest the time necessary to learn to code their own problems.
3. Full advantage should be taken of the recent advancements in coding techniques, particularly the use of regional assembly routines and automatic coding programs.
4. The important new concepts of design, such as rational optimization procedures, have now become a practical reality.

REFERENCES

1. R. K. Livesley; The Use of Digital Computers with Particular Reference to the Analysis of Structures, Trans. North East Coast Institution of Engineers and Shipbuilders, Vol. 71. p. 67, (1955).
2. J. Foulkes; The Minimum Weight Design of Structural Frames, Proc. Royal Society, Vol. 223, No. A1155, p. 482, (May 1954).
3. H. C. Martin; Truss Analysis by Stiffness Considerations, ASCE Proc. Vol. 82, No. EM4, (Oct. 1956).
4. E. P. Popov and E. G. Hirsch; Analysis of Arches by Finite Differences, ASCE Proc. Vol. 81, No. 829, (Nov. 1955).
5. C. L. Baker; The PACT I Coding System for the IBM type 701, Jr. of Assoc. for Computing Machinery, Vol. 3, No. 4, (Oct. 1956).
6. L. E. Grinter; "Automatic Design of Continuous Frames in Steel and Reinforced Concrete," The MacMillan Company, New York, 1939.

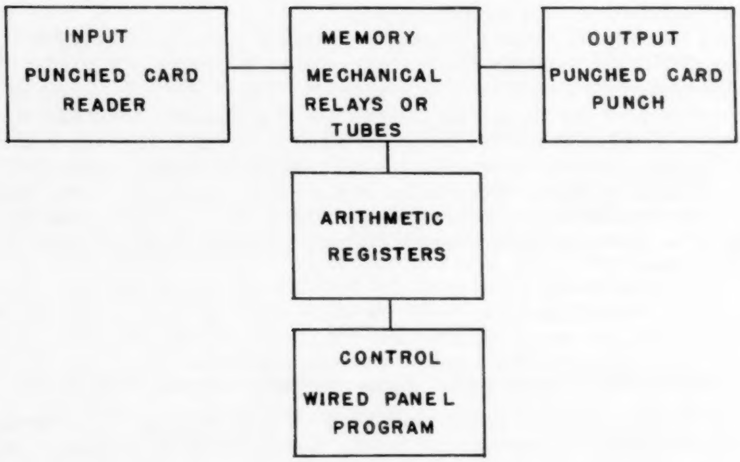

FIG. 1 COMPONENTS OF TYPICAL SMALL SCALE COMPUTER

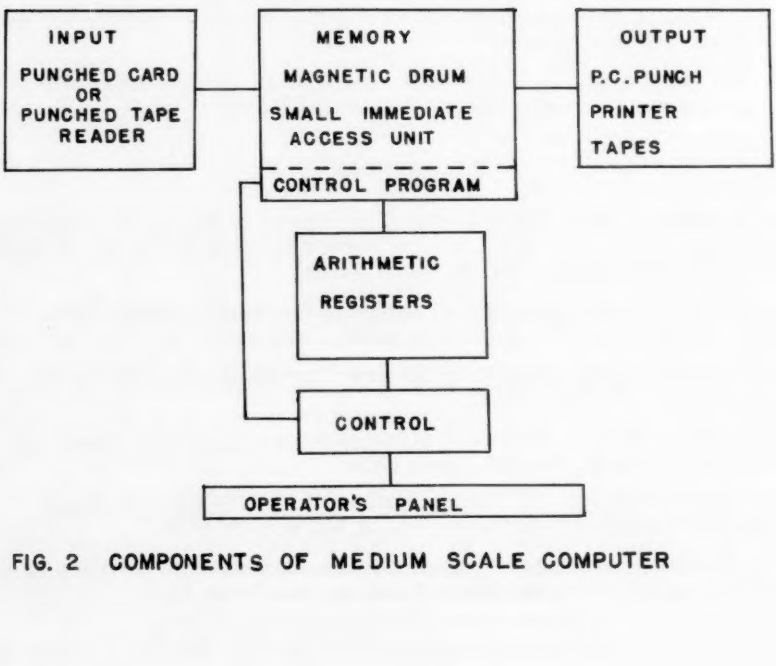

FIG. 2 COMPONENTS OF MEDIUM SCALE COMPUTER

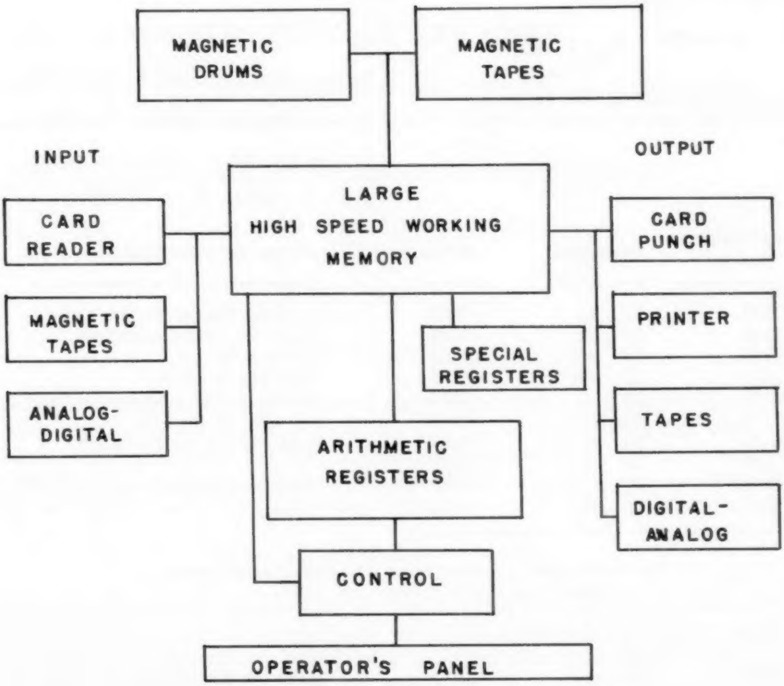

FIG. 3 TYPICAL LARGE SCALE COMPUTER

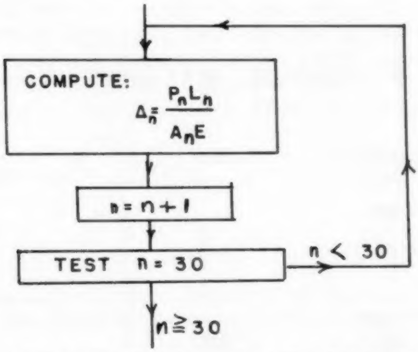

FIG. 4 PORTION OF TYPICAL FLOW CHART

Purpose of program:

Evaluate $\Delta_n = \frac{P_n L_n}{A_n E}$

For values of the index n from one to thirty.

Memory allocation:

$P_1 \ldots\ldots P_{30}$ in L(-0102) to(-0160)

$L_1 \ldots\ldots L_{30}$ in L(-0202) to(-0260)

$A_1 \ldots\ldots A_{30}$ in L(-0302) to(-0360)

1/E in L(-0050), -2 in L(-0060),

1 in L(-0062), 30 in L(-0070)

Instruction Number	Operation		Address	Action of instruction
1171	10	RA	-0202	Clears, Places L_1 in Acc.
1172	21	LR	0003	Shift to scale quotient.
1173	18	Div	-0302	Divides by A_1
1174	16	MPY	-0102	Multiplies by P_1
1175	21	LR	0035	Shifts to MQ, prepare to Mpy
1176	16	MPY	-0050	Multiplies by 1/E
1177	12	ST	-0402	Puts result in L(-0402)
1178	10	RA	1171	
1179	09	A	-0060	Advances address of Ins.1171
1180	13	SA	1171	

....

The next nine instructions advance the addresses of instructions 1173, 1174, and 1177.

....

1190	10	RA	-0070	Location initially contains 30
1191	05	SUB	-0062	Subtracts a 1
1192	12	ST	-0070	Stores count in L(-0070)
1193	03	TP	1171	Returns to beginning of loop.
1194	....		Goes to next section of Program	

FIG. 5 SAMPLE MACHINE LANGUAGE CODE

Step		Operation	Factor	S_1	S_2	Q	Constants
DEL	1.0	SET		i	= 1		
	2.0		L	i		7	
	3.0	/	A	i		7	
	4.0	X	P	i		7	
	5.0	X				7	1/E
	6.0	TEST		i	= 30		
	5.1	EQ	DEL	i		7	

FIG. 6 SAMPLE OF SYMBOLIC CODING

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ALUMINUM APPLICATIONS FOR HIGHWAY BRIDGES*

J. M. Pickett,[1] J.M. ASCE
(Proc. Paper 1312)

ABSTRACT

This paper includes a discussion of aluminum alloys recommended and used for highway bridges, a review of the aluminum accessories and appurtenances which are becoming standard usage of conventional bridges, and a summary of the larger bridge installations where aluminum components are being used as the main load-carrying members.

What are the facts? Should you, as a structural designer, be considering the use of more aluminum alloys on highway bridges and other related highway structures? Should you, admittedly swamped with both design and administrative problems of this new highway program, pause to re-evaluate the role aluminum alloys can perform in the great drama of highway bridge building?

Let's look at the record. Let's discover together why some of those same reasons that have made aluminum an everyday household word, can now be applied to the structural use of aluminum. Let's discuss the virtues of this light metal to discover together why aluminum is now ready, able and willing to assume its place alongside other bridge building materials.

What are the advantages of aluminum alloys? Here are the major ones:

1. Lightweight
2. Resistance to corrosion
3. Non-sparking properties
4. Reduced maintenance
5. Improved appearance

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6. Strength
7. Non-magnetic characteristics
8. High thermal conductivity
9. Non-toxicogenic
10. High electrical conductivity
11. Workability
12. High scrap value
13. Availability in all the various shapes and forms

Which of these advantages of aluminum alloys in general, are of particular interest to highway bridge designers and builders? Aluminum alloys are (1) strong, (2) lightweight, (3) resistant to corrosion, (4) workable and (5) available. What do these advantages mean; how can you as a designer and builder use them?

Aluminum Alloys are Strong

Two alloys which have been developed, for structural use by Civil Engineers, are 6061-T6 and 2014-T6. 6061-T6 is a moderate strength alloy with a high resistance to corrosion. This alloy has a recommended allowable design stress of 15 kips per square inch. 2014-T6 is a higher strength alloy having a recommended allowable design stress of 22 kips per square inch. Specifications for both of these alloys have been prepared by the Committee of the Structural Division On Design In Lightweight Structural Alloys, and are available from the Society as Proceedings Papers Nos. 970 and 971.

When the tensile properties of these two alloys are compared with those of steel, the tensile yield strength of 6061-T6 is slightly higher than that of structural carbon steel, and the yield strength of 2014-T6 is comparable to that of structural nickel steel. Also important to the designer in the design of bridges is the modulus of elasticity of aluminum alloys, which is roughly one-third that of steel. This lower modulus must be considered in determining the type and proportions of the bridge span as well as the design of compression members, compression flanges and girder webs. The accepted ratios of depth-to-span for the various types of bridges are usually increased to reduce deflections, and to give maximum economy of material.

Two other factors that must be considered with the design of aluminum structures are fatigue and thermal expansion. The number of cycles of load encountered in structures is usually small compared with those encountered in fatigue problems involving machine parts. Careful attention to details in design and fabrication can be expected to eliminate fatigue problems in structures. The coefficient of thermal expansion of aluminum is roughly twice that of steel or concrete, and must be considered in composite designs as well as in the design of expansion shoes and devices.

Aluminum Alloys are Lightweight

Both alloys 6061-T6 and 2014-T6 weigh approximately 35 per cent of the weight of structural steel. This lightweight characteristic of aluminum can result in considerable weight savings in structures. For small highway bridges in which the proportion of dead weight is low, weight savings of about 50 per cent can be expected. For a moderate length span, say six to seven

hundred feet, the weight savings would amount to about 60 per cent. And, in a really long span, where dead weight is a governing factor, even larger weight savings are possible. Another aspect of this pyramiding of weight saving occurs in bascule or lift bridges where machinery, counterweights and towers are involved.

Aluminum Alloys are Resistant to Corrosion

Alloy 6061-T6 members are not ordinarily painted except in areas where they are in contact with, or are fastened to, steel members, or other dissimilar materials. Alloy 2014-T6 structures are normally painted, but experience indicates that paint systems on this alloy can be expected to outlast comparable paint protection on steel members. Sheet and plate of 2014-T6 alloy is normally produced with a special surface cladding which is similar to alloy 6061-T6 in resistance to corrosion.

Aluminum Alloys Can be Fabricated

Structural aluminum can be handled by the ordinary steel fabricating shop with very little change in practice or procedure. Sawing and drilling of the thicker members is preferable to shearing and punching. The use of a cutting torch on aluminum is not acceptable because the material melts rather than burns, and the high temperature lowers the strength of the metal by eliminating the effect of previous heat treatment. The use of clean, polished tools is essential to good forming operations, and to facilitate bending, alloy 6061-T6 may be heated to a temperature not exceeding 400°F for a period of time not exceeding 30 minutes (the maximum heating time for alloy 2014-T6 is 15 minutes.)

Riveting is the preferred method of assembly for heat-treated alloys. Aluminum alloy rivets may be driven hot or cold, the selection depending on the alloy, the required shear strength, and accessibility. Cold driven rivets develop higher shear strengths, but are difficult to drive with pheumatic hammers if over 1/2 in. in diameter. A design strength in shear of 10 kips per square inch is generally used for cold driven 6061-T6 rivets in 6061-T6 structures. The same design stress is normally used for cold driven 2117-T3 rivets in 2014-T6 structures. Hot driven rivets of 6061-T43 are usually used for both 6061-T6 and 2014-T6 structures where hammers must be employed. These rivets should be heated in an air-circulating, controlled temperature furnace to a range of 990 to 1050°F before driving, and a design shear strength of 8 kips per square inch is generally used.

Aluminum Alloys are Available in Most Desired Shapes and Sizes

Most aluminum plates and shapes can currently be delivered to the fabricating shop within 4 to 6 weeks after receipt of order. Plates are available in thicknesses up to 3 in. and widths up to 10 ft. Lengths up to 39 ft. are available for the most commonly used widths and thicknesses. Standard structural shapes include sizes as large as 8-x 8-x 1- in. angles, 12-in. I-beams, and 15-in. channels. The most commonly used structural shapes are available in lengths up to 85 ft.

In addition to these standard structural shapes, there is available an infinite variety of special shapes which can be produced by the extrusion process. Dies can be provided at a cost which is very low compared to the investment required to roll a new shape, and of course, the time required to make a die is relatively small when compared to the time required to make new rolls. Important weight savings with resultant economies, can be obtained in structures by the use of special extruded shapes where the quantity involved justifies the extra tooling cost.

Having considered these advantages, the next question is: how can aluminum be used to advantage on highway bridge structures? Let's turn first to one of the smallest units used on a bridge, but certainly not the least important—signs. Flat sheet is used for regulatory signs, warning signs, guide signs, small informational signs and delineators, to control the flow of traffic on and off a bridge. Alloy 6061-T6 sheet is ideal for this application to resist wind loadings, as well as the common types of vandalism. Where flat sheet is inadequate without a supporting structural system, the use of extruded panels for this application, was first developed for the large informational signs on the Ohio Turnpike, and involves the use of a new sign construction technique. With this technique, informational signs of varying height and width may be built merely by using these extruded panels which are available in lengths up to about 28 ft.

Before leaving the sign subject, there are two other types of structures for which aluminum is ideally suited. The first, an overhead sign system, involves either the use of a single aluminum tubular member, or a welded assembly of smaller tubes supported at both ends by aluminum posts. Alloy 6063-T6 or 6061-T6 are recommended for these systems and offer the advantages of lightweight for easy and speedy installation as well as no interruption of traffic flow for maintenance. There is also a great deal of interest being shown in a similar type of structure—overhead traffic lane markers. These structures are fabricated of 6061-T6 alloy and erected in one piece at the job site. These structures are 85 to 90 ft. in span, and generally include a walkway between the modified plate girder type sections. Units of this type were first used about 7 years ago on the Philadelphia Camden Bridge. Similar structures have now been installed on the Delaware River Bridge connecting the New Jersey and Pennsylvania Turnpikes, and also on the Walt Whitman Bridge at Philadelphia.

No story on aluminum applications for Highway Bridges would be complete without including aluminum railings. The advantages of competitive first cost, maintenance-free service, and improved appearance is readily apparent to bridge designers. It is estimated that over 350 miles of aluminum railings have been installed. These aluminum railings may take the form of a full-height railing such as used on the Walt Whitman Bridge in Philadelphia, or as a parapet railing, like those used on the approaches to that bridge.

Aluminum lighting standards should be used on every bridge. The improved appearance of an installation of aluminum lighting standards is becoming synonymous with a modern bridge lighting system. Aluminum lighting standards perform well in all exposures, as demonstrated by the aluminum lighting standards on the Venetian Causeway, between Miami and Miami Beach. This example is particularly significant, because the excellent service of this installation in a salt water atmosphere has led to an outright specification for aluminum lighting standards in Miami Beach installations.

Aluminum grating for pedestrian service should be given full consideration

by designers. Approximately 40,000 sq. ft. of aluminum grating was used for emergency walkways on the Walt Whitman Bridge now being completed at Philadelphia. Where concrete sidewalks are used, vault frames of extruded aluminum sections work very well.

Another very interesting bridge accessory is the aluminum maintenance bridge used on the Carquinez Straits Bridge at San Francisco. These are traveling platforms and are used on the underside of the floor system for painting of the steel work, and for other maintenance of a steel structure that may be required from time to time.

Aluminum extruded cable separators and fillers can be used to advantage on suspension bridge cables. These extruded sections replace the redwood shims that were formerly used.

One of the most interesting new uses of aluminum in bridge appurtenances is the aluminum parapet to be used on the Fort Pitt Bridge in Pittsburgh. This bridge will have two levels of vehicular traffic, and to offset the unbalanced weight of one pedestrian sidewalk cantilevered out from one side of the bridge, an aluminum parapet will be fabricated. This parapet will be formed of 3/8-in. thick 6061-T6 alloy plate, sandblasted to provide a surface appearance simulating the concrete parapets. Each parapet section will span adjacent panel points and will be 25 ft. long.

The aluminum accessories or appurtenances for bridges, just described, indicate the infinite number of places on conventional bridges where aluminum can be used to advantage. Let's now take a look as to what can be accomplished using aluminum as the main load carrying members.

One of the earliest applications of aluminum alloys in bridge construction occurred during the fall of 1933, when the 50 year old Smithfield Street Bridge in Pittsburgh was rehabilitated. The heavy timber deck and steel floor system were entirely replaced with aluminum floorbeams, stringers, joists, and deck. A double track street railway, a two-lane highway, and two sidewalks are accommodated by this structure. A total of almost 700,000 lbs. of aluminum alloys was used on the two main spans of 360 ft. each. The old trusses were sufficiently unloaded by the use of a lightweight aluminum floor system, resulting in a weight savings of approximately one ton per linear foot, so that they can safely support the loadings imposed by modern traffic conditions.

The rebuilding of the Smithfield Street Bridge with aluminum alloys happened 24 years ago. Today the City of Milwaukee is completing a modernization program on their Sixth Street Viaduct by using aluminum to advantage. This Viaduct contains two double-leaf bascule bridges and in order to modernize these bridges to accommodate an increased roadway width, it was necessary to decrease the height of the girders below the roadway by replacing the top flange and a portion of the web of the girders. To keep the dead weight to a minimum on this project, it was decided to use aluminum structural shapes in the floor system, thereby reducing the weight on the rebuilt girders as well as reducing the need for more counterweight material. The use of aluminum enabled the City of Milwaukee to keep these bascule bridges in service to accommodate modern-day traffic needs.

Current delivery schedules of fabricated steel have led several state highway departments and consulting engineers to investigate the use of aluminum for small highway bridges. These bridges would be on the order of 80 to 100 ft. in span. This is comparable to the 100-foot aluminum plate girder span erected over the Grasse River Railroad Bridge at Massena, New York. This aluminum span was completely shop fabricated and erected in one complete

assembly weighing 53,000 lbs. This contrasts with the similar steel spans weighing 128,000 lbs. that had to be erected one girder at a time. All spans were designed for Cooper's E-60 loading and are of conventional design and construction. The aluminum girder span has successfully passed the most rigorous of tests, and shcws every promise of continuing to perform in a satisfactory manner.

In the field of movable bridges, two aluminum double leaf trunnion bascule bridges have been built in England. The first of these was erected at Hendon Dock Junction, Sunderland, England in 1948, having a length of 121 ft. between trunnion bearings. The second aluminum bascule bridge was completed in 1953, at Victoria Dock, Aberdeen Scotland, having a span between trunnions of 100 ft.

The town of Arvida, Canada, built an aluminum highway bridge over the Saguenay River in 1949. The central arch span is 290 ft. long with a 47 ft. 6 in. rise, and the total length, including approaches is 504 ft. This bridge accommodates a 24 ft. roadway plus two 4 ft. sidewalks, and required approximately 400,000 lbs. of aluminum alloys. A comparable steel span would have weighted twice as much.

One of the most interesting applications of aluminum in bridge building of recent times, was the use of aluminum falsework for the erection of the Richmond-San Rafael Bridge. This bridge contains, in part, thirty-six 289-foot steel truss spans. Twenty-eight of these 289-foot truss spans were erected entirely on the aluminum falsework, which consisted of two complete aluminum spans weighing about 225,000 lbs. each, and measuring 285 ft. long by 36 ft. wide by 42 ft. deep. These aluminum spans were floated to the job site and hoisted into position between piers by using two conventional derrick barges. The aluminum span, supported on temporary bents attached to the steel towers, supported the individual members of the steel truss spans until the steel truss spans were completely assembled and self-supported. This method of erection was at least partially responsible for the fact that the contractors were able to underbid their nearest rival by about 20 per cent.

What does the future hold for aluminum in the bridge building field? Present indications are that it may manifest itself in the near future in the form of small highway bridges because of the light metal's availability and advantages already mentioned—particularly maintenance economies. These bridges will be built up plate girder spans in which deflection limitations will be more influential in determining the girder section than allowable stresses. In spite of this relatively inefficient use of a basically higher priced material, the good delivery schedules of aluminum structural shapes and plate and particularly the savings expected from maintenance-free service, may be just the ticket needed to put some spark in delayed highway projects.

For a long time it has been known that the use of aluminum in long span truss bridges can result in economies of construction. In these structures, where the reduction in stiffness due to aluminum's lower modulus of elasticity may be, at least partially, compensated by increasing the depth of the trusses, this lightweight metal can be used to its full advantage. Because these structures have a primary function of supporting their own dead weight, the use of aluminum should result in compounded weight savings. For example, the use of aluminum in the suspended span of a cantilever bridge will effect economies in all of the other components of the structure: the cantilever arm, the anchor arm, the piers and the anchor supports. These savings taken altogether should result in sizeable savings in the over-all cost of bridges.

In conclusion, it is hoped that these examples of aluminum applications for highway bridges will stimulate your interest in aluminum as an effectual bridge-building material. Aluminum alloys can be designed into almost all portions of bridge structures with definite economic advantages. How about your future plans? Aluminum may be the answer to your bridge-building needs.

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SELECTION OF THE CROSS SECTION FOR A COMPOSITE T-BEAM*

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SYNOPSIS

The design of composite concrete and steel T-beams is usually based on allowable stresses for the component materials. It involves selection of the cross-section, evaluation of the section properties and computation of stresses. No comprehensive tables are available which would cover the practical ranges of all variables involved; therefore, the design of the section is usually accomplished by trial. The evaluation of the exact properties of a composite section requires tedious calculations: the repetition of such computations may be avoided by using the approximate method for selecting the trial section presented in this paper. The method is applicable to composite beams made with symmetrical rolled steel sections, with rolled steel sections with tension cover plates, or with built-up steel sections.

Derivations of exact formulas for the properties and of approximate equations for the selection of a composite cross-section are presented. The approximate equations are presented both in an albegraical and graphical form. Two designs of bridge beams illustrate the practical use of the method.

INTRODUCTION

A composite slab and stringer structure is made up of three essential elements: (1) the reinforced concrete slab which acts as a very effective cover plate on the compression side of the steel beams, (2) the steel beams which may be rolled beams, rolled beams with cover plates or built-up plate girders, and (3) shear connectors which provide the necessary horizontal and

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vertical connection between the slab and the beams. The design of the slab is independent of the composite action and is carried out in the same manner as for a noncomposite structure. The design of the slab precedes the design of the beams; the design of shear connectors, not covered in this paper, is carried out after the design of the beams.

The design of the composite beams is usually based on the allowable stresses for the component materials. Tests have shown that with properly designed shear connectors complete interaction between the concrete slab and the steel beam can be assumed.[3] Therefore, a composite T-beam may be designed by the theory of transformed section. It is usually most convenient to transform the effective area of the concrete slab into an equivalent steel area; this is done by dividing the effective slab area by the modular ratio n. The design calculations are then carried out as for a monolithic section.

Three principal types of loads are generally encountered in the design of composite T-beams: (1) noncomposite dead loads, (2) composite dead loads including the effects of creep of concrete, and (3) live loads. Deformational loads such as those caused by shrinkage and temperature are secondary loads. Higher allowable stresses are permitted by specifications when the effects of secondary loads are accounted for in the design. Ordinarily, the deformational loads do not result in the critical stress condition and, therefore, are not considered in the method of selecting the cross-section presented in this paper.

The noncomposite dead loads are loads applied to the steel beam before the slab has set, such as the weight of the steel beam and of the concrete slab. These loads are resisted by the steel beams alone.

The composite dead loads are loads placed on the structure after the slab has set, such as floor finish and permanent partitions in buildings or wearing surface, safety curbs and handrails in bridges. The composite dead loads set up permanent stresses in the slab causing the concrete to creep. The effect of creep is to relieve a part of the stresses in the concrete slab and to increase the stresses in the steel beams. The effects of creep on the steel stresses may be accounted for approximately by an increase of the modular ratio n.[4] Thus in computing the properties of and the stresses in the composite section for composite dead loads, the modular ratio n should be multiplied by a numerical factor k; value of $k = 3$ is used in this paper.

The live loads are usually temporary loads; as such, they do not cause any appreciable creep. Accordingly, the ordinary value of the modular ratio n should be used in computing the properties of cross-section and the stresses caused by live loads. If, however, the live loads are expected to be of long duration, such as loads in warehouses, the value of the modular ratio should be increased. In bridge design, the live loads should include an allowance for impact.

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3. "Studies of Slab and Beam Highway Bridges, Part IV: Full-Scale Tests of Channel Shear Connectors and Composite T-Beams," by I. M. Viest, C. P. Siess, J. H. Appleton and N. M. Newmark, University of Illinois Engineering Experiment Station Bulletin 405, 1952.
4. "Composite Construction for I-Beam Bridges," by C. P. Siess, Transactions, ASCE, Vol. 114, 1949, pp. 1023-1045.

Section Properties—Exact Formulas

The following formulas for the properties of composite T-beams are derived assuming linear stress distribution and no slip between the slab and the steel beam. The first assumption implies that the formulas are applicable only in the elastic range of stresses; a working load design falls into such range. The second assumption can be satisfied only approximately; it requires a shear connection which is capable to transfer horizontal shear at very small deformations of the shear connectors.

For beams having a concrete haunch or fillet between the steel beam and the slab proper, the contribution of the concrete in the haunch to the load resisting capacity of the T-beam is neglected to avoid unnecessary complexity of the resulting formulas.

Properties of Steel Section

A portion or all of the dead load is usually carried by the steel section alone. If the steel section consists of a rolled beam only, the section properties of the steel beam may be found in a steel handbook.[5] For a rolled beam with a tension cover plate and for an unsymmetrical built-up girder, often used for composite beams, the section properties must be computed.

Following the notation of Fig. 1 and considering the equilibrium of statical moments with respect to the neutral axis of the rolled beam, the distance $\overline{y}_s$ may be expressed as

$$\overline{y}_s = \frac{1}{2}(d + t_p)\frac{A_p}{A_B + A_p},$$

where A_B is the area of the rolled beam and A_p is the area of the cover plate. The moment of inertia with respect to the neutral axis of the unsymmetrical cross-section may then be expressed as

$$I_s = A_p\left[\frac{1}{2}(d + t_p)\right]^2 + I_B + I_p - (A_p + A_B)\,\overline{y}_s^{\,2},$$

where I_B and I_p are the moments of inertia of the rolled beam and the cover plate, respectively, about their neutral axes. Substitution for $\overline{y}_s$ and rearrangement leads to the following equation for the moment of inertia:

$$I_s = \left[\frac{1}{2}(d + t_p)\right]^2 \frac{A_B\,A_p}{A_B + A_p} + I_B + I_p.$$

Neglecting the small quantity I_p, observing the notation in Fig. 1, and letting

$$K_s = \frac{A_p}{A_B + A_p} = \frac{A_p}{A_s}, \tag{1a}$$

5. "Steel Construction," American Institute of Steel Construction, New York, N. Y.

the formulas for the properties of a rolled beam with tension cover plate may be expressed as follows:

$$\overline{y}_s = \frac{1}{2} (d + t_p) K_s \tag{1b}$$

$$I_s = \frac{1}{2} \overline{y}_s (d + t_p) A_B + I_B \tag{1c}$$

$$y_{ts} = \frac{1}{2} d + \overline{y}_s \tag{1d}$$

$$y_{bs} = \frac{1}{2} d + t_p - \overline{y}_s . \tag{1e}$$

The section modulus for the top and bottom extreme fibers and the stresses may be computed from the well known relationships $S = I/y$ and $s = M/S$ for the section modulus and stress, respectively.

The section properties of an unsymmetrical built-up girder may be evaluated in similar manner as for the rolled beam with tension cover plate. Following the notation of Fig. 2, the equilibrium of statical moments about the half-depth of the web gives the location of the neutral axis as

$$\overline{y}_s = \frac{(d_w + t_b) A_b - (d_w + t_t) A_t}{2 A_s} , \tag{2a}$$

where A_b is the area of the bottom flange, A_t the area of the top flange and A_s the area of the whole steel section. Neglecting the moments of inertia of the top and bottom flanges with respect to their own axes as very small quantities, the moment of inertia of the unsymmetrical built-up girder may be written as

$$I_s = \frac{1}{4} \left[(d_w+t_b)^2 A_b + (d_w+t_t)^2 A_t \right] + I_w - A_s \overline{y}_s^{\,2} , \tag{2b}$$

where I_w is the moment of inertia of the web with respect to its own neutral axis. The distances between the neutral axis of the whole cross-section and the extreme fibers are then

$$y_{ts} = \frac{1}{2} d_w + t_t + \overline{y}_s \tag{2c}$$

$$y_{bs} = \frac{1}{2} d_w + t_b - \overline{y}_s . \tag{2d}$$

The section moduli and stresses are computed in the same manner as for rolled beams with cover plates.

Properties of Composite Section

A portion of or all dead loads and all live loads are carried by the composite section. The properties of the composite section are a function of the

modular ratio n multiplied by the numerical factor k. The factor k is used to account for the effects of creep. For loads of short duration (temporary loads), such as moving vehicles, no creep effects are present, so that $k = 1$. For sustained loads, such as dead loads, the effects of creep may be accounted for by using $k = 3$.

In composite sections, the neutral axis may be located either below or in the slab. If it is located below the slab, as shown in Fig. 3, the full cross-section of the slab is effective in resisting the stresses. Neglecting the effects of the slab reinforcement, the transformed area of the slab is

$$A_c = \frac{bt}{kn} . \tag{3a}$$

Letting

$$K_c = \frac{A_c}{A_c + A_s} , \tag{3b}$$

the location of the neutral axis of the composite section may be found from the equilibrium of statical moments with respect to the neutral axis of the steel section:

$$\bar{y}_c = (y_{ts} + e_c)\, K_c . \tag{3c}$$

The moment of inertia of the composite section may be expressed as

$$I_c = A_c\, (y_{ts}+e_c)^2 + I_s + A_c\, \frac{t^2}{12} - (A_c+A_s)\, \bar{y}_c^{\,2} ,$$

where I_s is the moment of inertia of the steel section about its own neutral axis. Substituting for $\bar{y}_c$ from Eq. 3c and 3b and rearranging results in the following expression for the moment of inertia:

$$I_c = \bar{y}_c\, (y_{ts}+e_c)\, A_s + I_s + A_c\, \frac{t^2}{12} . \tag{3d}$$

The distances to the extreme fibers may be obtained from Fig. 3:

$$y_{tc} = y_{ts} - \bar{y}_c \tag{3e}$$

$$y_{bc} = y_{bs} + \bar{y}_c \tag{3f}$$

$$y_{cc} = y_{tc} + e_c + \frac{t}{2} . \tag{3g}$$

In the same manner as for the steel section, the section moduli and the

stresses in the composite section may be found from the relationships $S = I/y$ and $s = M/S$, respectively, by substituting the proper values of I (Eq. 3d), y (Eq. 3e-g) and M. It should be noted that slab stresses computed in this manner are those in the transformed section; to obtain the actual stresses in the concrete slab, the resulting values must be divided by the value kn.

Another section property needed in the design of composite structures is the statical moment m of the transformed area located at one side of the top beam surface. This quantity is needed in computing the horizontal shear which has to be transferred by shear connectors from the slab to the beam. Since the transformed area of the slab is A_c and the distance from the slab centroid to the neutral axis of the composite section is $(y_{tc} + e_c)$, the statical moment is

$$m = A_c\ (y_{tc} + e_c). \tag{3h}$$

In bridge designs, the neutral axis of the composite section is almost invariably located below the slab. In light beams with relatively heavy slabs, occasionally encountered in building construction, the neutral axis may be located in the slab. Assuming that the concrete is ineffective in resisting tension, neglecting the slab reinforcement and using the notation of Fig. 4, the position of the neutral axis may be found by writing static moments with respect to the neutral axis of the composite section:

$$y_{cc}\ \frac{b}{kn}\ \frac{y_{cc}}{2} = (y_{cs} - y_{cc})\ A_s.$$

The distance y_{cc} is the only unknown; it may be evaluated as

$$y_{cc} = \frac{kn\ A_s}{b}\left[\left(1 + \frac{2y_{cs}}{\frac{kn\ A_s}{b}}\right)^{\frac{1}{2}} - 1\right]. \tag{4a}$$

Letting

$$A_c = \frac{y_{cc}b}{kn}, \tag{4b}$$

the moment of inertia of the composite section may be written directly as

$$I_c = A_c\ \frac{(y_{cc})^2}{3} + A_s\ (\overline{y}_c)^2 + I_s \tag{4c}$$

where $\overline{y}_c = y_{cs} - y_{cc}$. The distances to the extreme fibers are then

$$y_{tc} = \overline{y}_c - y_{ts} \tag{4d}$$

$$y_{bc} = \overline{y}_c + y_{bs}. \tag{4e}$$

The statical moment of the transformed area located on one side of the top beam surface is then

$$m = \frac{1}{2} A_c y_{cc}. \tag{4f}$$

The section moduli and stresses are computed in the same manner as is described for the case of neutral axis in the slab.

Application of Eq. 3 and 4.

For any particular composite cross-section, the position of neutral axis may be determined from Eq. 3c or 4a. The section properties may then be computed from Eq. 3 if the neutral axis is located below the slab or from Eq. 4 if the neutral axis is located in the slab. The question under what conditions is the neutral axis located in the slab was investigated for cross-sections composed of a symmetrical steel beam and a slab resting directly on the top of the steel beam. If the steel beam is unsymmetrical or a fillet is inserted between the steel beam and the slab proper, the likelihood of the neutral axis entering the slab is decreased. The studies were extended also to the question of the magnitude of difference in the results given by Eq. 3 and 4 when the neutral axis is in the slab. The results are shown in Fig. 5.

From the standpoint of design, the most significant property of the cross-section is the section modulus of the bottom flange, S_{bc}. It can be shown that the ratio of the section modulus computed from Eq. 4, S^n_{bc}, to the section modulus computed from Eq. 3, S^g_{bc} is a function of two quantities only:

$$K_c = \frac{A_c}{A_c + A_s} \quad \text{and} \quad \frac{d}{t},$$

where $A_c = bt/kn$, d is the depth of the steel beam and t is the thickness of the slab.[6] The full dots in Fig. 5 represent the condition at which the neutral axis is located at the bottom face of the slab: they correspond to $K_c = 0.5$ for $d/t = 1.0$, $K_c = 0.667$ for $d/t = 2$ and $K_c = 0.75$ for $d/t = 3$. But $K_c = 0.5$, 0.667 and 0.75 correspond to $A_c/A_s = 1$, 2 and 3, respectively. That is, the neutral axis is located in the slab only when

$$\frac{d}{t} < \frac{A_c}{A_s}.$$

It can be seen in Fig. 5, that as K_c increases beyond the limiting value, the ratio S^n_{bc}/S^g_{bc} decreases from 1.0 slowly. For

6. Equations 3 involve the total, or gross, area of the slab: therefore the symbol S^g_{bc} is used. Equations 4 involve only the slab area subjected to compressive stresses, the so called net area: therefore S^n_{bc} is used.

$$\frac{d}{t} = \frac{1}{3}\ \frac{A_c}{A_s}\ ,$$

shown in Fig. 5 as circles and corresponding to $K_c = 0.75$ for $d/t = 1$, $K_c = 0.857$ for $d/t = 2$ and $K_c = 0.9$ for $d/t = 3$, the S^n_{bc} is only about 3.5 percent smaller than S^g_{bc}. Accordingly, it is sufficiently accurate for design purposes to compute the section properties from the simpler Eq. 3 whenever

$$\frac{d}{t} > \frac{1}{3}\ \frac{A_c}{A_s}\ . \tag{5}$$

The limitation given by Eq. 5 is invariably satisfied in bridge design. It is ordinarily satisfied also in building design so that Eq. 4 are needed only in very exceptional cases.

Approximate Formulas for Section Modulus

The design of a composite T-beam is governed by allowable stresses. The stresses in the slab are ordinarily substantially below the allowable value for concrete in compression; accordingly, the allowable steel stresses for the extreme top and bottom flange steel fibers govern the size of the cross-section. It has been pointed out that, in general, three types of loads must be considered. Thus, equating the actual stresses caused by the three types of load to the allowable value f_a, the following equation may be written:

$$f_a = \frac{M_{DLs}}{S_s} + \frac{M_{DLc}}{S_c} + \frac{M_{LL}}{S_c}\ ,$$

where S_s and S_c are the section moduli of the steel section and of the composite section, respectively. Multiplying both sides by the ratio A^s_s/f_a, where A^s_s is the area of a symmetrical steel section,[7] the area A^s_s may be expressed as the sum of three contributory areas

$$A^s_s = A_{DLs} + A_{DLc} + A_{LL}, \tag{6}$$

where

$$A_{DLs} = \frac{M_{DLs}}{f_a}\ \frac{A^s_s}{S_s}$$

7. The area A^s_s for various types of steel sections is defined in later portions of the paper.

$$A_{DLc} = \frac{M_{DLc}}{f_a} \frac{A_s^s}{S_c}$$

$$A_{LL} = \frac{M_{LL}}{f_a} \frac{A_s^s}{S_c} .$$

The terms M/f_a in Eq. 6 are known quantities. The terms A_s^s/S are not known but can be evaluated rapidly for a composite section of known dimensions as is shown below.

S/A_s^s—Values for Rolled Beams

For a composite beam with the neutral axis below the slab, shown in Fig. 6, the following dimensionless expressions may be written with the aid of Eq. 3:

$$\frac{\overline{y}_c}{d} = (\frac{1}{2} + \frac{e_c}{d}) K_c$$

$$\frac{I_c}{A_s^s d^2} = (\frac{1}{2} + \frac{e_c}{d})^2 K_c + \frac{I_s}{A_s^s d^2} + \frac{1}{12} \frac{A_c}{A_s^s} (\frac{t}{d})^2$$

$$\frac{S_{tc}}{A_s^s d} = \frac{1}{\frac{1}{2} - \frac{\overline{y}_c}{d}} \frac{I_c}{A_s^s d^2}; \quad \frac{S_{bc}}{A_s^s d} = \frac{1}{\frac{1}{2} + \frac{\overline{y}_c}{d}} \frac{I_c}{A_s^s d^2} .$$

The quantity $\frac{1}{12} \frac{A_c}{A_s^s} (\frac{t}{d})^2$ is small and may be neglected in preliminary calculations. The quantity $I_s/A_s^s d^2$ is the square of the ratio of the radius of gyration to the depth of the steel section. This ratio depends only on the shape of the cross-section so that for all steel sections of the same shape it is a constant quantity. For rolled sections, it is approximately equal to 0.165.

Using these simplifications and observing that for a symmetrical section $S = 2I/d$, the following approximate expressions may be written for the S/A_s^s ratios for a composite beam with rolled steel section:

$$\frac{S_{bs}}{A_s^s d} = \frac{S_{ts}}{A_s^s d} = \frac{S_s}{A_s^s d} = 2 \frac{I_s}{A_s^s d^2} \cong 0.330 \qquad \text{(7a)}$$

$$\frac{S_{tc}}{A_s^s d} \cong \frac{(\frac{1}{2} + \frac{e_c}{d})^2 K_c + 0.165}{\frac{1}{2} - (\frac{1}{2} + \frac{e_c}{d}) K_c} \qquad \text{(7b)}$$

$$\frac{S_{bc}}{A_s^s d} \cong \frac{(\frac{1}{2} + \frac{e_c}{d})^2 \quad K_c + 0.165}{\frac{1}{2} + (\frac{1}{2} + \frac{e_c}{d})\, K_c} \quad . \tag{7c}$$

The terms $\frac{e_c}{d}$ and $K_c = \frac{A_c}{A_c + A_s}$ in Eq. 7b and 7c may be evaluated if the depth and area of the steel section are known approximately. It is convenient to plot Eq. 7b and 7c as is shown in Fig. 7. The S/A_s^s—terms for Eq. 6 may then be evaluated rapidly by multiplying the values obtained from Eq. 7a and Fig. 7 with the depth of the steel section d.

S/A_s^s—Values for Built-up Girders

Equations 2 for the section properties of a built-up girder may be simplified without any significant decrease of accuracy if it is assumed that the top and bottom flanges have equal thickness. Letting

$$d' = d_w + \frac{t_t + t_b}{2}$$

and noting that for $t_b = t_t$

$$d_w + t_b = d_w + t_t = d' \ ,$$

Eq. 2 may be written in the following form:

$$\overline{y}_s = \frac{d'}{2} \frac{A_b - A_t}{A_s}$$

$$I_s = (\frac{d'}{2})^2 \left\{ \left[A_b + A_t + \frac{1}{3} A_w \, (\frac{d_w}{d'})^2 \right] A_s - (A_b - A_t)^2 \right\} \frac{1}{A_s}$$

$$y_{ts} = \frac{d'}{2} (\frac{d}{d'} + \frac{A_b - A_t}{A_s})$$

$$y_{bs} = \frac{d'}{2} (\frac{d}{d'} - \frac{A_b - A_t}{A_s}) \ .$$

From these, the section moduli for the top and the bottom flange may be expressed

$$S_{ts} = \frac{d'}{2} \frac{\left[A_b + A_t + \frac{1}{3} A_w (\frac{d_w}{d'})^2 \right] A_s - (A_b - A_t)^2}{A_s \frac{d}{d'} + (A_b - A_t)}$$

$$S_{bs} = \frac{d'}{2} \frac{\left[A_b + A_t + \frac{1}{3} A_w \, (\frac{d_w}{d'})^2 \right] A_s - (A_b - A_t)^2}{A_s \frac{d}{d'} - (A_b - A_t)} \ .$$

Noting that $d \cong d'$, $d_w \cong d'$ and $A_s = A_b + A_t + A_w$, the expressions for the section moduli may be simplified further and written as

$$S_{ts} \cong \frac{d'}{2} \frac{(A_b + A_t + \frac{1}{3} A_w) A_s - (A_b - A_t)^2}{A_w + 2A_b}$$

$$S_{bs} \cong \frac{d'}{2} \frac{(A_b + A_t + \frac{1}{3} A_w) A_s - (A_b - A_t)^2}{A_w + 2A_t} .$$

An examination of the substitutions shows that they increase the computed values of the section modulus. This increase may be compensated for by substituting $d_w/2$ for $d'/2$. Dividing the first equation by $(A_w + 2A_t)$ and the second equation by $(A_w + 2A_b)$ and rearranging gives the following approximate expressions for the S/A_s^s- terms for a built-up girder alone:

$$\frac{S_s}{A_s^s d_w} = \frac{S_{ts}}{(A_w + 2A_t) d_w} = \frac{S_{bs}}{(A_w + 2A_b) d_w} \cong \frac{1}{2} \left(\frac{\frac{1}{6} + \frac{A_t}{A_w}}{1 + 2\frac{A_t}{A_w}} + \frac{\frac{1}{6} + \frac{A_b}{A_w}}{1 + 2\frac{A_b}{A_w}} \right) \tag{8a}$$

The section moduli of a composite section made with a built-up girder may be computed from Eq. 3 in combination with Eq. 2. Using again the depth d' as described above and neglecting the small quantity $A_c t^2/12$, the properties of the composite section may be evaluated from the following equations:

$$\bar{y}_c = \left[\frac{d'}{2} \left(\frac{d}{d'} + \frac{A_b - A_t}{A_s} \right) + e_c \right] K_c$$

$$I_c = \left[\frac{d'}{2} \left(\frac{d}{d'} + \frac{A_b - A_t}{A_s} \right) + e_c \right]^2 K_c A_s + S_{ts} \frac{d'}{2} \left(\frac{d}{d'} + \frac{A_b - A_t}{A_s} \right)$$

$$= \left[\frac{d'}{2} \left(\frac{d}{d'} + \frac{A_b - A_t}{A_s} \right) + e_c \right]^2 K_c A_s + S_{bs} \frac{d'}{2} \left(\frac{d}{d'} - \frac{A_b - A_t}{A_s} \right)$$

$$y_{tc} = \frac{d'}{2} \left(\frac{d}{d'} + \frac{A_b - A_t}{A_s} \right) - \left[\frac{d'}{2} \left(\frac{d}{d'} + \frac{A_b - A_t}{A_s} \right) + e_c \right] K_c$$

$$y_{bc} = \frac{d'}{2} \left(\frac{d}{d'} - \frac{A_b - A_t}{A_s} \right) + \left[\frac{d'}{2} \left(\frac{d}{d'} + \frac{A_b - A_t}{A_s} \right) + e_c \right] K_c .$$

The corresponding expressions for the section moduli are then

$$S_{tc} = d' \frac{\left[\frac{1}{2}\left(\frac{d}{d'} + \frac{A_b - A_t}{A_s}\right) + \frac{e_c}{d'}\right]^2 K_c A_s + \frac{S_{ts}}{2d'}\left(\frac{d}{d'} + \frac{A_b - A_t}{A_s}\right)}{\frac{1}{2}\left(\frac{d}{d'} + \frac{A_b - A_t}{A_s}\right) - \left[\frac{1}{2}\left(\frac{d}{d'} + \frac{A_b - A_t}{A_s}\right) + \frac{e_c}{d'}\right] K_c}$$

$$S_{bc} = d' \frac{\left[\frac{1}{2}\left(\frac{d}{d'} + \frac{A_b - A_t}{A_s}\right) + \frac{e_c}{d'}\right]^2 K_c A_s + \frac{S_{bs}}{2d'}\left(\frac{d}{d'} - \frac{A_b - A_t}{A_s}\right)}{\frac{1}{2}\left(\frac{d}{d'} - \frac{A_b - A_t}{A_s}\right) + \left[\frac{1}{2}\left(\frac{d}{d'} + \frac{A_b - A_t}{A_s}\right) + \frac{e_c}{d'}\right] K_c} .$$

An examination of the expressions for S_{tc} and S_{bc} shows that setting simultaneously $d/d' = 1.0$ and $d' = d_w$ does not affect significantly the resulting values. Introducing both of these approximations, dividing the S_{tc} equation by $(A_w + 2A_t)\, d_w$ and the S_{bc} equation by $(A_w + 2A_b)\, d_w$, and rearranging leads to the following approximate S/A_s^s- expressions for a composite beam with built-up steel girder:

$$\frac{S_{tc}}{A_s^s d_w} = \frac{S_{tc}}{(A_w + 2A_t) d_w} \cong \frac{\left(\frac{1}{2} K_b + \frac{e_c}{d_w}\right)^2 \frac{K_c}{K_t} + \frac{S_{ts}}{2(A_w + 2A_t) d_w} K_b}{\frac{1}{2} K_b - \left(\frac{1}{2} K_b + \frac{e_c}{d_w}\right) K_c} \tag{8b}$$

$$\frac{S_{bc}}{A_s^s d_w} = \frac{S_{bc}}{(A_w + 2A_b) d_w} \cong \frac{\left(\frac{1}{2} K_b + \frac{e_c}{d_w}\right)^2 \frac{K_c}{K_b} + \frac{S_{bs}}{2(A_w + 2A_b) d_w} K_t}{\frac{1}{2} K_t + \left(\frac{1}{2} K_b + \frac{e_c}{d_w}\right) K_c} , \tag{8c}$$

where

$$K_b = \frac{1 + 2\frac{A_b}{A_w}}{1 + \frac{A_b}{A_w} + \frac{A_t}{A_w}}$$

$$K_t = \frac{1 + 2\frac{A_t}{A_w}}{1 + \frac{A_b}{A_w} + \frac{A_t}{A_w}}$$

$$K_c = \frac{A_c}{A_c + A_s}$$

It may be noted that Eq. 8 may be evaluated if the area of the web and the ratios A_b/A_w, A_t/A_w are known approximately. Although the equations are cumbersome, they can be used readily if presented in graphical form in terms of the dimensionless parameters K_c, e_c/d_w, A_b/A_w and A_t/A_w as shown in Fig. 8.

The S/A_s^s- terms for Eq. 6 may be evaluated rapidly from graphs, such as those shown in Fig. 8, by multiplication with the depth of the web d_w. The curves in Fig. 8a cover the full practical ranges of all variables. The curves in Fig. 8b and 8c pertain only to $A_t/A_w = 0.3$ and $A_b/A_w = 1.1$. In using a set of graphs having A_t/A_w and A_b/A_w in increments of 0.3, no interpolation is necessary between the graphs. For e_c/d_w in any particular graph, straight line interpolation is sufficiently accurate, if e_c/d_w is plotted in 0.1 increments.

Selection of Cross-Section

The slab of a composite cross-section is designed for the transverse load distribution so that its dimensions are known in advance of the design of the composite T-beam. Thus only the steel section needs to be selected. In general, the selection involves three steps: (1) assume a steel section (or some of its characteristics), (2) compute the necessary steel area from Eq. 6 with the aid of S/A_s^s- terms evaluated as described in the preceding section, and (3) complete or revise the assumed steel section according to the computed area. If the computed area differs radically from the assumed one, the procedure should be repeated with the computed area.

The stresses in the selected cross-section should be computed with the exact formulas. Ordinarily, the resulting stresses are close to the allowable values so that no further revisions of the cross-section are necessary.

Rolled Beams Without Cover Plate

The size of a composite section composed of a symmetrical rolled section and of a slab on the compression side is governed by the tensile flange steel stresses. The neutral axis is located closer to the compression flange of the steel beam, so that the stresses in that flange are smaller than those in the tension flange.

To select the rolled section, first the required section moduli, M/f_a, are computed for all three types of loads (see terms in Eq. 6). Next, assume the size of the rolled beam area $A_B = A_s = A_s^s$ and the depth d;[8] these, with the known values of e_c and A_c, give the parameters e_c/d and K_c.[9] The corresponding $S_{bc}/A_s^s d$ values are obtained from Fig. 7, the necessary steel area computed from Eq. 6 and a beam having the assumed depth and the computed

8. It is convenient and usually sufficiently accurate to select a rolled beam two or three sizes smaller than that needed for a noncomposite beam.
9. There are two K_c parameters: one for composite dead loads (k = 3) and the other for live loads (k = 1).

area selected. As long as the difference between the assumed and computed steel area does not exceed about 20 percent, there is no need for repeating the approximate calculations.

Rolled Beams with Cover Plate

The selection of the composite section having a rolled beam with steel cover plate on the tension side is governed by both the maximum tensile and maximum compressive steel stresses. The size of the rolled section is usually selected on the basis of the top flange (compressive) stress and the size of the cover plate is chosen on the basis of the bottom flange stress.

In the same way as for a rolled beam without cover plate, the M/f_a- terms are computed, the area of the rolled beam A_B and the depth d assumed,[10] and the parameters e_c/d and K_c computed.[11] The corresponding $S_{tc}/A_s^s d$ values are obtained from Fig. 7 and the necessary area computed from Eq. 6. A rolled section having the assumed depth and the computed area is then selected.

The needed area of the cover plate is found by computing the area of a symmetrical steel section corresponding to the allowable bottom flange stress, i.e., by using $S_{bc}/A_s^s d$ values from Fig. 7. Then the difference (A_s^s - A_B) corresponds to the increase of the steel section over that necessary for the top flange stress. Since the cover plate is needed only on the bottom flange, its area is

$$A_p = \frac{1}{2} (A_s^s - A_B). \tag{9}$$

The procedure is based on the observation that the quantities $S_{bs}/A_s^s d$ and $S_{bc}/A_s^s d$ are affected very little by the addition of a cover plate. In designs requiring a relatively large cover plate, the error caused by neglecting the effect of the cover plate on the S/A_s^s- values may be a few percent; if desired, it may be compensated for by choosing a cover plate size 1/16 in. thinner than that indicated by the computations.

The values of $S_{ts}/A_s^s d$ and $S_{tc}/A_s^s d$ are more sensitive to the addition of the cover plate. As a result, the preliminary design for the top flange is not as accurate as for the bottom flange. The inaccuracy, however, is on the conservative side and a correction can be introduced if the designer so desires.

The procedure for selecting the cross-section of a composite beam made up of a slab and a rolled steel beam with cover plate is illustrated in Example No. 1 in Appendix I.

Built-up Girders

The selection of the cross-section of a built-up beam for a composite structure is usually governed by the maximum compressive and maximum tensile steel stresses. The compressive stress determines the size of the top flange and the tensile stress determines the size of the bottom flange.

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10. The size of the rolled beam is usually four or more sizes smaller than that needed for a noncomposite beam.

11. It is sufficiently accurate to use $K_c = \dfrac{A_c}{A_c + A_B}$.

After computing the required section moduli, M/f_a, it is convenient to assume the depth of the web and the ratios of the top flange to web and of the bottom flange to web areas.[12] Selecting the web thickness in relation to the assumed depth permits computation of the web area, of the assumed total steel area A_s, and of parameters e_c/d_w and K_c. The corresponding S/A_s^s-values can then be evaluated from graphs such as those shown in Fig. 8.

Using Fig. 8a and 8b, for built-up girders the quantity A_s^s in Eq. 6 is equal to $(A_w + 2A_b)$. The necessary area for the bottom flange may, therefore, be computed as

$$A_b = \frac{1}{2}\left[(A_w + 2A_b) - A_w\right]. \tag{10a}$$

Similarly, using Fig. 8a and 8c, $A_s^s = (A_w + 2A_t)$ and the necessary area of the top flange is equal to

$$A_t = \frac{1}{2}\left[(A_w + 2A_t) - A_w\right]. \tag{10b}$$

The procedure for selecting the size of a plate girder is illustrated in Example No. 2 in Appendix I.

CONCLUDING REMARKS

The method presented in this paper permits a rapid selection of the steel section for a composite concrete and steel T-beam. It is based on simplified general formulas for the section properties presented in the form of graphs.

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APPENDIX I - DESIGN EXAMPLES

Example No. 1. - This example represents the design of an interior girder of an eighty-foot simple span highway bridge subjected to H20-S16 loading. The cross-section of the T-beam is shown in Fig. 9: the effective slab is 72 x 6 in., the steel beam a rolled section with tension cover plate.

12. For bridge plate girders of spans longer than 80 ft. $A_t/A_w = 0.3$ and $A_b/A_w = 0.8$ usually represent good first estimates. For shorter spans, $A_t/A_w = 0.3$ and $A_b/A_w = 1.1$ are preferable.

$M_{DLs} = 571 \text{ k-ft.} \times \frac{12}{18} = 381 \text{ in}^3$ $n = 10$

$M_{DLc} = 144 \text{ k-ft.} \times \frac{12}{18} = 96 \text{ in}^3$ $f_c = 1200$ psi

$M_{LL} + \text{Impact} = 876 \text{ k-ft.} \times \frac{12}{18} = 584 \text{ in}^3$ $f_s = 18{,}000$ psi

Assume 36 WF 150: $A_s = 44.16 \text{ in.}^2$, $d = 35.84$ in.,
$I_s = 9012.1 \text{ in.}^4$, $S_s = 502.9 \text{ in.}^3$

$\frac{e_c}{d} = \frac{3.00}{35.84} = 0.0837$ $\frac{S_s}{A_s} = \frac{502.9}{44.16} = 11.39$

$k_{DLc} = 3$ $k_{LL} = 1$

$A_c = \frac{72.00 \times 6.00}{3 \times 10} = 14.40 \text{ in}^2$ $A_c = \frac{72.00 \times 6.00}{1 \times 10} = 43.20 \text{ in}^2$

$K_c = \frac{14.40}{14.40 + 44.16} = 0.246$ $K_c = \frac{43.20}{43.20 + 44.16} = 0.495$

From Figure 7

$\frac{A_s^s d}{S_{tc}} = \quad = 1.43$ $\frac{A_s^s d}{S_{tc}} = \quad = 0.64$

$\frac{S_{bc}}{A_s^s d} = \quad = 0.387$ $\frac{S_{bc}}{A_s^s d} = \quad = 0.423$

Top flange Bottom flange

$A_{DLs} = \frac{381}{11.39} = 33.4 \text{ in}^2$ $A_{DLs} = \frac{381}{11.39} = 33.4 \text{ in}^2$

$A_{DLc} = \frac{96 \times 1.43}{35.84} = 3.8 \text{ in}^2$ $A_{DLc} = \frac{96}{0.387 \times 35.84} = 6.9 \text{ in}^2$

$A_{LL} = \frac{584 \times 0.64}{35.84} = \underline{10.4 \text{ in}^2}$ $A_{LL} = \frac{584}{0.423 \times 35.84} = \underline{38.4 \text{ in}^2}$

Req. $A_B = 47.6 \text{ in}^2$ Req. $A_s^s = 78.7 \text{ in}^2$

Try 36 WF 150: $A_B = 44.2 \text{ in}^2$ $\quad A_B =$ $\quad -\,44.2 \text{ in}^2$

34.5 in^2

$$\text{Req. } A_p = \frac{34.5}{2} = 17.2 \text{ in}^2$$

Try 10 1/2" x 1 9/16":

$A_p = \quad 16.41 \text{ in}^2$

Properties of trial section

Steel section:

$$K_s = \frac{16.41}{16.41 + 44.16} = 0.271$$

$$\bar{y}_s = \frac{1}{2}(35.84+1.56)\ 0.271 = 5.07 \text{ in}$$

$$I_s = (5.07)(18.70)\ 44.16+9012.1 = 13{,}200 \text{ in}^4$$

$y_{ts} = 17.92 + 5.07 = 22.99$ in $\qquad S_{ts} = 574 \text{ in}^3$

$y_{bs} = 17.92 - 5.07 + 1.56 = 14.41$ in $\qquad S_{bs} = 916 \text{ in}^3$

Composite section: $k = 3$

$$K_c = \frac{14.40}{14.40 + 60.57} = 0.1920$$

$$\bar{y}_c = (22.99 + 3.00)\ 0.1920 = 4.99 \text{ in}$$

$$I_c = (4.99)(25.99)\ 60.57+13200+ \frac{14.40x(6.00)^2}{12} = 21{,}100 \text{ in}^4$$

$y_{tc} = 22.99 - 4.99 = 18.00$ in $\qquad S_{tc} = 1172 \text{ in}^3$

$y_{bc} = 14.41 + 4.99 = 19.40$ in $\qquad S_{bc} = 1088 \text{ in}^3$

$y_{cc} = 22.99 - 4.99 + 6.00 = 24.00$ in $\qquad S_{cc} = 879 \text{ in}^3$

Composite section: $k = 1$

$$K_c = \frac{43.20}{43.20 + 60.57} = 0.416$$

$\overline{y}_c = (25.99)\ 0.416 \qquad = 10.82 \text{ in}$

$I_c = (10.82)(25.99)\ 60.57 + 13200 + \dfrac{43.20 \times (6.00)^2}{12} = 30{,}400 \text{ in}^4$

$y_{tc} = 22.99 - 10.82 \qquad = 12.17 \text{ in} \qquad S_{tc} = 2500 \text{ in}^3$

$y_{bc} = 14.41 + 10.82 \qquad = 25.23 \text{ in} \qquad S_{bc} = 1203 \text{ in}^3$

$y_{cc} = 22.99 - 10.82 + 6.00 \qquad = 18.17 \text{ in} \qquad S_{cc} = 1671 \text{ in}^3$

Stresses	f_c	f_t	f_b
D.L.s.	$= 0$	$\dfrac{571 \times 12}{574} = 11.93$	$\dfrac{6850}{916} = 7.48$
D.L.c.	$\dfrac{144 \times 12}{879 \times 30} = 0.07$	$\dfrac{1723}{1172} = 1.47$	$\dfrac{1723}{1088} = 1.58$
L.L.	$\dfrac{876 \times 12}{167 \times 10} = 0.63$	$\dfrac{10510}{2500} = 4.21$	$\dfrac{10510}{1203} = 8.74$
	0.70 ksi	17.61 ksi	17.80 ksi

Example No. 2. - This example represents the same problem as in Example No. 1 but the T-beam is designed with an unsymmetrical welded plate girder. The cross-section of the T-beam is shown in Fig. 10. Except for noncomposite dead load (M_{DLs} = 544 k-ft., required section modulus = 363 in^3), the moments, required section moduli and allowable stresses are the same as in Example No. 1.

Assume: $d_w = 48 \text{in.}$, $t_w = 3/8 \text{ in.}$, $\dfrac{A_t}{A_w} = 0.3$, $\dfrac{A_b}{A_w} = 1.1$,
$A_w = 18.00 \text{ in}^2$

$$A_s = (1 + 0.3 + 1.1)\ 18.00 = 43.20 \text{ in}^2$$

From Figure 8a

$$\frac{e_c}{d_w} = \frac{3.00}{48.00} = 0.0625 \qquad \frac{S_{ts}}{(A_w + 2A_t)\ d_w} = 0.343$$

$k_{DLc} = 3 \qquad k_{LL} = 1$

$A_c = 14.40 \text{ in}^2$ $\quad$ $A_c = 43.20 \text{ in}^2$

$K_c = \dfrac{14.40}{14.40+43.20} = 0.250$ $\quad$ $K_c = \dfrac{43.20}{43.20+43.20} = 0.500$

From Figure 8

$\dfrac{S_{bc}}{(A_w + 2A_b)\, d_w} = 0.430$ $\quad$ $\dfrac{S_{bc}}{(A_w + 2A_b)\, d_w} = 0.453$

$\dfrac{(A_w + 2A_t)\, d_w}{S_{tc}} = 1.15$ $\quad$ $\dfrac{(A_w + 2A_t)\, d_w}{S_{tc}} = 0.49$

Top Flange	Bottom Flange
$A_{DLs} = \dfrac{363}{0.343 \times 48} = 22.1 \text{ in}^2$	$A_{DLs} = \dfrac{363}{0.343 \times 48} = 22.1 \text{ in}^2$
$A_{DLc} = \dfrac{96 \times 1.15}{48} = 2.3 \text{ in}^2$	$A_{DLc} = \dfrac{96}{0.430 \times 48} = 4.6 \text{ in}^2$
$A_{LL} = \dfrac{584 \times 0.49}{48} = 6.0 \text{ in}^2$	$A_{LL} = \dfrac{584}{0.453 \times 48} = 26.8 \text{ in}^2$
$A_w + 2A_t = 30.4 \text{ in}^2$	$A_w + 2A_b = 53.5 \text{ in}^2$
$A_w = -18.0 \text{ in}^2$	$A_w = -18.0 \text{ in}^2$
12.4 in^2	35.5 in^2
$A_t = \dfrac{12.4}{2} = 6.2 \text{ in}^2$	$A_b = \dfrac{35.5}{2} = 17.8 \text{ in}^2$
$\dfrac{A_t}{A_w} = \dfrac{6.2}{18.0} = 0.344$	$\dfrac{A_b}{A_w} = \dfrac{17.8}{18.0} = 0.989$
Try 12" x 1/2" top flange	Try 18" x 1" bottom flange

$A_s = 6.00 + 18.00 + 18.00 = 42.00 \text{ in}^2$

Computed values of $\frac{A_t}{A_w}$ and $\frac{A_b}{A_w}$ are close enough to the assumed values so that there is no need for a second computation of the approximate flange areas.

Properties of trial section:

Steel section:

$$\bar{y}_s = \frac{(48.00+1.00)\ 18.00-(48.00+0.50)\ 6.00}{2 \times 42.00} = 7.036 \text{ in}$$

$$I_s = \frac{1}{4}(48.00+1.00)^2\ 18.00+(48.00+0.50)^2\ 6.00 + \frac{18.00x(48.00)^2}{12} - 42.00(7.036)^2 = 15{,}710 \text{ in}^4$$

$y_{ts} = 24.00+7.04+0.50 = 31.54$ in $\qquad S_{ts} = 498.1$ in^3

$y_{bs} = 24.00-7.04+1.00 = 17.96$ in $\qquad S_{bs} = 874.7$ in^3

Composite section: $k = 3$

$$K_c = \frac{14.40}{14.40 + 42.00} = 0.2553$$

$$\bar{y}_c = (31.54 + 3.00)\ 0.2553 = 8.82 \text{ in}$$

$$I_c = (8.82)(34.54)\ 42.00+15710+ \frac{14.40x(6.00)^2}{12} = 28{,}550 \text{ in}^4$$

$y_{tc} = 31.54 - 8.82 = 22.72$ in $\qquad S_{tc} = 1257$ in^3

$y_{bc} = 17.96 + 8.82 = 26.78$ in $\qquad S_{bc} = 1066$ in^3

$y_{cc} = 31.54-8.82+6.00 = 28.72$ in $\qquad S_{cc} = 994.1$ in^3

Composite section: $k = 1$

$$K_c = \frac{43.20}{43.20 + 42.00} = 0.5070$$

$$\bar{y}_c = (31.54 + 3.00)\ 0.5070 = 17.51 \text{ in}$$

$$I_c = (17.51)(34.43)\ 42.00+15710+ \frac{43.20x(6.00)^2}{12} = 41{,}240 \text{ in}^4$$

$y_{tc} = 31.54 - 17.51 = 14.03$ in $\quad S_{tc} = 2939$ in^3

$y_{bc} = 17.96 + 17.51 = 35.47$ in $\quad S_{bc} = 1163$ in^3

$y_{cc} = 31.54 - 17.51 + 6.00 = 20.03$ in $\quad S_{cc} = 2059$ in^3

$m = 43.20\ (14.03 + 3.00) = 735.7$ in^3

Stresses:

	f_c	f_t	f_b
D.L.s.	0	$\frac{544 \times 12}{498.1} = 13.11$	$\frac{6528}{874.7} = 7.46$
D.L.c.	$\frac{144 \times 12}{994.1 \times 30} = 0.06$	$\frac{1728}{1257} = 1.34$	$\frac{1728}{1066} = 1.62$
L.L.	$\frac{876 \times 12}{2059 \times 10} = 0.51$	$\frac{10510}{2939} = 3.58$	$\frac{10510}{1163} = 9.04$
	0.57 ksi	18.03 ksi	18.12 ksi

APPENDIX II - NOTATIONS

A_B = Area of rolled steel beam.

A_s^s = Area of symmetrical steel section used in selecting the cross-section of a composite T-beam.

A_b = Area of bottom flange of plate girder.

A_c = Transformed effective area of concrete slab.

A_{DLs} = Steel area required to resist dead loads acting on the steel section alone.

A_{DLc} = Steel area required to resist dead loads acting on the composite section.

A_{LL} = Steel area required to resist live loads.

A_p = Area of steel cover plate.

A_s = Total area of steel section.

A_t = Area of top flange of plate girder.

A_w = Web area of plate girder.

b = Effective width of concrete slab.

d = Depth of rolled steel beam or plate girder.

d' = Distance from center of gravity of top flange to center of gravity of bottom flange.

d_w = Depth of web of plate girder.

e_c = Distance from top surface of steel beam or girder to center of gravity of the effective concrete slab.

E_c = Modulus of elasticity of concrete.

E_s = Modulus of elasticity of steel.

f_c = Stress at outermost top concrete fiber.

f_b = Stress at outermost bottom steel fiber.

f_t = Stress at outermost top steel fiber.

I_B = Moment of inertia of rolled steel beam.

I_c = Moment of inertia of the composite section.

I_p = Moment of inertia of cover plate.

I_s = Moment of inertia of the steel section.

I_w = Moment of inertia of web of plate girder.

k = Numerical factor, depending on the type of loading. equal to 1 for temporary loads and 3 for sustained loads.

K_s $= \frac{A_p}{A_s}$.

K_c $= \frac{A_c}{A_c + A_s}$.

m = Statical moment of the transformed compressive concrete area about the neutral axis of the composite section.

M_{DLc} = Dead load moment acting on the composite section.

M_{DLs} = Dead load moment acting on the steel section alone.

M_{LL} = Live load moment.

n = Modular ratio $\frac{E_s}{E_c}$.

S_{cc} = Section modulus of outermost concrete fiber for composite section.

S_{bc} = Section modulus of outermost bottom steel fiber for composite section.

S_{bs} = Section modulus of outermost bottom steel fiber for steel section.

S_s = Section modulus of outermost bottom and top steel fiber for symmetrical steel section.

S_{tc} = Section modulus of outermost top steel fiber for composite section.

S_{ts} = Section modulus of outermost top steel fiber for steel section.

t = Thickness of concrete slab.

t_b = Thickness of bottom steel flange of plate girder.

t_t = Thickness of top steel flange of plate girder.

y_{bc} = Distance from N.A. of composite section to outermost bottom steel fiber.

y_{bs} = Distance from N.A. of steel section to outermost bottom steel fiber.

$\bar{y}_c$ = Shift in N.A. from addition of concrete slab.

y_{cc} = Distance from N.A. of composite section to outermost top concrete fiber.

y_{cs} = Distance from N.A. of steel section to outermost top concrete fiber.

$\bar{y}_s$ = Shift in N.A. from addition of a steel cover plate.

y_{tc} = Distance from N.A. of composite section to outermost top steel fiber.

y_{ts} = Distance from N.A. of steel section to outermost top steel fiber.

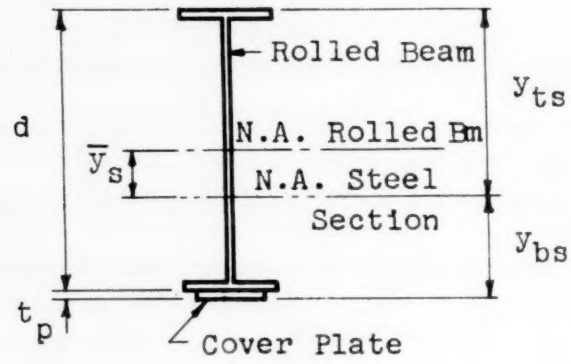

FIG. 1 - STEEL BEAM WITH COVER PLATE

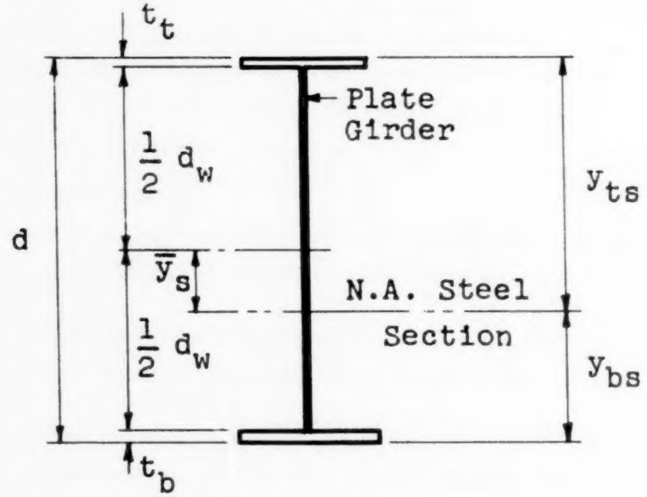

FIG. 2 - UNSYMMETRICAL PLATE GIRDER

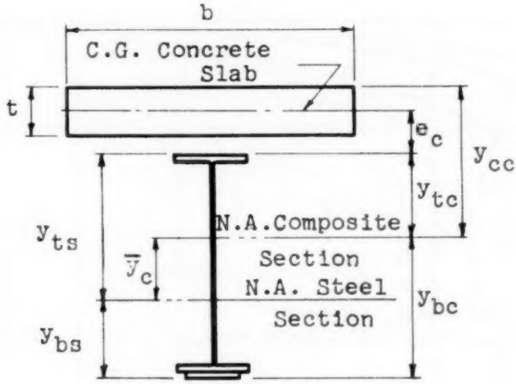

FIG. 3 - COMPOSITE BEAM WITH NEUTRAL AXIS BELOW THE SLAB

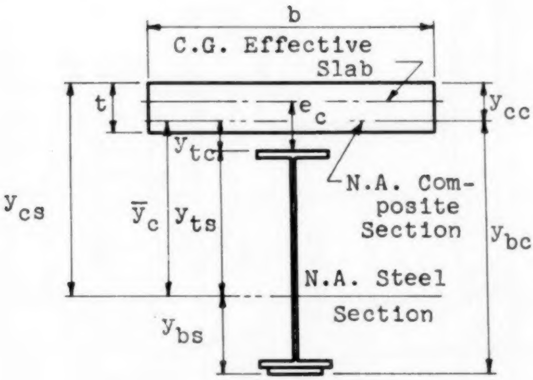

FIG. 4 - COMPOSITE BEAM WITH NEUTRAL AXIS IN THE SLAB

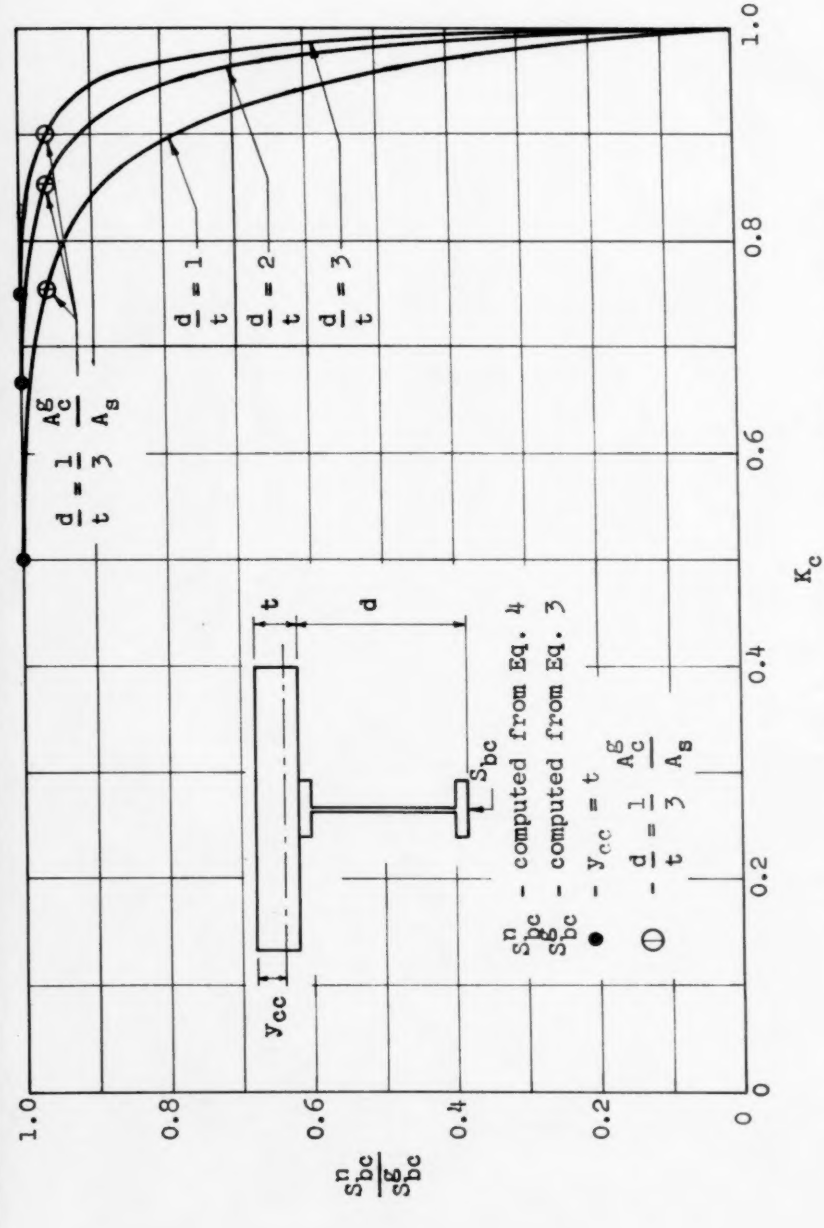

FIG. 5 RANGE OF APPLICABILITY OF EQUATIONS 3

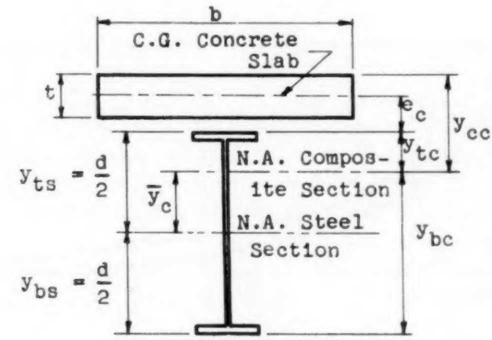

FIG. 6 - COMPOSITE BEAM WITH SYMMETRICAL STEEL SECTION

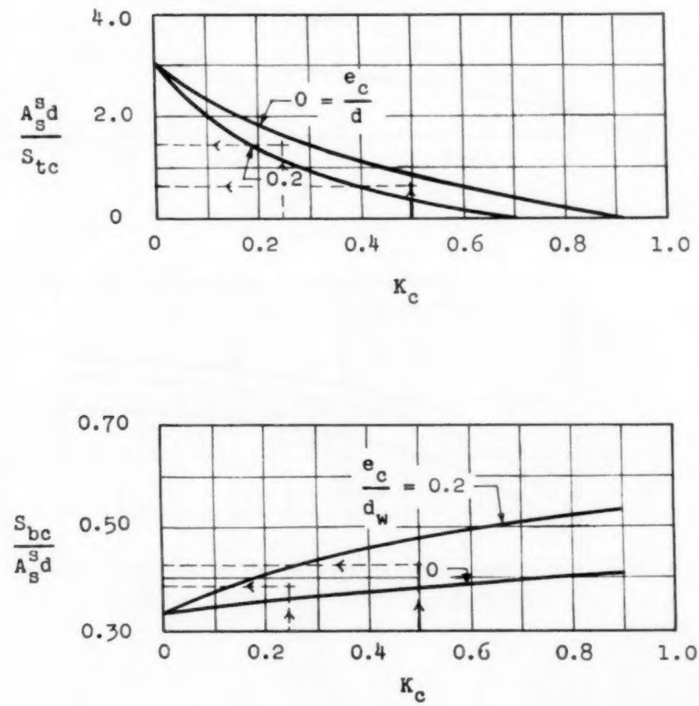

FIG. 7 - APPROXIMATE SECTION MODULI FOR BEAMS WITH ROLLED SECTIONS

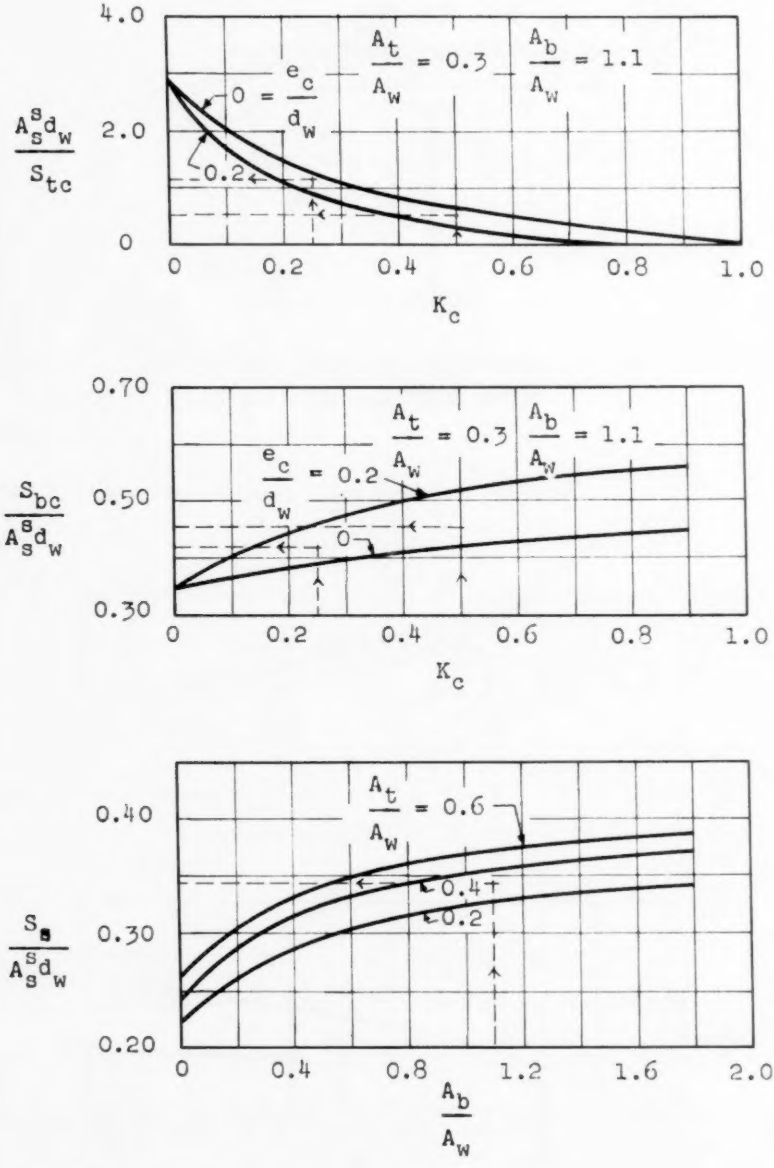

FIG. 8 - APPROXIMATE SECTION MODULI FOR BEAMS WITH BUILT UP GIRDERS

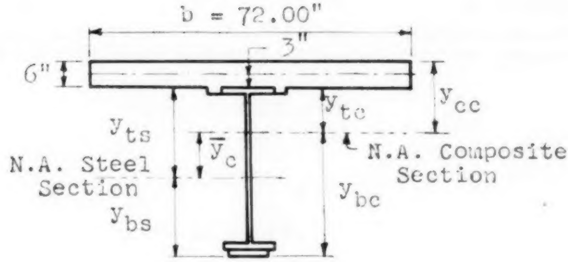

FIG. 9 - BEAM SECTION FOR EXAMPLE No. 1

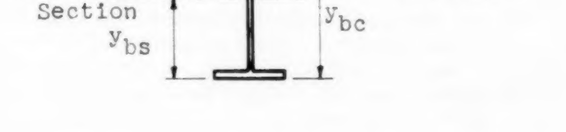

FIG. 10 - BEAM SECTION FOR EXAMPLE No. 2

Journal of the
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HIGHWAY BRIDGE LIVE LOADS BASED ON LAWS OF CHANCE*

Henson K. Stephenson**
(Proc. Paper 1314)

SYNOPSIS

A new method, based on elementary probability theory, has been developed for estimating live load frequencies on highway bridges that may be expected from various types and levels of heavy motor vehicle operation. The main objective of the method is to provide a relatively simple mathematical basis for estimating approximately how often any specified sequence or group of two or more vehicles might be expected to occur on any particular part or length of bridge as a result of given or anticipated compositions, volumes, and speeds of traffic. In addition to making use of the frequency distributions of heavy vehicle loads obtained from loadometer surveys, the new method provides the means for estimating the frequencies of various intensities of live load which result from the change grouping of two or more heavy vehicles on a given part or length of bridge at the same time.

INTRODUCTION

This paper is concerned with a study of live load frequencies on highway bridges which result from the chance grouping of vehicles in traffic. Its object is to present a new method for estimating the frequencies of various intensities of these loadings; and to show how such loadings for a given span may be related to their stress producing characteristics and effects. It deals with the problem of vehicle grouping from a mathematical standpoint, based on the same elementary laws of chance or probability that have already been used successfully for solving many types of frequency problems encountered in the various branches of science and engineering.(1) It presents:

1. A discussion of the factors in highway traffic which influence the spacings and frequencies of individual vehicles and vehicle groups.

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2. The development of the mathematical equations for estimating specified vehicle group frequencies and a discussion on their uses.

3. A few selected graphs and tables, covering typical problems relating to vehicle grouping, which illustrate how the method may be used for estimating approximately how often various sequences or groups of specified vehicles might be expected to occur within specified lengths of time, or distance along the highway.

Need of Method for Analyzing Bridge Loading Frequencies

The selection of the proper live load to be used for the design of various parts and types of highway bridges represents one of the most important as well as difficult problems encountered by those responsible for the planning of such structures. In large measure, the choice of a design live load not only determines the maximum sizes and weights of vehicles but also their speeds, spacings and other operating conditions necessary to insure that a given bridge will perform safely and economically the functions for which it was intended.

The successful planning of any particular bridge requires that the engineer have adequate information concerning the site and foundation conditions; a thorough knowledge of bridge design procedures and how they are related to the physical properties of the materials to be used in its construction; and last, but perhaps most important, he must somehow arrive at a design live load that will be commensurate with present as well as anticipated traffic conditions. Satisfactory procedures are presently available for performing each of the several operations involved in the planning of a bridge, except for that of determining the live load for which it should be designed. As a partial contribution toward the fulfillment of this need, this paper presents a new method for analyzing the frequencies of heavy vehicle loadings which provides a simple yet rational mathematical procedure for selecting a design live load consistent with the other requirements which may obtain for any given structure. The method also provides the means for investigating the adequacy of existing bridges of given design designation.

Mathematical Basis for Study of Vehicle Group Frequencies

1. General Discussion

The proper design live load for highway bridges is not only a function of the sizes, weights and frequencies of individual heavy vehicles found on the highways, but also of the frequencies of various intensities of loading that might be expected to occur on a given part or length of bridge, as a result of the chance grouping of two or more of these heavy vehicles in traffic. Fortunately it is only necessary to make a few simplifying assumptions concerning the behavior of highway traffic in order to apply the theory of probability to the chance grouping of vehicles and the frequency of specified vehicle groups. These assumptions may be stated as follows:

a) That vehicles, both individually and by types, are distributed at random in ordinary highway traffic.

b) That the average composition, volume and speed of traffic remain constant during the time period under consideration.

The first assumption means that the time and distance spacings of vehicles occur entirely by chance and not as a result of artificial control. Similarly, it means that the various vehicle types—such as automobiles, busses and trucks—occur entirely by chance throughout the traffic stream. The second assumption merely means that the time period under consideration must be of short enough duration to insure that the average composition, volume and speed of the traffic remain constant during that time. At certain times this time period could be several hours; but at others when the characteristics of the traffic are changing rapidly, the time period may be only one half or quarter hour.

Numerous studies by the author and others have demonstrated that the above assumptions approximate the actual behavior of ordinary highway traffic sufficiently close for solving many types of traffic problems now thought to be incapable of solution by mathematical means. Moreover these studies have shown that the time and distance spacings of vehicles—both individually and by groups—in ordinary traffic agree rather closely with the distributions given by the Poisson frequency distribution formula; also known as Poisson's law. This means therefore that the probability of vehicle groups of unspecified types occurring within specified lengths of time or distance can be estimated mathematically by use of Poisson's law. Once this probability has been determined, the probability that the group consists of certain specified vehicles or that they are arranged in some particular order may be found by use of the basic theorems for calculating simple and compound probabilities. It should be mentioned also that Poisson's law has also been found to provide a very good estimate of the frequency distribution of various intensities of heavy vehicle loads measured in terms of their H truck loading equivalencies on a given span. (See pages 427-438, Ref. 2)

2. Basic Theorems for Calculating Simple and Compound Probabilities

The fundamental theorems for calculating simple and compound probabilities are fully explained in almost any book on college algebra. For this reason it will only be necessary here to state these theorems and illustrate how they may be applied to a few simple situations to show how they lead more or less automatically to the Binomial and Poisson frequency distributions. Special emphasis is placed on the Poisson frequency distribution because it is the limit of the Binomial distribution and also because it is the simpler of the two to use in many cases.

Fundamental Theorems

Events of a set are usually classified as being independent, dependent, or mutually exclusive. The theorems corresponding with these classifications are, respectively:

Theorem 1 - The probability that all of a set of independent events will happen on a given occasion when each of them is possible is the product of their separate probabilities of occurrence.

Theorem 2 - If the probability of a first event is P_1, and if, after this has happened, the probability of a second event is P_2; then the probability that both events will happen in the order specified is P_1P_2 (the obvious extension of this to m events would result in the probability, $P_1P_2 \ldots P_m$).

Theorem 3 - The probability that one or the other of a set of mutually exclusive events will occur is the sum of the probabilities of occurrence for the separate events.

3. The Binomial Distribution

The binomial distribution is given by the successive terms of the expansion of the binomial:

$$(q+p)^m = C_m^m q^m p^0 + C_m^{(m-1)} q^{(m-1)} p^1 + C_m^{(m-2)} q^{(m-2)} p^2 + \cdots + C_m^0 q^0 p^m \tag{1}$$

in which p = probability of success on any one trial
q = probability of failure on any one trial
and m = number of trials (sample size or lot size)
also $p < 1$, and $q = 1-p$

In this binomial expansion, the symbol C_m^n means the number of combinations of m things taken n at a time. This may be expressed algebraically as follows:

$$C_m^n = \frac{m!}{n!(m-n)!} \tag{2}$$

This may be illustrated by inquiring the number of 3 letter combinations that can be obtained from the 4 letters; a, b, c, and d. This may be done in the following 4 ways:

abc, abd, acd, and bcd

and by the above algebraic expression, this would be determined as follows:

$$C_m^n = C_4^3 = \frac{4\cdot3\cdot2\cdot1}{3\cdot2\cdot1(1)} = 4 \tag{3}$$

With this in mind, it may now be explained that each term in the above binomial expansion gives the probability of exactly n successes in a set of m trials and each term may be written thus:

$$P_m(n) = C_m^n q^{(m-n)} p^n \tag{4}$$

in which the symbol $P_m(n)$ means the probability of n successes in a given sample of m trials, where $n = 0,1,2,3,\ldots,m$. In other words, the first term gives the probability of no successes in m trials; the second term, the probability of 1 success in m trials; and so on to the last term which gives the probability of m successes in m trials. In this connection, it should be noted that any given sequence or set of m trials each may be thought of as a sample of size m or a lot of size m.

Perhaps the simplest way to explain the meaning of the binomial

distribution is to apply it to the tossing of one or more coins. On a single toss of a coin it can fall in 2 ways, either a head or a tail, each of which is equally likely. Now if 2 coins are tossed at the same time (or one coin tossed twice in succession) they may fall in any one of the following 4 equally likely ways: TT, TH, HT, HH. Here, it will be noted that 1 of the 4 ways is favorable to 2 tails (no heads); 2 of the 4 ways are favorable to 1 head and 1 tail (one head); and 1 of the 4 ways is favorable to 2 heads.

Now if the tossing of a head is considered a success and a tail considered a failure, then according to the above nomenclature: $p = .5$ and $q = .5$, from which it will be seen that the binomial expansion

$$(q+p)^2 = q^2+2qp+p^2 \tag{5}$$

gives the same results as were obtained by enumerating all the different combinations that could be obtained from the tossing of a single coin twice in succession (or the tossing of 2 coins simultaneously). The first term of this expansion means that the probability of no successes (2 tails) is q^2; the probability of 1 success (1 head and 1 tail) is $2pq$; and the probability of 2 successes (no tails) is p^2.

Similarly the probabilities of obtaining no heads, 1 head, 2 heads, and 3 heads in any 3 tosses of a single coin (or a single toss of 3 coins) would be given by the 4 respective terms of the binomial expansion for 3 trials per sample or sample size of $m = 3$, thus:

$$(q+p)^3 = q^3+3q^2p+3qp^2+p^3 \tag{6}$$

$$= .125+.375+.375+.125 \tag{6a}$$

This means that the probability of getting no heads (3 tails) is .125; the probability of getting 1 head is .375; the probability of 2 heads is .375; and the probability of getting 3 heads is .125.

From this it will be seen that the calculation of values for the successive terms in a binomial becomes quite laborious when m is large. A better appreciation of the time required to make such calculations may be obtained by examining the binomial expansion for $m = 5$, which is as follows:

$$(q+p)^5 = q^5+5q^4p+10q^3p^2 +10q^2p^3+5qp^4+p^5 \tag{7}$$

Now if the number of trials or sample size, m, were increased, to say 100, it will be seen that the time required to evaluate the 101 terms of such a binomial distribution would be considerable to say the least. It is for this reason that resort is made to approximations of the binomial distribution in many practical problems where the number of trials per sample or sample size is large.

The Poisson distribution, for example, is used in many practical situations to approximate the values of a specific binomial distribution, particularly in cases where the sample size is large. The agreement between the

binomial and the Poisson distributions, however, increases as the sample size increases. In fact, the binomial distribution tends to approach the Poisson distribution as a limit as the number of trials or sample size becomes very large.

Use of Binomial Distribution for Sampling

In order to simulate a continuous process, suppose that a large bin is continuously being supplied or filled as needed with balls which are identical in every respect except that 80 per cent of them are white and 20 per cent of them are black. Now, if these balls are withdrawn at random from the bin and put into boxes containing 5 balls each, what proportion of the boxes would be expected to contain n black balls, where n = 0,1,2,3,4, and 5, respectively?

If a single ball is withdrawn, the probability of its being black would be p =.2, and similarly the probability of its being white would be q = .8. Under these conditions, the expected frequency of appearance of 0,1,2,3,4, and 5 black balls among the boxes of 5 balls each (sample size m = 5) can be calculated by evaluating the successive terms of the expansion of the binomial.

$$(.8 + .2)^5 = .3277 + .4096 + .2048 + .0512 + .0064 + .0003 \tag{8}$$

This means that 32.77 per cent of the boxes would be expected to contain no black balls; 40.96 per cent, 1 black ball; 20.48 per cent, 2 black balls; 5.12 per cent, 3 black balls; 0.64 per cent, 4 black balls; and only about 3 of each 10,000 boxes would be expected to contain 5 black balls.

Comment

If the drawing of a black ball is considered a success, and the letter K is used to indicate the average number of successes per sample or box of 5 balls each, then

$$K = mp \tag{9}$$

$$= (5)(0.2) = 1 \tag{9a}$$

which means that the average number of successes (black balls) per sample would be 1. In general, this means that the average number of successes, K, expected per sample is equal to the probability of success on a single trial, p, times the number of trials per sample or sample size m.

4. Development of the Poisson Distribution

In the preceding discussion it was explained that each term in the binomial expansion gives the probability of exactly n successes in a set of m trials and may be written thus:

$$P_m(n) = C_m^n q^{(m-n)} p^n \tag{4}$$

in which the symbol P_m (n) means the probability of n successes in a given sample of m trials where n = 0,1,2, . . . , m.

In the case of the binomial law, it was shown that the average number of successes, K, expected per sample (expectation of n) is equal to K = mp.

With this information, it can now be shown that the binomial distribution approaches the Poisson distribution as a limit as the number of trials m become very large. This development is accomplished by first noting that the probability p may be determined thus:

$$p = \frac{K}{m} \tag{10}$$

and if the value of p is now substituted in the above equation, it becomes:

$$P_m(n) = C_m^n \left(\frac{K}{m}\right)^n \left(1 - \frac{K}{m}\right)^{(m-n)} \tag{11}$$

Now if the operations indicated in this equation are carried out and the intermediate steps are omitted, it can be shown (see page 214, Ref. 1 or page 372, Ref. 2) that:

$$P_m(n) = \left[\left(1-\frac{1}{m}\right)\left(1-\frac{2}{m}\right)\cdots\left(1-\frac{n-1}{m}\right)\right] \left[\left(1-\frac{K}{m}\right)^{-n}\right]\left[\left(1-\frac{K}{m}\right)^{m}\right]\left(\frac{K^n}{n!}\right) \tag{12}$$

By remembering that p is rather small, it is obvious that only those values of n are of consequence which are very small as compared to m which is very large. On this basis, therefore, each of the factors enclosed within the first set of brackets becomes approximately equal to unity, as m becomes larger and larger compared with n. The same is true of the quantity 1 - (K/m) which occurs in the secondand third brackets, because K/m, or p, is very small. Therefore, since there are comparatively few of these factors in the first 2 sets of brackets, it follows that their product is also not greatly different from unity and actually approaches unity as m becomes very large compared with n.

The same line of reasoning cannot be applied to the factor within the third bracket, however, owing to the fact that the quantity 1 - (K/m) is raised to a very large power. By consulting almost any text on algebra or calculus, it will be found that the expression in the third bracket is equal to e^{-K}, or

$$\left(1-\frac{K}{m}\right)^m = e^{-K} \tag{13}$$

in which e = 2.71828 (Base of Napierian or natural logarithms).

On the basis of this line of reasoning, therefore, one would be justified in concluding that in the limit

$$P_m(n) = \frac{K^n e^{-K}}{n!} \tag{14}$$

which is known as the Poisson distribution or Poisson's law. The important thing to note here is that the binomial law approaches the Poisson law as a limit as m becomes very large. The successive terms of the binomical expansion therefore have as their limits the corresponding terms in the Poisson distribution, as follows:

$$P(n) = e^{-K} + Ke^{-K} + \frac{K^2e^{-K}}{2!} + \frac{K^3e^{-K}}{3!} + \cdots = 1 \tag{15}$$

for n = 0,1,2,3. . .
The successive terms in this series may be interpreted as the proportion of samples in which 0,1,2,3, . . . of some specified event would be expected to occur when the average number of occurrences per sample is K = mp.

Comment

One of the principal advantages of using the Poisson distribution as an approximation to a specific binomial distribution is the comparative ease with which the successive terms of the Poisson series may be evaluated. Actually, though, there is rarely ever any occasion for making such calculations since tables (See Ref. 3, or pages 380-384, Ref. 2) are available that cover a wide range of values for K = mp.

Frequency of Specified Vehicle Groups Occurring Within Specified Lengths

1. General Discussion

Assuming the average composition, volume and speed of traffic remains constant during the time period under consideration, the problem of estimating the frequency of specified vehicle groups occurring within specified lengths of time or distance is most conveniently handled by breaking it down into the following three parts:

a) First Part - Perhaps the most common situation requiring consideration in the first part consists of calculating the probability of n vehicles, unspecified as to type, occurring at a given location in any manner in either or both directions of travel, within a specified interval of t seconds or a specified length of X feet (such as a bridge) along the highway. The next most important situation no doubt consists of calculating the probability of n unspecified vehicles occurring simultaneously in each direction of travel, within a specified interval of t seconds or x feet; when the traffic volume and speed is the same in each direction. Many other situations could be defined involving different vehicle group sizes n, as well as different average volumes and speeds of traffic in each direction; but these will not be considered here owing to space limitations. It should be added, though, that these situations can be calculated quite as easily and in the same way as the more important situations mentioned above. Once these probabilities have been found, the frequencies of the events under consideration can be readily determined.

b) Second Part - The second part consists of calculating the probability of and group of n unspecified vehicles, selected in a random manner from the

traffic stream (such as the group of n unspecified vehicles in the preceding step) occurring as to type or arrangement as previously specified. Once this probability has been found, the frequency of the event can be easily determined.

c) Third Part - The third part consists of calculating the combined probabilities or frequencies from those found in the first and second parts. This gives the desired information concerning the frequency of specified vehicle groups occurring within specified lengths of time or distance.

2. First Part of Problem

a) Occurrence of n Unspecified Vehicles in Either or Both Directions

At any particular location on a highway and for any given average composition, volume and speed of traffic, the probability of n vehicles, unspecified as to type, occurring in any manner in either or both directions of travel within a time interval of t seconds or a distance of X feet is given by Poisson's formula as follows:

$$P(n, X; \tfrac{a}{2}) = \frac{K^n e^{-K}}{n!} \tag{16}$$

in which K is the average number of vehicles expected within the distance X, based on the total number of vehicles per hour (both directions) at the given location. Thus,

$$\begin{aligned} K &= \left(\frac{\text{Number of vehicles per hour}}{\text{Average speed in miles per hour}}\right)\left(\frac{X}{5280}\right) \\ &= \left(\frac{R}{D}\right)\left(\frac{X}{5280}\right) = \frac{RX}{5280D} \end{aligned} \tag{17}$$

If time instead of distance is used to measure the interval in which n vehicles is to occur in any manner in either or both directions, the probability that they will occur within t seconds is also given by Poisson's formula as follows:

$$P(n, t; \tfrac{a}{2}) = \frac{K^n e^{-K}}{n!} \tag{18}$$

This equation would be read: The probability of n vehicles occurring in any manner in either or both directions in t seconds is given by the Poisson formula in which K is the average number of vehicles per time interval of t seconds. Equation (16) would be read similarly using distance X instead of t seconds; and using K as the average number of vehicles expected in X distance; or the average number of vehicles per cell of X feet in length along the highway.

To illustrate the use of Eqs. (16) and (18) suppose a traffic volume, R, of 500 vehicles per hour (12,000 vehicles per day) with an average speed, D, of 39.457 m.p.h. is used and it is desired to know the probability of n vehicles

occurring within a length, X of 125 ft. or in a time interval corresponding to the number of seconds required to travel 125 ft. at 39.457 m.p.h., or a time, t, of 2.16 seconds. Thus

$$\text{for } X = 125 \text{ ft.} \qquad K = \frac{500 \times 125}{5280 \times 39.457} = .3$$

$$\text{and for } t = 2.16 \text{ sec.} \quad K = (500 \times 2.16)/3600 = .3$$

The average number of vehicles expected within the 125 ft. length, therefore, is 0.3, and the average number of vehicles expected in each 2.16 sec. interval is also 0.3.

Suppose it is desired to know the probability that no vehicles will occur within any 2.16 sec. interval. This is given by Eq. (18)

$$P(0, 2.16 \text{ sec.}; a/2) = (.3^0 \times e^{-.3})/0! = e^{-.3} = .7408182$$

This means that 74.08% of the 2.16 sec. time intervals will contain no vehicles. Solved for other values of n, Eq. (18) gives the following results for K = .3.

Table 1

for K = .3

n	Individual Terms	Cumulative Terms
0	.7408182	1.0000000
1	.2222455	.2591818
2	.0333368	.0369363
3	.0033337	.0035995
4	.0002500	.0002658
5	.0000150	.0000158
6	.0000008	.0000008
Total	1.0000000	

From the individual terms it will be seen that 74.08% of these time intervals would contain no vehicles; 22.22% of them would contain one vehicle; 3.33% would contain 2 vehicles and so on. If distance instead of time were considered, the probability of n vehicles occurring in X distance of 125 ft. (K = .3) would be given by Eq. (16) and would be the same as shown for t = 2.16 sec. (K = .3).

The cumulative terms on the right are also informative. They show that 100% of the time intervals (t = 2.16 sec.) will contain none or more vehicles; 25.92% of them will contain one or more; 3.69% will contain 2 vehicles or more, and so on.

If the above frequency distribution for K = .3 were applied to a very large number of intervals (observations, samples, or trials)—say ten million—the total number of vehicles involved would be 10,000,000 x .3 = 3,000,000. The

distribution of these 3,000,000 vehicles among the 10,000,000 intervals would be as follows:

Table 2

Distribution of Three Million Vehicles among Ten Million Intervals for K = .3

No. of Vehicles n	No. of Intervals with n Vehicles	Total No. of Vehicles
0	7,408,182	0
1	2,222,455	2,222,455
2	333,368	666,736
3	33,337	100,011
4	2,500	10,000
5	150	750
6	8	48
	10,000,000	3,000,000

From these figures it will be seen that 2,222,455/3,000,000 = 74.08 per cent of the vehicles occur on the length X = 125 feet (or t = 2.16 sec.) one at a time; and, similarly, 22.22 per cent are on the 125 ft. length 2 at a time; and 3.33 per cent are on it 3 at a time and so on.

Note, for example, that 33,337 of the intervals contained 3 vehicles each; and since there are a total of three million vehicles in all the intervals, this means that on the average 3,000,000/33,337 = 90 vehicles would pass for each time that 3 vehicles occurred simultaneously within the interval. In this case 90 is the vehicle interval, V.

In the 2nd column of Table 2, it will be noted that if the decimal point is moved 7 places to the left, the numbers will be the same as the probability values given by the individual terms of the distribution in the 2nd column of Table 1. Also if the decimal in the 3rd column of Table 2 is moved 7 places to the left, it will be noted that the total number of vehicles would be 0.3, which is the same as K. This shows that the vehicle interval, V, required for each occurrence of n vehicles within the defined interval is found by dividing the average number, K, per interval by the probability of n occurring.

This means that the number of vehicles, on the average, that would be required to pass for each occurrence of a given event—that is, the vehicle interval would be calculated thus

$$V(n, X; a/2) = K/P(n, X; a/2)$$
$$\text{or} \quad V(n, t; a/2) = K/P(n, t; a/2)$$

depending on whether the length is measured in time or distance.

For example, with a total traffic volume of 500 vehicles per hour, the

vehicle interval required, on the average, between occurrences of 3 vehicles within the 125 ft. length (K = .3) would be

$$V(3, 125; a/2) = 0.3/.0033337 = 90$$

The time interval, on the average, between the occurrences of 3 vehicles within X = 125 ft. for the traffic conditions defined above, would be the vehicle interval, V, divided by the rate, R, thus:

$$\begin{aligned} T(n, X; a/2) &= V(n, X; a/2)/R \\ &= 90/500 = .18 \text{ hrs.} \end{aligned}$$

For given averages volumes and speeds of traffic, the above procedure provides the means for determining the probabilities, vehicle intervals and time intervals associated with the occurrence of n unspecified vehicles, in any manner in either or both directions of travel, within a specified time interval of t seconds or X feet.

b) Occurrence of n Unspecified Vehicles in Each Direction When Average Volumes and Speeds of Traffic are the Same in Each Direction

The determination of probabilities, vehicle intervals, and time intervals, for events involving the occurrence of n unspecified vehicles in each of the two directions of travel, is very similar to that given above for events involving the occurrence of n unspecified vehicles in any manner in either or both directions of travel. In the case of n vehicles occurring in each direction, it is only necessary to determine the average number of vehicles per time or distance interval for each direction individually. And if the traffic in each direction is the same then k_1 in direction 1 is equal to k_2 in direction 2; and if they were different k_1 would not be the same as k_2. The present discussion though is confined to situations where $k_1 = k_2$.

Therefore the probability of n unspecified vehicles occurring in each direction within an interval of t seconds or X feet is given by the product of the separate probabilities indicated by Poisson's formula for each direction, individually, as follows:

$$P(n, X; 2) = (k^n e^{-k}/n!)^2$$

In this case k is the average number of vehicles per interval or cell in each of the two directions.

And as previously explained, the vehicle interval will be

$$V(n, X; 2) = 2k/(k^n e^{-k}/n!)^2$$

in which 2 k is the average number of vehicles per cell for total traffic.

Also as previously shown, the time interval would be the vehicle interval divided by the number of vehicles per unit length of time, or

$$T(n, X; 2) = V(n, X; 2)/R$$

3. Second Part of Problem

The probability of n unspecified vehicles, selected in a random manner from the traffic stream, occurring as to type or arrangement as previously specified is found by use of the simple and compound probability theorems given in Part IV of this paper. A more complete discussion of them though may be found in almost any book on college algebra.

For example, consider a traffic composition consisting of 75% M, 20% L

and 5% H. If a group of 2 vehicles are selected at random, what is the probability that they both are H, or heavy vehicles? It would be calculated thus

$$P(2H) = (.05)(.05) = .0025$$

The frequency with which this event will occur is the number of trials required on the average for each success. In this case

$$E(2H) = 1.0/P(2H) = 1.0/.0025 = 400$$

If it were desired to investigate groups of vehicles containing two or more types, such as H and L vehicles, the probabilities associated with various permutations and combinations can be calculated without great difficulty but space does not permit a discussion of them here. This type of calculation can also be found in almost any college algebra.

So, for present purposes it is believed that the above illustrations will suffice to calculate the probabilities and frequencies pertaining to specified vehicle groups.

4. Third Part of Problem

For given traffic conditions, the probability of n <u>specified</u> vehicles occurring within <u>specified</u> intervals of time or distance is merely the product of the two separate probabilities calculated in the two preceding parts of the problem, respectively.

For example, suppose the traffic conditions are as follows: 500 vehicles per hour (12,000 vehicles per day) equally divided between the two directions; average speed of 39.46 mph; and traffic composition 75% M, 20% L, and 5% H vehicles.

For these conditions, suppose it is desired to know how often 4 heavy vehicles will occur on a 125 ft. span; with K = .3 and k - .15. From Table 1 it will be found that the probability of 4 unspecified vehicles occurring on the 125 ft. span is

$$P(4, 500; a/2) = .00025$$

from which the vehicle interval is determined, thus

$$V(4, 500; a/2) = .3/.00025 = 1200$$

which results in the time interval

$$T(4, 500; a/2) = 1200/500 = 2.4 \text{ hours}$$

Then the probability that these 4 vehicles will be heavy vehicles is calculated thus

$$P(4H) = (.05)^4 = .000,006,25$$

and the number of trials or events required for each success is given by

$$E(4H) = 1.0/.000,006,25 = 160,000$$

Therefore the time interval required for each occurrence of 4H on the 125 ft. span would be

$$T(4H, 125; a/2) = 2.4 \text{ hrs.} \times 160,000 = 384,000 \text{ hrs.} = 44.9 \text{ yrs.}$$

These illustrations will suffice to indicate how the frequencies, vehicle intervals and time intervals for the occurrence of <u>specified</u> vehicle groups within

specified intervals of time or distance may be evaluated.

For the same traffic conditions, if it were now desired to know how often 2 heavy vehicles will occur in each direction simultaneously on this 125 ft. span, the results for the unspecified vehicles (omitting the detail calculations) would be:

$$P(2,\ 125;\ 2) = .000{,}093{,}76$$
$$V(2,\ 125;\ 2) = 3200$$
$$T(2,\ 125;\ 2) = 3200/500 = 6.4 \text{ hours}$$

and for the specified vehicle groups the time interval would be

$$T(2H,\ 125;\ 2) = 6.4 \times 160{,}000 = 1{,}025{,}000 \text{ hrs.} = 117 \text{ yrs.}$$

Other time intervals for the same traffic composition and varying numbers of unspecified and specified vehicle groups on spans from 10 to 500 feet in length are given by the graphs in Figures 1 and 3 for 250 vehicles per hour (6000 vehicles per day) and Figures 2 and 4 for 500 vehicles per hour (12,000 vehicles per day).

Heavy Vehicle Frequencies Related to Design Stresses in Bridges

If one considers the simple situation of an ordinary bridge on a main rural highway where the traffic may be considered distributed at random, it will be found that two or more heavy vehicles (those weighing in excess of about 13 tons) in each of the two directions of travel would occur so seldom on bridges of 500 feet or less in length that the effects of such loadings might be neglected in so far as their effects on design stresses are concerned.

For ordinary highway bridges, therefore, the most severe loading condition that need be considered (at normal service load allowable stresses) is for one heavy vehicle to occur in each of the two directions of travel at the same time. For example, if one considered a traffic volume of 500 vehicles per hour or 12,000 vehicles per day containing 5 per cent heavy vehicles, it will be found from Figure 4 that one heavy vehicle would occur in each of the two directions of a 50 ft. span, within a critical 10 or 12 ft. length at or near the mid-span about 80 times per year; and, for this same traffic, one heavy vehicle would occur in each of the two directions of a 100 ft. span, within a critical 20 to 25 ft. length at or near the mid-span, about 120 times per year.

Similarly, if a traffic volume of 250 vehicles per hour or 6000 vehicles per day containing 5 per cent heavy vehicles is considered (which is a very high volume for main rural roads and also an extremely high concentration of heavy vehicles) it will be found from Figure 3 that one heavy vehicle would occur in each of the two directions on a 50 ft. span, within a critical 10 or 12 ft. length at or near the mid-span, about 20 times a year. And, for this same traffic, one heavy vehicle would occur in each of the two directions of a 100 ft. span, within a critical 20 or 25 ft. length at or near the mid-span, 35 times a year.

But even though two heavy vehicles do occur within a critical distance, at or near the mid-span of a given bridge, several times a year, the probability that both vehicles would either be the least or the greatest H-equivalency encountered in such traffic is so remote that it may be neglected. In fact, it can be shown that the two heaviest vehicles likely to occur on a 50 ft. bridge at the same time would produce less stress than a single vehicle with one of the higher H-equivalencies.

In order to illustrate some of the implications of the above discussion, the loading and stress frequencies, resulting from a traffic volume of 500 vehicles per hour (12,000 vehicles per day) with 5 per cent heavy trucks, will be considered in both a 50 ft. and a 100 ft. simple span bridge. And since the stress effects of overload are greater in bridges with the smaller ratios of dead load to design load stresses, the lightest type of ordinary construction will be considered; namely, bridges with a minimum thickness concrete deck supported by longitudinal steel beams or girders. The ratios of dead load stress to total design stress, and live load plus impact stress to total design stress for spans of various lengths are presently available (see page 17, Ref. 4) for bridges of this type of H 15-44 design.

If the dead load and live load plus impact stress ratios given on page 17 of Ref. 4 are used, and it is further assumed that a single vehicle in one lane will produce a maximum bending stress equal to about 75 per cent of that produced by identical vehicles in each lane, one can then develop some rather interesting stress frequency relationships. For these purposes it will be assumed, for example, that a single H 15 truck in one lane will ordinarily produce about 75 per cent as much live load moment in a stringer or girder as an H 15 truck in each of the two adjacent lanes. Another way of saying this is that two H trucks of given designation (one in each lane) will produce about 4/3 as much live load stress in the most critically stressed interior stringer as a single H truck of the same designation.

Based on these assumptions, together with the dead and live load plus impact ratios given in Reference 4, and the frequency distribution of H-equivalencies as found from the national loadometer survey of 1942 (see p. 409, Ref. 2), the frequencies of stress repetitions in a 50 ft. bridge and a 100 ft. bridge for an assumed life of 50 years would be approximately as shown in Fig. 5 and Fig. 6, respectively. The amazing thing about these figures is that, even with full allowance for impact, there is such a small number of stress repetitions in excess of the allowable design stresses that would result from a continuously flowing traffic volume of 500 vehicles per hour or 12,000 vehicles per day containing 5 per cent heavy trucks for the full 50 years useful life of each bridge.

Much more could be said about Figures 5 and 6, of course, but it is believed that the implications are sufficiently clear without burdening the reader with further explanation or discussion. It might be pointed out in closing though that in no case do the maximum bending stresses produced by legal loads approach values that would be considered critical.

CONCLUSIONS

Based on the previously substantiated fact that vehicles, both individually and by types, are distributed at random in ordinary highway traffic, the paper shows that the frequencies with which specified heavy vehicle groups might be expected to occur on various parts or lengths of bridge can be analyzed mathematically. This together with highway loading frequencies, measured in terms of equivalent H or H-S standard design trucks or any other convenient equivalent design loads (see pages 390-438, Ref. 2) and the stress producing effects of such loads (see Ref. 4) provides the means for estimating the number of repetitions of various intensities of stress that might be expected at any point in a given bridge during its useful life. Two typical examples of

this kind are shown in Figure 5 and Figure 6, respectively. Although the mathematics involved in the method presented is quite simple, most of the situations that would be of interest to the engineer could be reduced to charts similar to Figures 1-4 inclusive. At a glance, for 6000 and 12000 vehicles per day, these charts will give the time interval for both unspecified vehicle groups occurring within specified lengths on a 2-directional highway. For example, Figure 4 shows that for 12000 vehicles per day with 5% heavy trucks, 2 heavy trucks in each direction on a 100 ft. span would occur about once in each 250 years. Time intervals for many other situations can be found in a similar manner from Figures 1-4, inclusive.

NOMENCLATURE AND DEFINITIONS

M represents one miscellaneous vehicle (automobile or bus).

L represents one light freight vehicle.

H represents one heavy freight vehicle (weight in excess of 13 tons).

X = length of section or distance in feet along highway (distance interval) which the grouping of vehicles is to occur.

t = length of time in seconds (time interval) within which the grouping of vehicles is to occur.

R = average number of vehicles per hour in any one designated direction or total traffic in both directions as may be specified.

D = average speed of traffic in any designated direction.

n = number of vehicles in a group or sequence but unassigned as to class or type.

n! = factorial n. For example, factorial 4 = 4! = 1 x 2 x 3 x 4 = 24

e = exponential base, 2.718,281 . . .

K = average number of vehicles expected within a specified length of X feet or a specified time of t seconds, based on total traffic in both directions. For a specified length of X feet; $K = RX/5280D$ and for a specified time of t seconds; $K = Rt/3600$

k = average number of vehicles expected within a length of X feet or a time of t seconds in one designated lane, based on the number of vehicles per hour, (R_1), and average speed of vehicles, (D), in that lane.

P represents a general term used to indicate the probability that an event (to be defined) will occur as specified.

E = number of events or trials between occurrences of vehicle groups as may be defined.

V = vehicle interval between occurrences of certain specified events to be defined.

T = time interval between occurrences of certain specified events to be defined.

P(4H, X; 2) = probability of the group, 4H, occurring within X feet in each of the 2 directions.

P(G, X; a/2) = probability of the group, G, occurring within X feet in any manner in either or both directions.

E(n, X; 2) = number of events between occurrences of n vehicles in each of 2 lanes within X feet.

V(G, X; a/2) = vehicle interval between occurrences of the group, G, in any manner in either or both directions within X feet.

T(G, X; a/2) = time interval between occurrences of the group, G, within X feet in either or both directions.

The terms given above do not show all the possible combinations of symbols for describing conditions associated with vehicle groups on a two or more lane highway. Those shown, however, are typical: other combinations can be selected suitable for describing the particular operation under consideration.

REFERENCES

1. T. C. Fry, "Probability and Its Engineering Uses," D. Van Nostrand Co., New York, 1928.
2. Henson K. Stephenson and Kriss Cloninger, "Method of Converting Heavy Motor Vehicle Loads into Equivalent Design Loads on the Basis of Maximum Bending Moments," Texas Engineering Experiment Station Bul. 127, Texas A & M College, College Station, Texas, 1952.
3. E. C. Molina, "Poisson's Exponential Binomial Limit," D. Van Nostrand Co., New York, 1943.
4. Henson K. Stephenson and Kriss Cloninger, "Stress Producing Effects of Equivalent Design Loads on Modern Highway Bridges," Texas Engineering Experiment Station Bul. 132, Texas A & M College, College Station, Texas, 1953.
5. Sven O. Asplund, "Probabilities of Traffic Loads on Bridges," ASCE Proc., Vol. 81, Separate 585, Jan. 1955.
6. John G. Wardrop, "Some Theoretical Aspects of Road Traffic Research," Road Paper No. 36, Vol. 1, Part No. 2, Engineering Divisions, Proc., Inst. of Civil Engineers, June 1952, London.
7. B. D. Greenshields, D. Schapiro and E. L. Ericksen, "Traffic Performance at Urban Street Intersections," Yale Univ. Bur. of Highway Traffic, Tech. Rep. No. 1, New Haven, Conn., 1947.
8. B. D. Greenshields and F. M. Weida, "Statistics with Applications to Highway Traffic Analyses," Eno Foundation for Traffic Control, Saugatuck, Conn., 1952.

TIME INTERVAL FOR TYPICAL UNSPECIFIED VEHICLE GROUPS OCCURRING WITHIN SPECIFIED LENGTHS
Based on 250 Vehicles Per Hour (6000 Per Day) at Average Speed of 39.5 M.P.H.

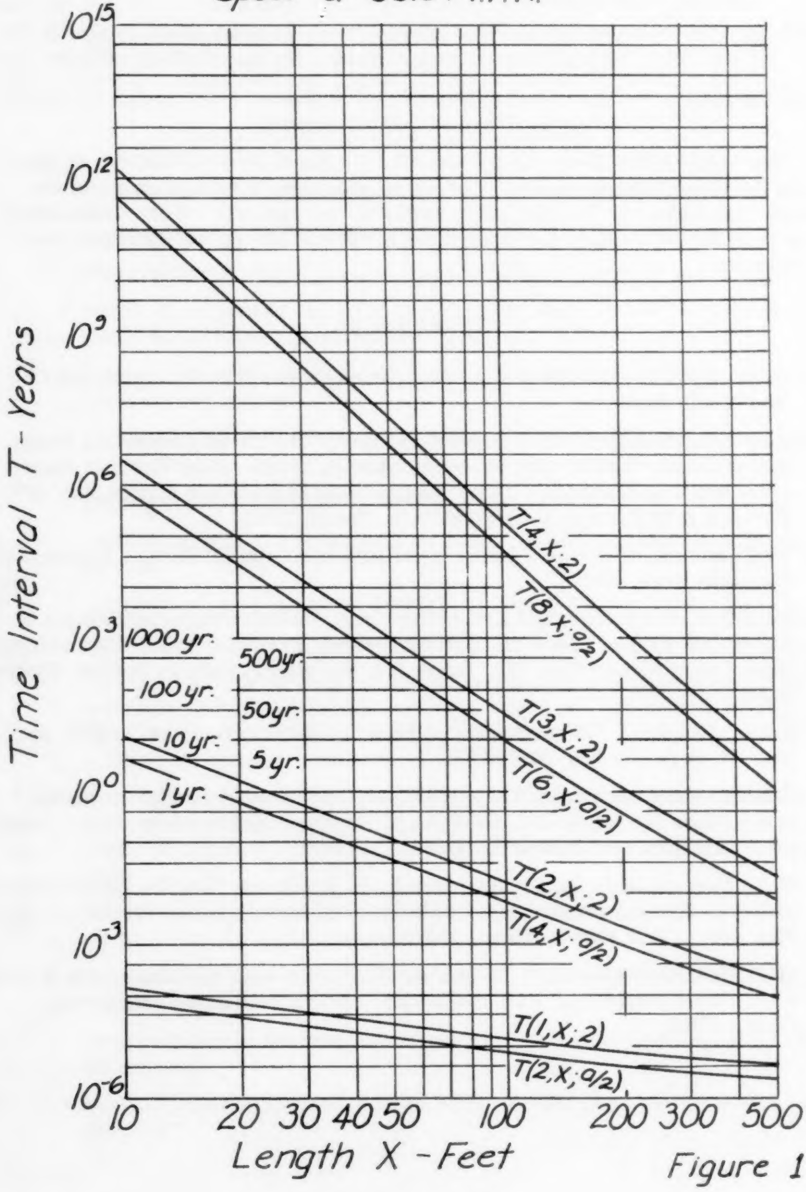

Figure 1

TIME INTERVAL FOR TYPICAL UNSPECIFIED VEHICLE GROUPS OCCURRING WITHIN SPECIFIED LENGTHS

Based on 500 Vehicles Per Hour (12,000 Per Day) at Average Speed of 39.5 M.P.H.

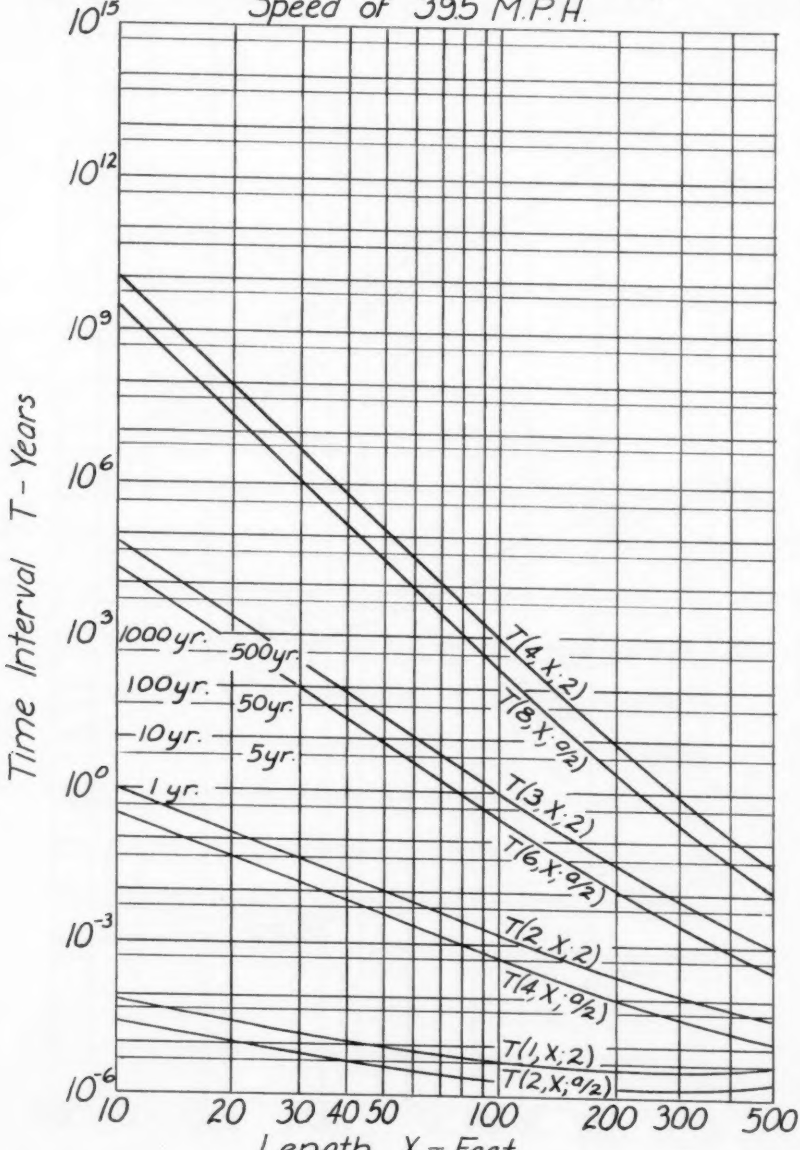

Figure 2

TIME INTERVAL FOR TYPICAL SPECIFIED VEHICLE GROUPS OCCURRING WITHIN SPECIFIED LENGTHS

Based on 250 Vehicles Per Hour (6000 Per Day) at Average Speed of 39.5 M.P.H. Consisting of 75% M, 20% L, and 5% H

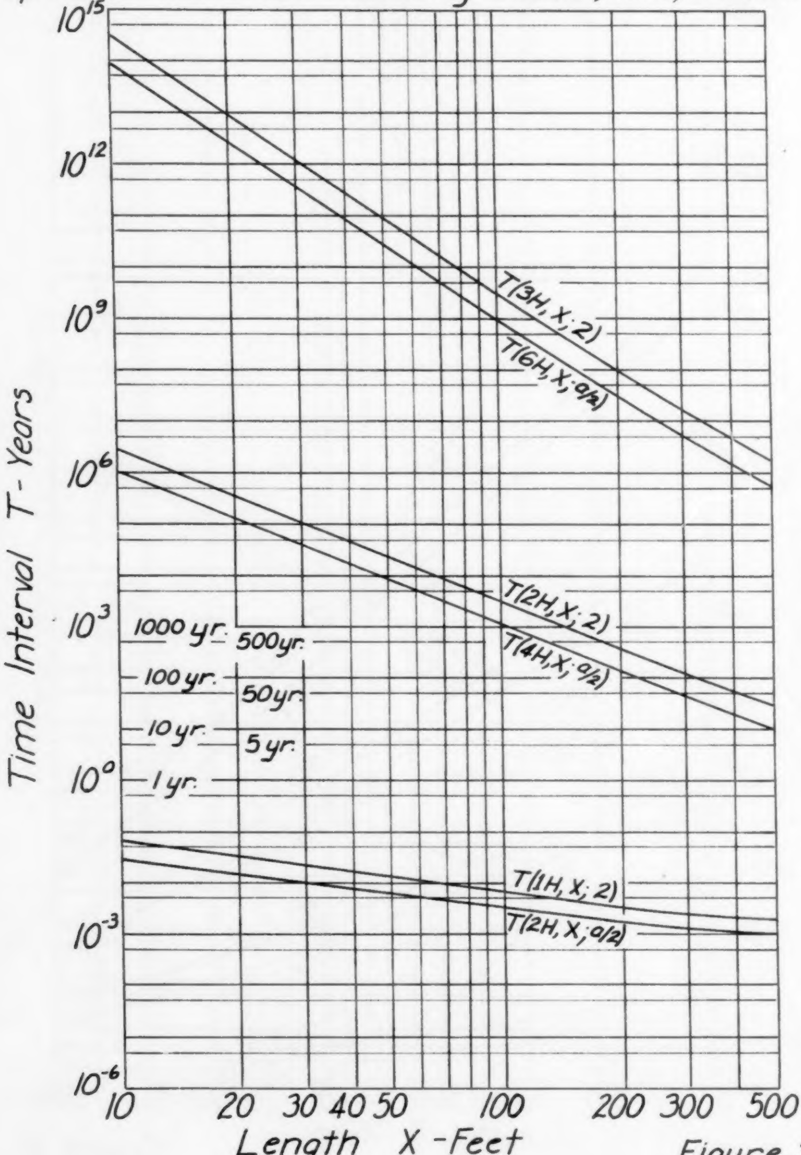

Figure 3

TIME INTERVAL FOR TYPICAL SPECIFIED VEHICLE GROUPS OCCURRING WITHIN SPECIFIED LENGTHS

Based on 500 Vehicles Per Hour (12,000 Per Day) at Average Speed of 39.5 M.P.H. Consisting of 75% M, 20% L, and 5% H

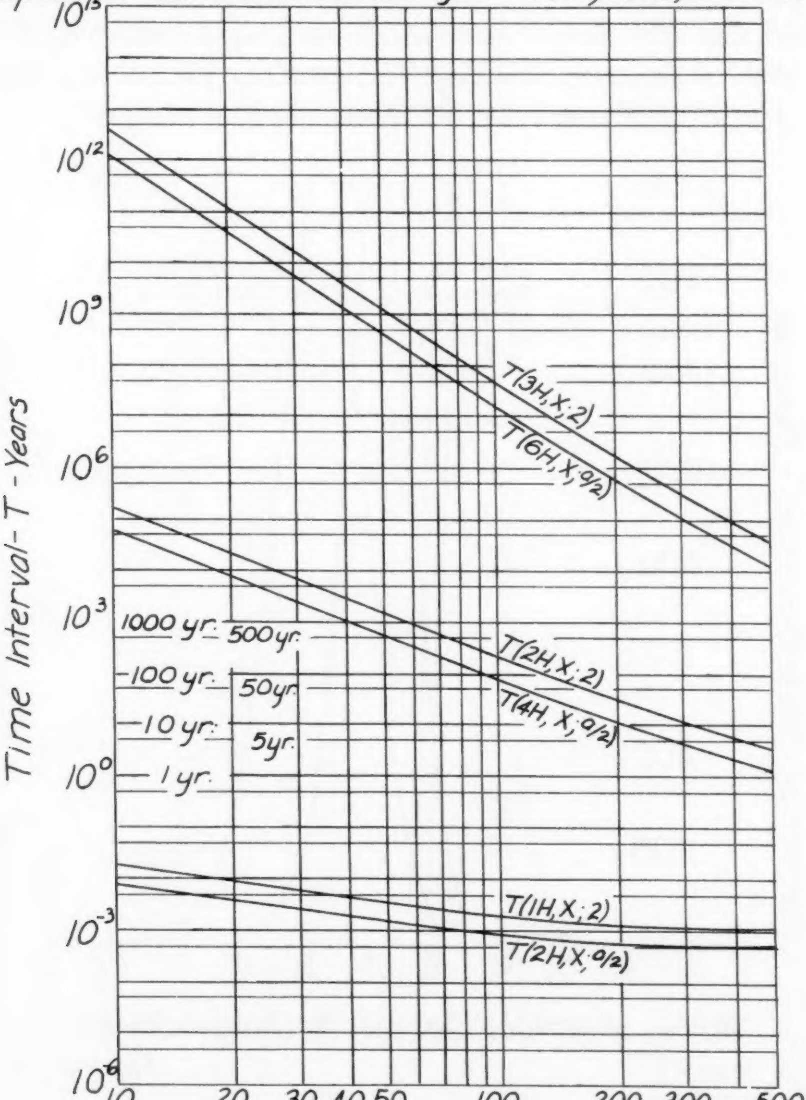

Figure 4

NUMBER OF STRESS REPETITIONS PRODUCED IN A 50 FOOT SIMPLE SPAN BRIDGE OF H 15-44 DESIGN DURING AN ASSUMED USEFUL LIFE OF 50 YEARS

Stress Effects Are Based on Continuous Traffic Volume of 500 Vehicles Per Hour (12,000 Per Day) Containing 5% Heavy Vehicles. Heavy Vehicles by Definition Are Those in Excess of 13 Tons Gross Weight.

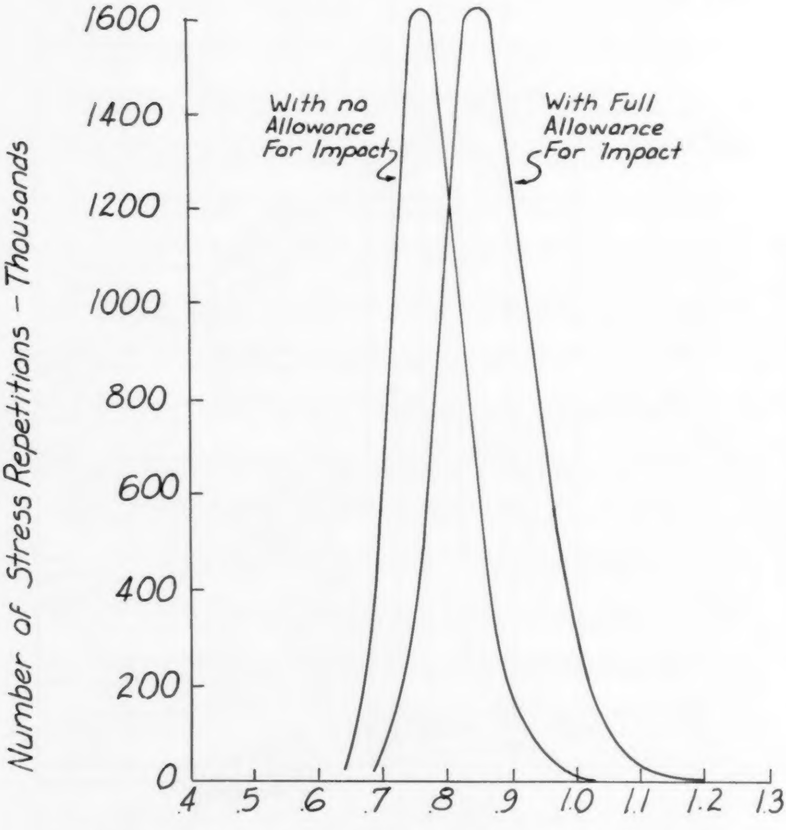

Figure 5

NUMBER OF STRESS REPETITIONS PRODUCED IN A 100 FOOT SIMPLE SPAN BRIDGE OF H15-44 DESIGN DURING AN ASSUMED USEFUL LIFE OF 50 YEARS

Stress Effects Are Based on Continuous Traffic Volume of 500 Vehicles Per Hour (12,000 Per Day) Containing 5% Heavy Vehicles. Heavy Vehicles by Definition Are Those in Excess of 13 Tons Gross Weight.

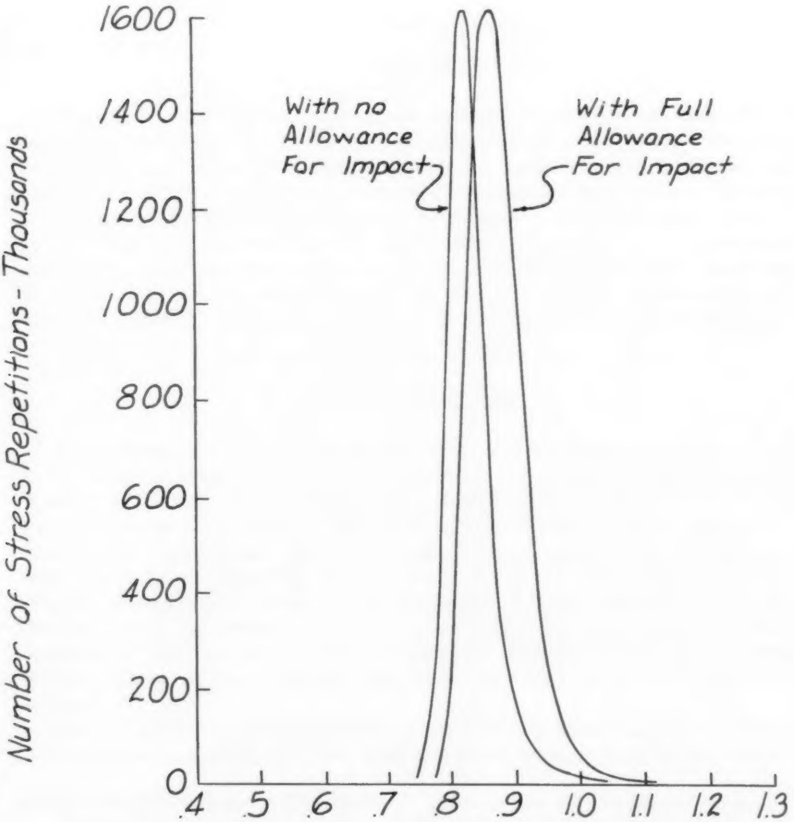

Figure 6

Journal of the
STRUCTURAL DIVISION
Proceedings of the American Society of Civil Engineers

LOAD FACTORS FOR PRESTRESSED CONCRETE BRIDGES*

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(Proc. Paper 1315)

SYNOPSIS

The use of load factors in ultimate design for prestressed concrete bridges is compared to the elastic theory and allowable stress method. Variables affecting the load factor of bridge sections are analyzed, indicating the wide divergence when designed on the basis of no tension in concrete. In addition to the load factors for flexural strength of simple and continuous spans, factors for strength at transfer, for shear, and for tension and compression members, are discussed. The significance of load distribution, load repetition, and the meaning of yield point and usable strengths are brought out. Load factors for total load are listed, as well as for dead, live and wind loads.

INTRODUCTION

Load factor may be defined as the ratio of the collapse load for a structure to its working load. The use of load factors for bridge design serves two purposes. When applied to the total load, including both dead and live loads, they are primarily intended to result in the proper behavior of the structure under the working conditions. Evidently, these factors must be appreciably greater than one because under the service load, our structures should not be anywhere near collapse. In the 1956 ACI Building Code, this is termed as the load factor K, specified at 1.8 (D + L) or 2 (D + L), where D stands for dead load and L for live load. In the Criteria for Prestressed Concrete Bridges, by the Bureau of Public Roads, this is set at 2 (D + L) where L includes the impact.

Often times, the specified factors for live and dead load differ, because the chances of increase in live load are much higher than the increase in

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dead load. The Bureau of Public Roads Criteria call for load factors of $D + 3L$. This indicates that the bridge will not collapse until the actual live load and impact approaches three times the design live load and impact. It will be interesting to note that the 1956 ACI Building Code for Reinforced Concrete specifies load factors of $1.2D + 2.4L$. The factor $2.4L$ may be justified since the probability of excessive live loads is lower in buildings than in bridges, while the factor $1.2D$ may be based on the possibility of some increase in dead load, or the desirability of providing some margin of safety so that the collapse load will never be approached. For similar reasons, the December 1956 report of ACI-ASCE Joint Committee on Prestressed Concrete recommends $1.4D + 2.3L$ for buildings and $1.6D + 2.4L$ for bridges.

It is therefore observed that while it is possible to use one set of load factors in ultimate design, two sets are often specified—one is the factor K, in the formula $K(D + L)$, to be applied to the total working load, another is the combination of a factor K_D for the dead load with another factor K_L to be applied to the live load and impact thus, $K_D D + K_L L$. Load factors currently specified or suggested are listed in Table 1.

Ultimate vs Elastic Design

Design on the elastic theory is made on the basis of the working load, limited by a set of allowable stresses. A margin of safety is thus provided solely in the allowable stresses, with no direct allowance for the possibility of excessive overloads.

In ultimate design, the working load is multiplied by a load factor and equated to the ultimate load capacity of the structure. Thus a margin of safety is provided entirely in the load factor.

For prestressed concrete bridges, the application of the elastic theory is somewhat hampered by the non-proportionality of stresses to the applied loads. Stresses generally are not a correct measure of strength. Hence to base the design on a set of fixed allowable stresses will yield structures of widely divergent strengths. This is one reason why the design of prestressed concrete started in the direction of ultimate theories even at an early stage.

Should we, then, completely discard the elastic theory in the design of prestressed concrete bridges? The answer is no. The elastic theory describes rather accurately the behaviour of prestressed concrete before the start of cracking. It can be applied for the control of camber, deflections, creep, vibrations, and the limitation of cracks, as far as the working load is concerned. It cannot be used as a measure of overload capacity, unless an extensive set of allowable values be made for different kinds of members under varying conditions.

Can we, then, use ultimate strength as the only method for design? With qualifications, the answer is yes. So far as the overload capacity is concerned, the use of a proper load factor definitely insures the ability of the structure to carry overloads. But it is not always a guarantee of the proper behaviour of the structure under the working conditions. Certain structures can be far from total collapse but may already deflect too much, vibrate too much, crack too much, or creep too much. This indicates that a comprehensive set of load factors must be worked out to suit all conditions. In other words, the value K must be varied if proper behavior under the service load is to be obtained.

Some engineers advocate the use of elastic design subject to checking for the ultimate strength. This is a safe procedure, but may not always yield consistent results or economical designs. Furthermore, we only have the elastic theory for flexural strength but none for the shear strength, bond strength, or the strength of non-prestressed reinforcements. We have not standardized our method of stress analysis for composite sections, continuous beams made up of precast elements, prestressed slabs in two directions, etc. Designs based on fixed allowable stresses could sometimes lead to load factors of 5 or 7, much too high to be reasonable.

It is often a simpler procedure to design by the ultimate theory and check for the elastic behaviour. Unfortunately, we do not have sufficient knowledge of the ultimate strength of many types of prestressed construction, e.g., combined direct and bending loads, combined shear and flexural strength, and bond strength near cracked sections. Hence it is not yet possible to design all portions of a structure by the ultimate theory.

Flexural Strength of Members Designed for No Tension

One usual criterion for elastic design of prestressed concrete bridges is the condition of no tension under the working load. While this value of no tension is often convenient for analysis and design calculations, it will be shown that such an arbitrary rule results in members of divergent margins of safety. The load factor K will vary with the shape of the section, the location of the tendons, and the amount of prestress on the section. As plotted in Figs. 1-4, it is seen that the load factor K for solid rectangular sections varies from 2.4 to 5.4, Fig. 1; for a given hollow cored section from 2.4 to 4.6, Fig. 2; for a modified T-section from 2.4 to 3.3, Fig. 3; for a tee, a channel, a box, and a composite section, from 2.2 to 3.0, Fig. 4. It will be noticed, that in general the load factor increases with decreasing value of f_c/f_c' (the compressive stress ratio in the concrete) and with increasing amount of d'/h, (concrete protection ratio measured to the c.g.s.). It is evident from these figures that the criterion of no tension is not a fair measure of the ultimate flexural strength of a member.

Figs. 1-4 are plotted on the following assumptions:

1. The ultimate strength of the tendons is twice their effective prestress; the stress-strain relation of the tendons is such that the ultimate strength of the steel is developed at rupture of the beam (this is true except for certain cases of over-reinforcement.)
2. The tendons are bonded to the concrete. If unbonded, the load factor will be correspondingly decreased, perhaps by 10 to 30%.
3. No mild steel or other non-prestressed reinforcement are added to the section. It is known that, by adding such reinforcement, the ultimate strength can be easily increased by 5 to 50%.

Under the most adverse conditions, using sections in Fig. 4, with $f_c/f_c' = 0.4$, $d'/h = 0.1$, if the ultimate strength of the tendons is only 1.6 times the effective prestress, or if the tendons are unbonded and no mild steel is added, the load factor of such a simple beam can be 1.7 or lower even though the section is designed for no tension under the working load.

On the other hand, under the most favorable conditions, for a rectangular solid section, Fig. 1, concentrically prestressed ($d'/h = 0.5$) and with low

average prestress, if non-prestressed steel is added, the load factor K can be 7.0 or higher, though it has the same elastic strength as the section in the last paragraph.

Hence the question is: should both of these two members be designed on the basis of no tension? If so, their load factor K will vary from 1.7 to 7.0.

Tensile Stress in Members Designed for Load Factor K = 2

Since elastic design yields divergent margins of safety, suppose we proceed on the basis of a constant load factor, e.g., K = 2. It is at once evident that varying tensile stresses will obtain under the design load. This is shown in Fig. 5 which gives the magnitude of resulting tensile stresses in the bottom fiber if designed for a load factor K = 2. It is further observed that the resulting tensile stress varies with the compressive stress ratio f_c/f_c' and with the shape of section; but it does not vary with the location of the c.g.s.

Fig. 5 was plotted on the same three assumptions as listed in the previous section. In addition the beams are assumed to be not cracked despite the high tensile stresses. Actually, when the tension exceeds about $0.12f_c'$, cracking would start and the computed tensile stress would have no meaning except to indicate the appearance of such cracks.

A study of Fig. 5 at once reveals the problem encountered when using a constant load factor of K = 2 or any other fixed factor. It would be quite satisfactory for a box section where the tensile stress will not exceed $0.07f_c'$ and hence no cracks should occur under the design load, but may result in excessive cracking and deflections for certain tee and rectangular sections where the tensile stress under working loads will exceed the modulus of rupture. Cracking may or may not be objectionable depending upon the circumstances.

Hence if the use of a load factor K is to insure proper behaviour of the structure under service conditions, then K must be a varying value depending upon the shape of section, the compressive stress ratio, the ratio of ultimate stress to effective prestress in tendons, whether the tendons are bonded, whether there are any non-prestressed reinforcements and whether cracking is objectionable. To use a fixed factor K will yield just as inconsistent result as to use a fixed allowable stress, such as no tension.

Continuous Spans

While the above discussion holds true for simple spans, another problem enters into the picture for continuous ones. In a simple span, the strength is determined by one critical section; in a continuous one, the ultimate load is not reached until a sufficient number of plastic hinges are formed. By virtue of its redundancy, there is an additional margin of safety in continuous construction. That additional safety can be utilized in design, as is advocated for similar structures of steel and reinforced concrete.

For the purpose of discussion, let us consider the typical interior span of a continuous beam over an infinite number of equal spans, all loaded with uniform loads. In addition to those which affect the load factor for simple spans, (such as the compressive stress ratio, the location of c.g.s., the shape of section, the presence of non-prestressed steel), the concordancy of cables and the variation of moment of inertia will affect the load factor for continuous spans. Since it will be a major job to list all possible cases, only some are considered as follows.

Case A—Fig. 6 (a), for solid rectangular section with uniform thickness, a concordant cable is located as shown. Assuming $d'/h = 0.1$ and $f_c/f_c' = 0.4$ over the support, the load factor K for that section is 2.3 (from Fig. 1). According to the plastic hinge theory, owing to the excess strength at the midspan section, the load factor considering the beam as a whole is 2.6.

Case B—Fig. 6 (b), a beam similar to Case A but having smaller eccentricity for the cables (as may be required by the low ratio of dead to live load), the load factor K for the section over the support is 3.7 (assuming $f_c/f_c' = 0.1$). By the plastic hinge theory, the load factor for the beam as whole is 4.6.

Case C—Fig. 7, a continuous beam with a varying box section, if designed for no tension, would have a factor K = 2.4 for either the midspan section or the section over the support. Hence the factor of safety for the over-all structure is no greater than 2.4.

If the above beams are all designed for a factor K = 2, the value of maximum tensile stress would vary appreciably. For example, the maximum tensile stress in Case A would be 0.192 f_c' and in Case C would be $0.072f_c'$. It is therefore apparent that neither a fixed allowable stress nor a fixed load factor K will yield consistent results.

Some continuous bridges make use of precast prestressed sections joined together with non-prestressed reinforcement over the supports. When properly designed, a high load factor is obtained for this type of construction. But there is certainly no way to eliminate tension, since the non-prestressed steel would not act until the concrete cracks. Designs similar to this type cannot be checked by the elastic theory with fixed allowable stresses; neither can they be designed by the ultimate theory with fixed load factors. Proper insight and understanding is required, rather than a set of arbitrary rules.

Load Factor at Transfer

The problem of proper load factors at transfer is an even more serious one, because so little work has been done to determine the ultimate strength at transfer. Enough is known, however, to show that conventional design on the basis of a fixed allowable stress will yield ridiculously divergent margins of safety. For example, consider a rectangular section as shown in Fig. 8 (a), designed for top fiber tensile stress of $0.05f_{ci}'$, it will require 4 times the prestress or 1.8 times the eccentric moment to fail that section. For the T section, in Fig. 8 (b), also designed for top fiber tension of $0.05f_c'$, there is practically no margin of safety. Because as soon as the top fiber starts to crack, the section could collapse under the prestress.

Tests are now being conducted at the University of California, in a program sponsored by the California State Division of Architecture, to determine the behaviour and strength of concrete beams under excessive prestress. It is known that the prestress ratio in concrete, the location of c.g.s., the shape of section, the amount of external moment, and the addition of non-prestressed steel will all affect the strength of the beam. The elastic theory using a fixed allowable stress in tension is certainly not the right solution, but what load factors and how they should be applied remain to be determined. For example, the possibility of mishandling the beam, the probability of mislocation of tendons and the chances of over-prestressing will all affect the value of the load factor.

Load Factor for Shear

The elastic analysis for shearing or principal stresses in prestressed concrete is believed to be fairly accurate so long as the beam has not cracked. After cracking, and when considering the effect of web reinforcement, the logical approach to design is the ultimate strength method. Since shear failure often occurs suddenly, a load factor slightly higher than that for flexure may be desired.

A critical problem in shear failure is a combination of moment and shear. If moment is small at the section of high shear, no flexural cracking can occur and the entire section of concrete is effective in resisting shear. On the other hand, if flexural cracks occur, the concrete section will be reduced, and the shear resistance decreased. Hence when investigating a section for shear strength, it is important to note the part which moment plays in that section. The load factor should often be applied to both the moment and the shear.

It is now known that, for continuous beams of reinforced concrete, there is a possibility of shear failure at points of inflection. This is not so critical in prestressed construction, since at least a portion of the concrete section will be under compression and help to carry the shear.

For punching shear in slabs, data are also lacking. A program is now being carried on at the University of California. Load factors cannot be assigned until we know the relation between the behavior and the ultimate strength of slabs under concentrated loads, the effect of moment on shear strength, and other details.

Load Factors for Tension and Compression Members

Tension and compression members of prestressed concrete are being used for bridgework. For example, tie rods for arches can be advantageously prestressed; prestressed piles are driven to serve as members carrying direct load with or without bending. The application of the elastic theory with a fixed allowable stress is again an unreliable method of approach.

Consider a prestressed concrete tie rod; if designed for no tension, its load factor K can be barely unity. For example, if the concrete has cracked before prestressing, then as soon as the working load is exceeded, the steel tendons will elongate unduly and failure of other parts of the structure may result from this excessive deformation. If a factor of $K = 2$ is required, the tie rod will have to be highly prestressed and will maintain a high residual compression. Such high compression may be entirely unnecessary and may result in excessive creep of the member. Hence neither a factor of $K = 2$ nor an allowable stress of zero is a safe criterion, but the engineer should use his proper judgment.

Consider a compression member under bending, say a pile subject to eccentric loads. The presence of direct external compression on the pile increases its elastic flexural strength, if that strength is computed on the basis of a fixed allowable tensile stress. On the other hand, its ultimate flexural strength is decreased by external compression. The choice of a proper load factor will therefore depend on many things, among them the significance of tensile stress and the chances of overloading for that particular member.

Usable Strength, Fatigue Strength

As commonly applied, the term load factor indicates the ratio of the ultimate collapse load to the working load. Since a bridge may be rendered unserviceable even if it has not collapsed, some engineers believe that usable strength, (or serviceable strength), rather than the collapse strength, should be used as the measure, and all load factors should be referred to the usable strength as the basis.

It is, however, a rather difficult problem to define and determine the usable strength. A noticeable permanent set or objectionable cracks may be the usable limit for certain members. For shear failure, the usable strength may coincide with the ultimate. If such limit is used as the reference rather than the collapse load, the load factor may have to be modified accordingly.

Another important consideration is the fatigue limit of the bridge structure. Evidently, the fatigue limit is lower than the collapse strength under a single loading. Hence the load factor, if referred to the fatigue strength, can be lower than that referred to a single loading strength.

Load Factors for Live, Dead, and Wind Loads

Since the chances of increase in live load on highway bridges are higher than increase in dead load, it is logical to use a higher load factor for live load than for dead load. Such a higher factor for live load will also strengthen the shorter spans where repeated loadings and high overloads are more frequent. This view is shared by many engineers. For example, the Bureau of Public Roads recommends load factors of D + 3L, in addition to the over-all load factor of 2 (D + L).

In view of the heavy vehicles now on the highways, far exceeding the design load of H20-S16, it is considered logical to employ a higher factor for live load. The December, 1956 report of the ACI-ASCE Committee on Prestressed Concrete, recommends 1.6D + 2.4L. The factor of 1.6D is to insure that the loads on the bridge will never approach the ultimate, it is not intended to imply that the dead load can be actually increased by as much as 60%. The factor of 2.4L is perhaps somewhat low, in view of the desire to leave a reasonable margin between the maximum vehicle and the ultimate load. It is possible that factors of 1.5D + 2.5L would be slightly more logical.

The factors of 1.6D + 2.4L, while likely sufficient for portions of the bridge under a high LL/DL ratio, may not yield a satisfactory structure if dead load predominates. For example, a beam carrying dead load only would possess a margin of safety of 0.6 which could indicate the existence of objectionable cracking of the prestressed beam. Hence it is often necessary to check the behavior of the bridge under service conditions, even though it may meet the requirement of load factors.

Another approach is to use a variable load factor K, depending upon the ratio of dead load to total load, DL/TL. From Fig. 9, it is seen that the requirement of D + 3L is the same as a varying K from 3 to 1 for DL/TL ratio from 0 to 1. Thus, instead of two criteria "D + 3L" and "2D + 2L," the solid line ABC in Fig. 8 would give values of K for different ratios of DL/TL. For example, suppose DL is 200 kips, and TL is 800 kips, DL/TL = 0.25 and K is 2.5 as indicated by line ABC.

When the dead load stress is of opposite sign to the live load, it is

generally suggested that only 80% of the dead load should be considered as effective in counter-acting the live load effects. The inclusion of wind load presents another problem. The ACI-ASCE recommendation of 1.6D + 1.0L + 2.0W is perhaps a reasonable approach; although when compared to steel bridges, factors of 1.5D + 1.5L + 1.5W would be more compatible.

Behavior under Service Conditions

The use of load factors in design may not always insure the proper behavior of a bridge under service conditions. A bridge may have sufficient overload capacity but may camber too much, may deflect and vibrate too much, may creep and shorten too much, or may crack too much. It is often necessary to check the design by the elastic theory, but the permissible stresses must be varied to fit different conditions. For example, high tensile or high compressive stresses may not be objectionable if localized in a given area where it will do no harm.

Another factor that must be considered is the actual load distribution in a slab or girder bridge. When elements of a bridge are rigidly connected together, such as by a sufficient amount of transverse post-tensioning, any localized load may be more evenly distributed than indicated by our empirical formulas, in both the elastic and the ultimate ranges. Thus the correctness of assumptions used for load distribution may affect the choice of load factors.

CONCLUSION

It is of paramount importance that proper load factors be selected for the design of prestressed concrete bridges. This is especially true since stresses given by the elastic theory do not always present a true picture of the reserve strength in the structure. However, it is not always possible to obtain consistent or optimum designs by using a simple set of load factors.

For bridges of the usual type and proportions, the relation between the behavior under service and the ultimate strength is pretty well known. Hence the usual load factors such as $K = 2$ or 2.5 would yield satisfactory results, so far as flexural strength is concerned. Not enough is known of the shear strength, combined shear and flexural strength, combined direct load and bending, etc., so that load factors cannot be definitely recommended for them. If the satisfactory behavior of the bridge under working loads can be assured, load factors such as 1.5D + 2.5L would give sufficient protection so far as occasional overloads are concerned.

For bridges with non-prestressed reinforcements and for continuous bridges, high load factors are often obtained if designed by the elastic theory using the ordinary allowable stresses. On the other hand, the use of a fixed factor such as $K = 2$ could yield a bridge that does not behave satisfactorily under service conditions. There are also cases such as tie rods for arches, where the load factor can be low without producing undesirable effects.

It must be concluded, therefore, that at this stage of our knowledge concerning prestressed concrete, we cannot set up a complete set of load factors applicable to members of all types and proportions. We have a lot to learn even for the simple case of flexural strength in simple beams. Safe values can be assigned, but they may not always yield economical or consistent designs. It is still necessary to use good judgment in the choice of load factors. This is perhaps true not only of prestressed concrete, but also of reinforced concrete and steel designs.

TABLE 1 — LOAD FACTORS FOR PRESTRESSED CONCRETE BRIDGES

Source	Description	D	L	D+L	D	L	W
England	Steel Concrete			2 2.5			
France		1	3	2			
Germany	Steel Concrete			1.75 2.6			
Switzerland		1.5	2.5	2.0			
BPR		1	3	2			
ACI	R.C. Bldgs	1.2	2.4	1.8 to 2.0*	1.2 1.2	2.4 0.6	0.6 2.4
ACI - ASCE Dec. 1956	Bldgs Bridges	1.4 1.6	2.3 2.4		1.4 1.6	1.0 1.0	1.8 2.0

Notes: D = dead load
L = live load plus impact
W = wind load or earthquake load
*Also for $D + L + \frac{W}{2}$ and for $D + \frac{L}{2} + W$

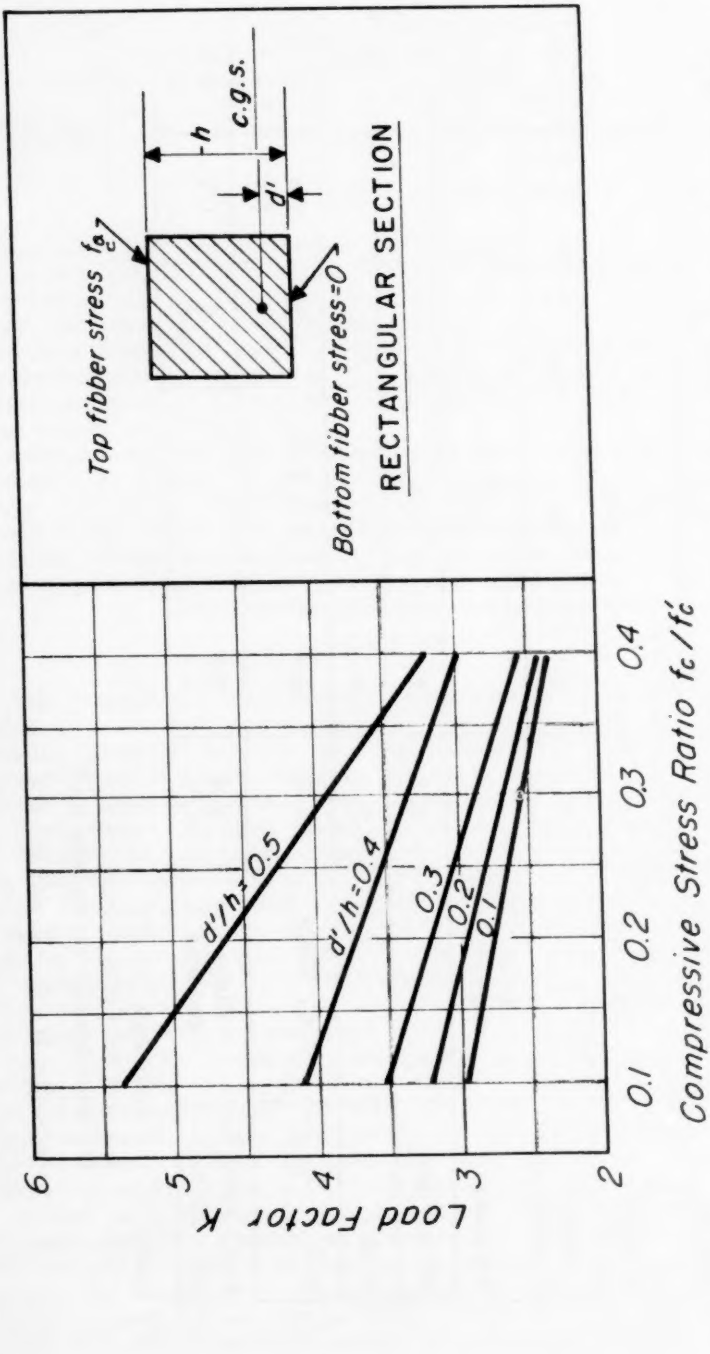


Fig. 1-LOAD FACTORS OF RECTANGULAR SECTIONS DESIGNED FOR NO TENSION.

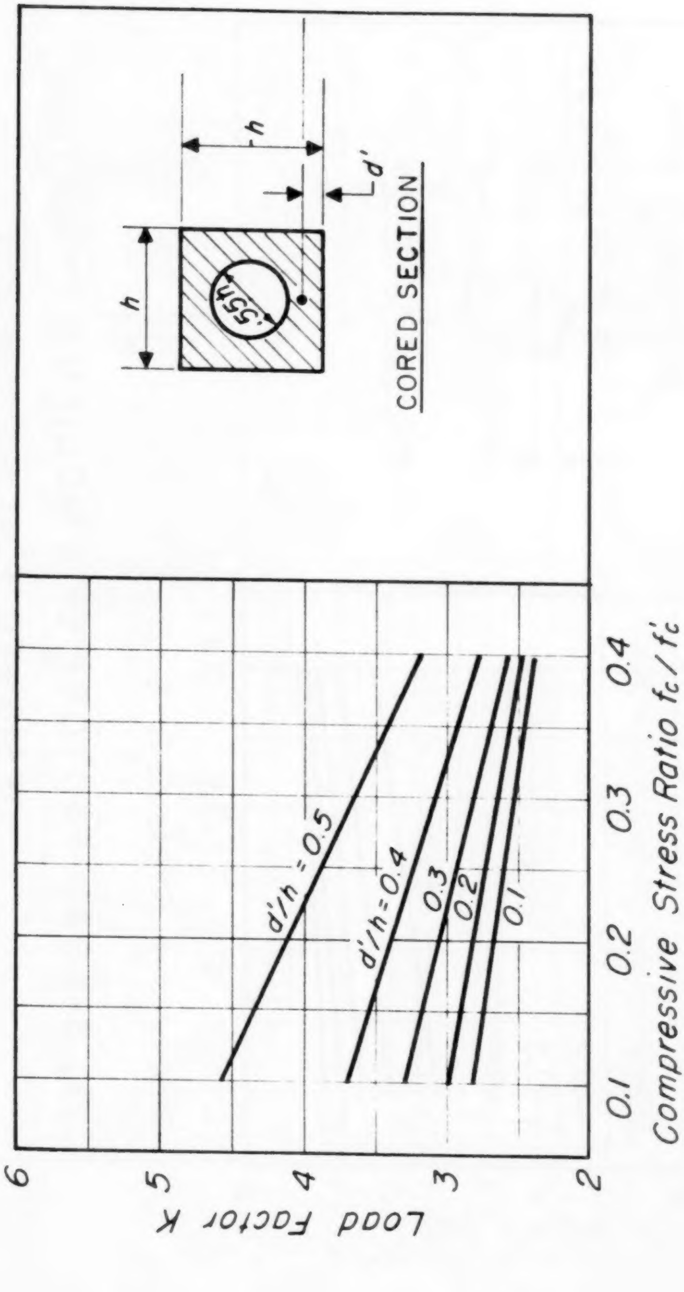


Fig.2 - LOAD FACTORS OF A CORED SECTION DESIGNED FOR NO TENSION.

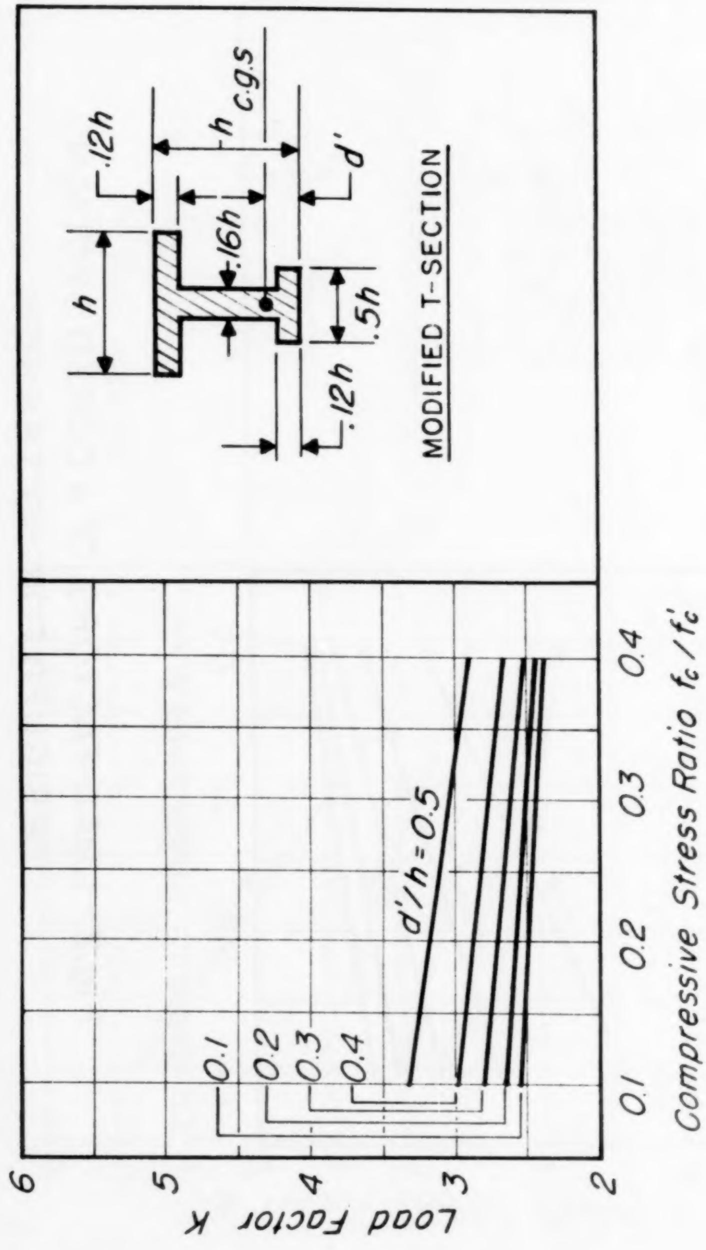


Fig. 3–LOAD FACTORS OF A MODIFIED T–SECTION DESIGNED FOR NO TENSION.

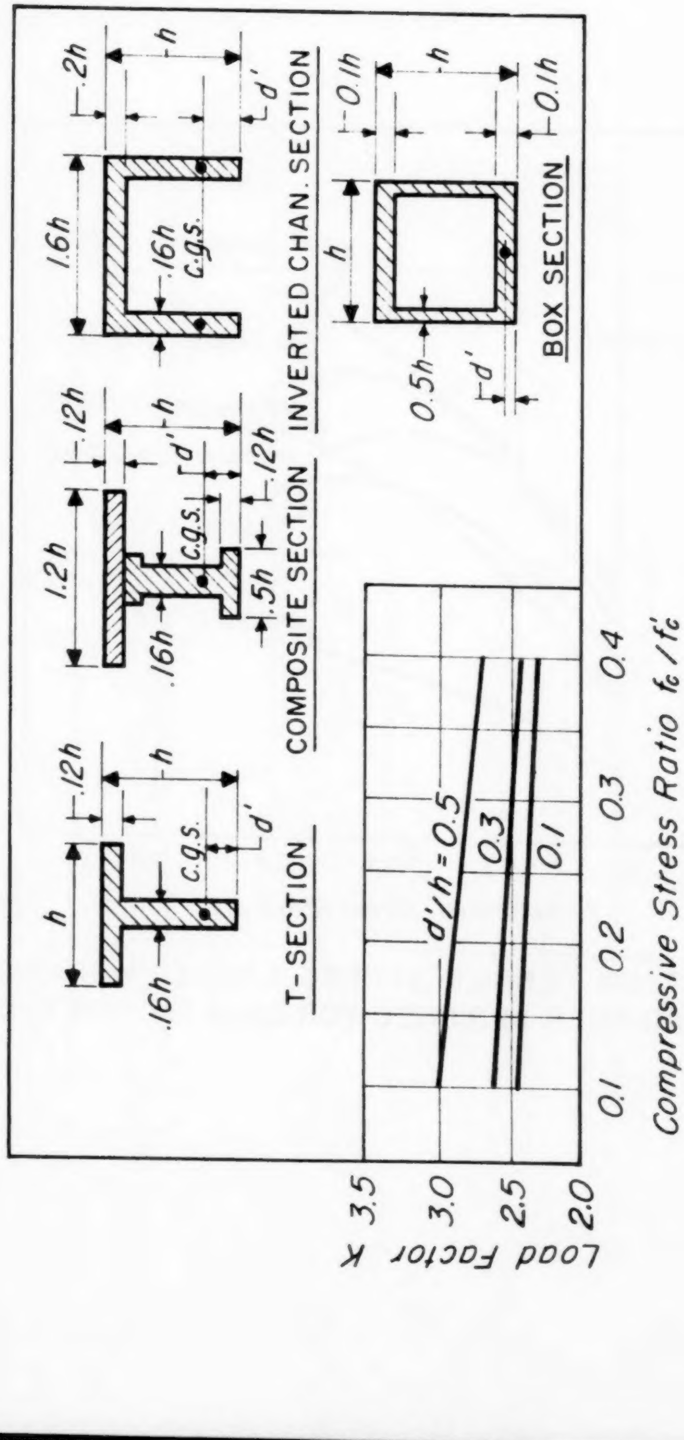


Fig. 4 - LOAD FACTORS OF SEVERAL BRIDGE SECTIONS DESIGNED FOR NO TENSION.

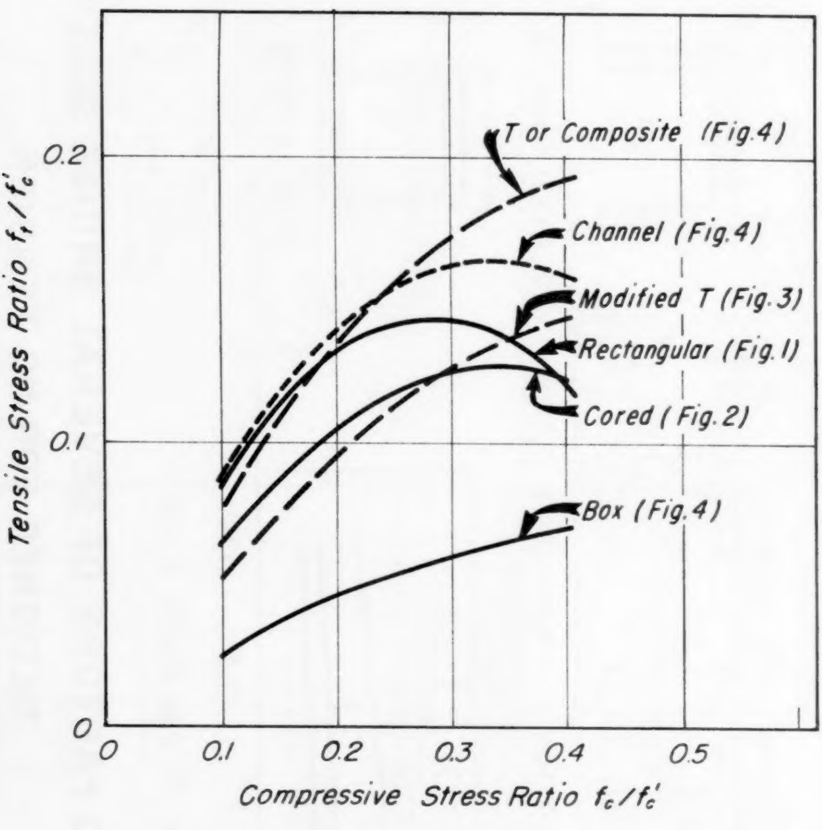


Fig. 5 - BOTTOM TENSILE STRESS f_t UNDER WORKING LOAD WHEN DESIGNED FOR LOAD FACTOR K=2

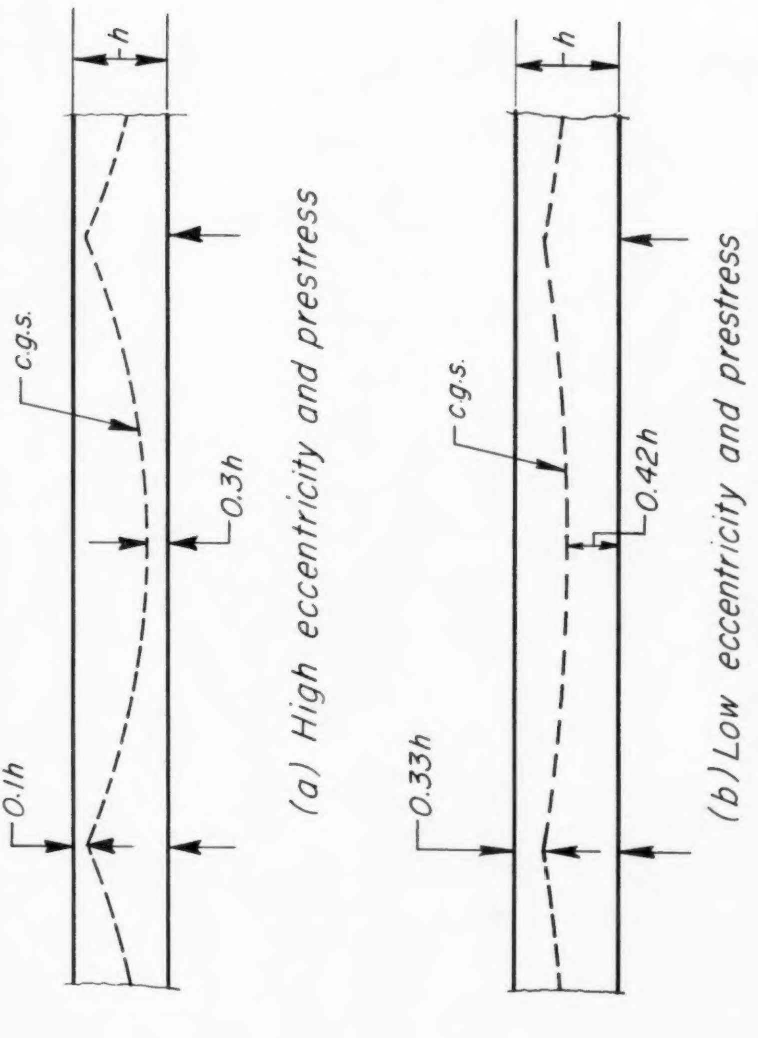


Fig.6 - CONTINUOUS BEAMS OF UNIFORM RECTANGULAR SECTION

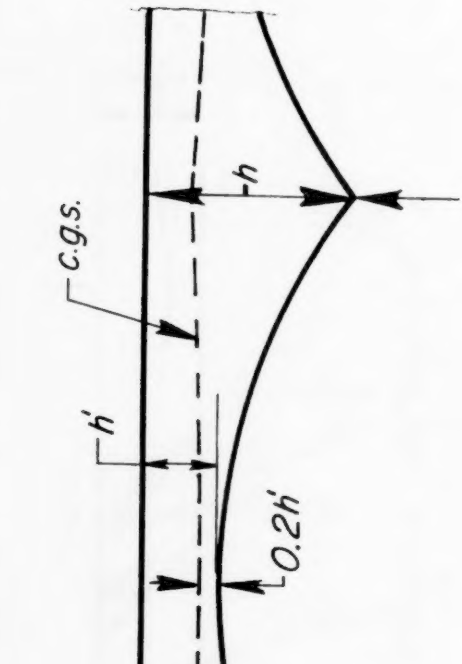
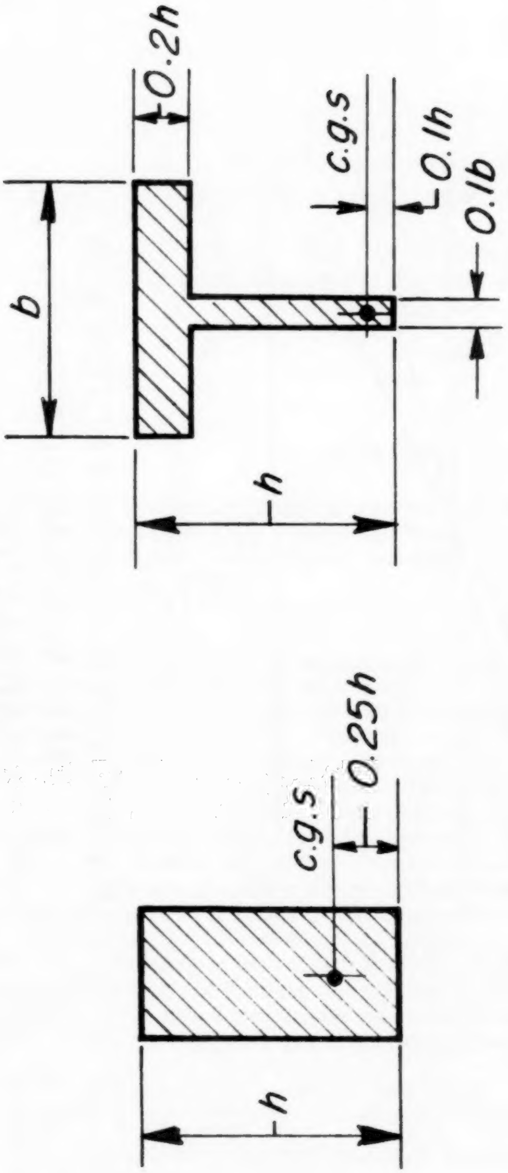


Fig. 7- CONTINUOUS BEAM OF VARYING BOX SECTION.



(a) A Rectangular Section

(b) A Tee Section

Fig. 8-ULTIMATE STRENGTH OF PRESTRESSED BEAMS AT TRANSFER.

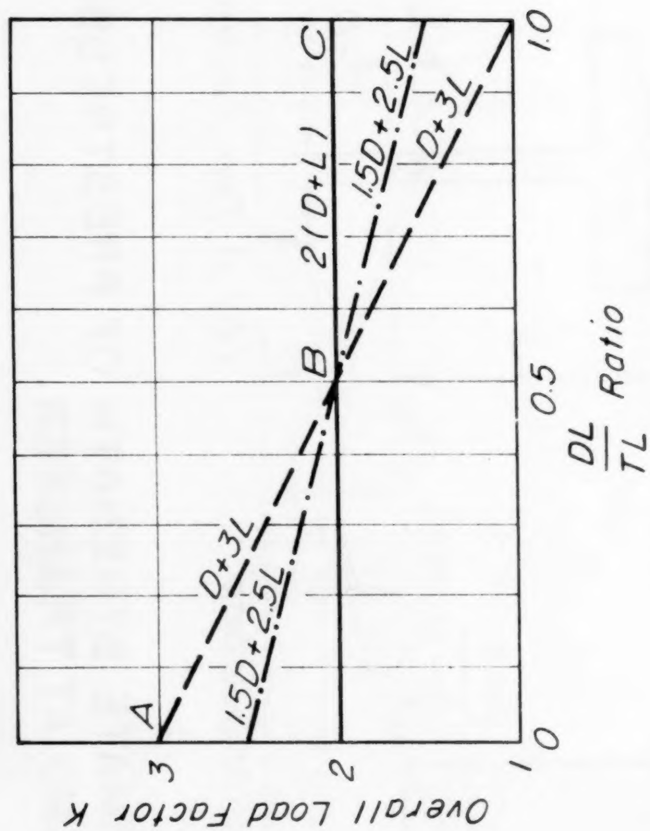


Fig. 9-OVERALL LOAD FACTOR FOR DIFFERENTIAL DEAD AND LIVE LOAD FACTOR.

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SYNOPSIS OF FIRST PROGRESS REPORT OF COMMITTEE ON FACTORS OF SAFETY*

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(Proc. Paper 1316)

ABSTRACT

This paper gives (1) definitions, (2) some statistical data regarding variations in physical properties of steel and concrete, and (3) tables and graphs for correlating factors of safety and probabilities of failure.

This synopsis aims to present a sketchy view of the work of the Committee on Factors of Safety, the purpose of which is to, (a) define the term "factor of safety" and related terms as applied to bridges, buildings, and similar structures; (b) correlate such terms with the corresponding probabilities; (c) survey the field as to currently employed factors; and (d) recommend forms of these factors to be employed in the future.

It should be understood clearly that such terms as factor of safety are without real meaning unless they are correlated with corresponding probabilities. The resisting properties of materials and the maximum load-effects to which structures are subjected vary. Even if one were cognitive enough, had sufficient time to calculate resistances of structures based on test values of materials exactly, and the construction were perfect, there would be considerable uncertainty in the ratio of the resistance of a given structure to the loading-effect to which it might be subjected in service. In the limit, the resistance of the structure may approach any positive value less than the calculated value, also the loading-effect in many cases may possibly increase

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** Chairman of Committee on Factors of Safety, Head of Structural Division, Jackson & Moreland, Inc., Engineers, Boston, Mass.

Note: Although this synopsis is believed to reflect the sense of the Committee's views, the full report which has not as yet been voted on may reflect views which vary in detail from those expressed here.

indefinitely beyond the value estimated. Even if the load-effects could be determined exactly, which practically speaking is the case for some types of structures, the variations in the resistance will be finite—not zero. By increasing the probable resistance, the factor of safety can be increased indefinitely and the probability of failure made extremely small, say 10^{-7}. However, it will still exist, be finite—not zero. It is imperative to realize that, regardless of how conservative the design and how well a structure is built, there is in the nature of things some probability of its failure.

The correlation of factors of safety and factors of serviceability with probabilities of survival and serviceability for each individual structure designed is impracticable. However, it is believed to be practicable to consider such correlation in the framing of design rules and regulations. It is highly desirable that such rules and regulations plainly state the factors and corresponding probabilities which they imply. To some, the probabilities convey much more meaning than the factors. Insurance underwriters having actuarial training and being realistic would be apt to disregard the factors and inquire: "What is the probability of loss?" It appears advisable to add that some members of the committee incline to the view that the committee is overemphasizing the importance of statistical and probability studies. Such studies, like other mathematical tools can be only guides which in all cases must be supplemented by the application of common sense and engineering judgment. However, full advantage should be taken of available mathematical tools to sharpen judgment and minimize the extent to which guessing must be employed.

Following the lead of A. M. Freudenthal, M. ASCE, as outlined in "Safety and the Probability of Structural Failure," published in Transactions ASCE Vol. 121, 1956, and earlier papers, considerable work has been done. However, it has become evident that this work will be of little avail until such time as structural engineers have acquired, (a) a statistical background regarding the resistance of materials and structures, including time-yield, dynamic and fatigue effects; (b) a similar background regarding load-effects; and (c) the necessary competence in the calculus of probability, which includes the elements of statistical analysis. Considerable educational effort is required within the profession. A not inconsiderable part of the committee's work has been self-education.

Unfortunately there does not appear to be unanimity, even within the profession, as to what is meant by the term "factor of safety" This has led to many semantic blocks, misunderstandings and false impressions. After considerable discussion extending over a number of years, it has been decided by the committee that two definitions are desirable. These are:

1. Minimum required factor of safety to assure that a given probability of failure P_F of the structure is not exceeded, is defined as the ratio (greater than unity) of R_o the mean[2] estimated resistance to collapse during the anticipated life of a large number of structures meant to be identical with the subject structure, and W_o the mean-load effect for which the subject structure is designed.

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2. The term "mean" is synonymous with "arithmetic average." It should not be confused with "median" (see Figs. 1 to 6) which for a group of measurements is the middle measurement and for the log-normal distribution is the geometric mean. (See footnote 4)

2. Minimum required factor of serviceability to assure that a given probability of the structure becoming unserviceable, for the purpose and during the anticipated life for which it is designed, is not exceeded, is defined as a similar ratio but with respect to serviceability rather than collapse.

It will be noted that these definitions have to do with the structure rather than with individual members of the structure; this because the resistance of a structure is not necessarily governed by the strength of individual members. For example, a structure which has redundant members and restraints may be safe and serviceable after any number of redundants have failed or become inoperative. It will also be noted that the definitions are framed in terms of resistance rather than in terms of stress; this because rational analysis indicates that stress per se plays a relatively minor part in the safety of structures. However, for structures subjected to pulsating loading, such as bridges, machine supports, etc., fatigue phenomena are of prime importance, and the strength of a member or of the structure may be governed by the fatigue stress applicable to the range of load-effects, time-rate of load applications, and the anticipated life of the structure. The term load-effect, rather than load, is used to take account of dynamic and other effects which depend upon properties of the structure as well as the load and time-rate of application. In many cases a load-effect may be due to time-yield, temperature changes, differential temperature, or movements of foundations.

Two simple examples of unserviceability as compared to failure by collapse may be helpful. Assume that a high office building is so constructed that the probability of collapse is negligible. However, during gusty wind storms, which are prevalent in that location, it vibrates to such an extent that the tenants become alarmed and vacate. It is "safe" but "unserviceable," because it does not fulfill the purpose for which it was designed. For the second example, assume that the pedestal for a turbo-generator is so constructed that, although the probability of collapse is almost nil, portions of the pedestal flex to such an extent during operation as to render proper operation of the machine difficult and unduly costly. Again, the structure is "safe" but "unserviceable."

Ordinarily one is warranted in choosing a considerably greater probability for the structure becoming unserviceable than for failure by collapse, and letting the factor of serviceability be somewhat smaller than the factor of safety. In case the principle of superposition applies, an adequate design for serviceability will usually result in an adequate design for safety. However, it should be emphasized that in case the principle of superposition does not apply, as will often be the case for members carrying compressive loads, an adequate design for either serviceability or safety will not assure an adequate design for the other. Failure to realize this simple fact may account for the preponderance of compression member failures. A safe rule is to calculate both factors independently.

In the definitions given, the terms defined are associated with given probabilities. When selecting these probabilities, consideration should be given to at least the following:

a) The type of failure—will it be without warning as for example the tensile failure of brittle material such as concrete; or on the other hand will increases in deformation give warning of impending danger?

b) The value of human lives which may be lost, in case of failure.

c) The importance of the structure and its cost, including costs other than of the structure itself incurred on account of it being out of service.

d) The capitalized cost of maintaining the structure in serviceable condition.

e) The replacement cost in case of failure.

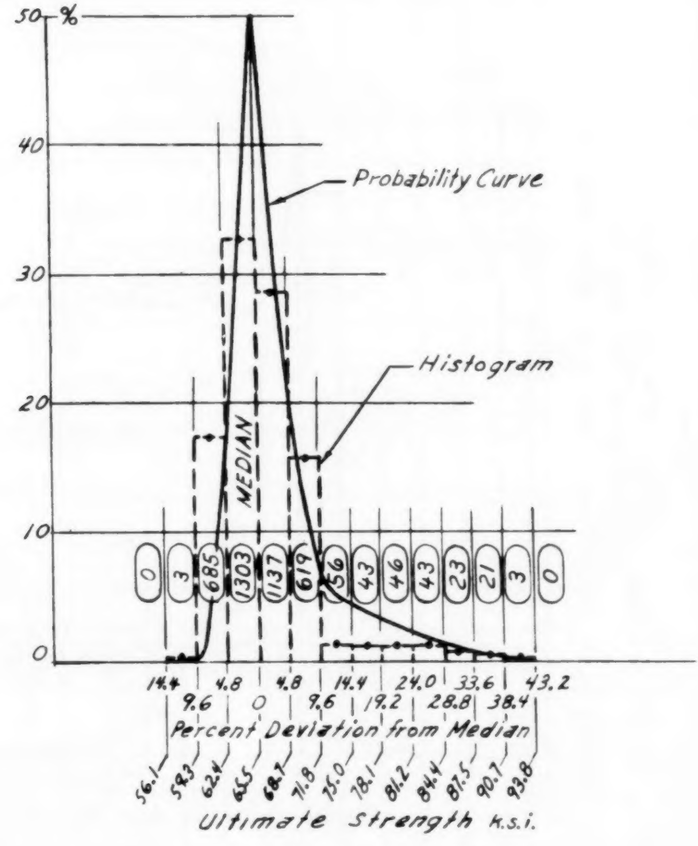
f) A charge against the structure equal to the capitalized total cost of failure multiplied by the probability of its occurrence.

It will be noted that these items overlap somewhat; also that a number of them raise questions which must be answered largely on the basis of judgment.

The factor of safety and the factor of serviceability can be considered as "load factors" by which the mean of the design load-effect is multiplied to equal the mean calculated resistance which may in some cases correspond to and be limited by a comparatively small deformation required to render the structure unserviceable, while in other cases it may be almost as high as the ultimate strength. Corresponding to each of these and to intermediate cases, there are limiting stresses which should not be exceeded. Provided suitable, "interaction formulas" applicable to cases, such as flexure combined with axial load stresses, are available, the design or analysis can be executed in accordance with the limiting stress concept. Alternatively, they can be executed in accordance with the limiting strength concept. In either case, due account must be taken of deformations which may render the structure unserviceable.

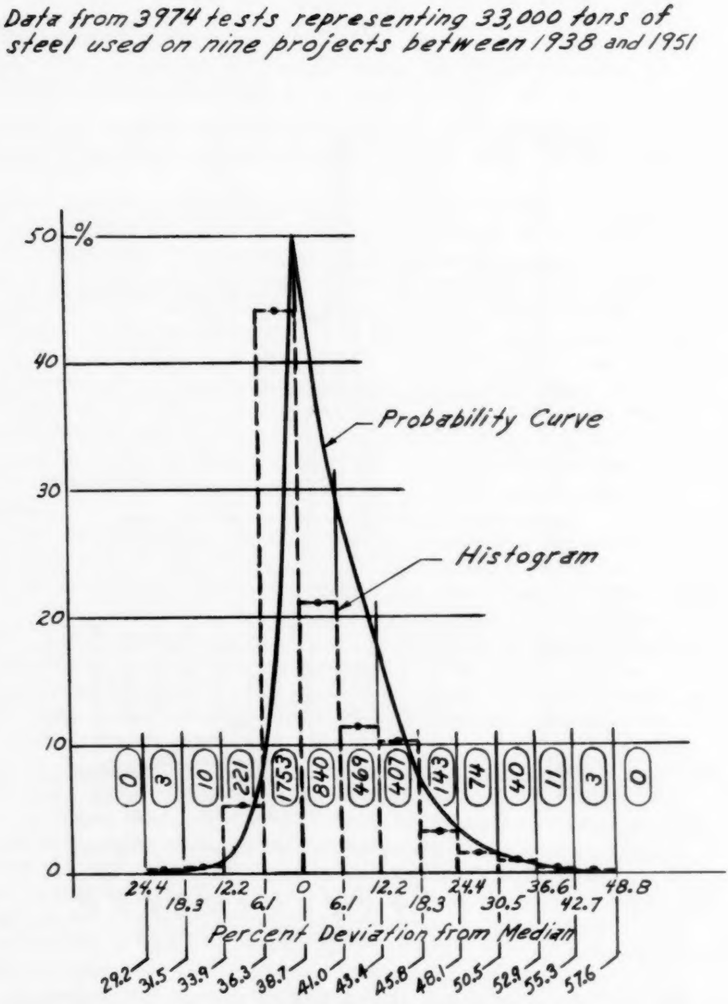
In order that the resistance of a structure in the design stage can be estimated, it is necessary to have statistical data pertaining to the in-place strength, lower yield point, and fatigue strength of materials similar to those which will be used. These data should preferably include the results of a large number of tests and should be in such form that the following quantities and graphs can be determined and prepared for each material involved: median and average strengths, coefficients of variation, values of lowest and highest tests, histograms and cumulative frequency graphs indicating the statistical distribution of test results. Figures 1 to 5 and Table 1 indicate some such data for structural steel, concrete, and concrete reinforcing steel. Fig. 1 has to do with the ultimate strength and Fig. 2 pertains to the yield strength of structural steel. The ordinates of the histograms shown as cells drawn in dash lines represent the relative frequency of the test results occurring within the cell widths indicated by the abscissas at their bases. In order to have a standard for comparison, the width of the cells in these and the following figures has been arbitrarily chosen as one probable deviation according to the Gauss-Laplace normal distribution;[1,2] and the graphs in terms of percentages have all been drawn to the same scale. It will be recalled that for the normal distribution, a probable deviation is the deviation which is as likely to be exceeded as not, and equals 0.6745 of a standard deviation; also that for any distribution, a standard deviation equals the root mean square (or in the language of mechanics the radius of gyration) of the test results with respect to the mean. The figures shown in circles indicate the number of test results pertaining to each cell of the histograms. The curves drawn in solid lines represent the cumulative frequency or probability of occurrence. The ordinates pertaining to these curves indicate: (a) to the left of the median, the percentage of total number of samples testing below the value indicated by the corresponding abscissa; and (b) to the right of the

Data from 3982 tests representing 33,000 tons of steel used on nine projects between 1938 and 1951.



HISTOGRAM & PROBABILITY CURVE OF ULTIMATE STRENGTH OF ASTM A-7 STRUCTURAL STEEL FROM MILL TEST REPORTS

FIG. 1



HISTOGRAM & PROBABILITY CURVE OF YIELD STRENGTH OF ASTM A-7 STRUCTURAL STEEL FROM MILL TEST REPORTS

FIG. 2

TABLE I

	Structural Steel Ultimate	Structural Steel Yield Point	Concrete Ultimate Good Control	Concrete Ultimate Poor Control	Concrete Reinf. Steel Yield Point
Fig. No.	1	2	3	4	5
Standard Deviation	4.66	3.5	0.54	1.54	5.95
Coefficient of Variation	0.071	0.087	0.104	0.263	0.124
Highest Test Strength	91.14	56.6	6.98	9.10	64.9
Mean " "	66.27	40.0	5.18	5.85	47.74
Lowest " "	57.36	31.1	3.81	1.50	38.95
Desired Minimum Strength	60.0	33.0	3.83	4.3	40.0

Standard deviation & strength values in kips per sq. in.

$$\text{Coefficient of variation} = \frac{\text{Standard deviation}}{\text{Mean test strength}}$$

median, the percentage of total number of samples testing above the value indicated by the corresponding abscissa.

The strength f_c' of the concrete represented by Fig. 3 used in design of the structures in which the concrete was placed was 3.83 ksi. However, the control of the concrete was aimed at a minimum strength of 4.50 ksi. It will be noted that approximately 10% of the test results fell below this latter figure which is midway between the median and the lowest test value, and that only a negligible portion fell below the f_c' value used in design. Experience indicates that such values are typical for cases in which considerable case is taken as to control. Such control may be characterized as "good."

Fig. 4 in contrast to Fig. 3 indicates the variations in concrete strength which may be expected if the control is poor. Fig. 5 indicates the variations in yield strength of new billet steel reinforcing bars. Pertinent data in addition to those shown in Figs. 1 to 5 are given in Table 1.

The data shown for yield strengths of structural steel and for concrete under good control have been exhibited by Freudenthal in Transactions Vol. 121, 1956, on log-normal[4] probability paper and the data pertaining to strength of concrete under poor control on "extremal"(4) probability paper. Such presentations have the distinct advantage of greatly facilitating extrapolation and determining which type of known statistical distributions the data fits best.

Figs. 1 and 2 pertaining to structural steel were made from data as reported in mill test reports. There is considerable evidence that such reports reflect strengths somewhat higher than would pertain to the material under service conditions. The reasons for this, together with the probable degree of difference as to yield strength, are as follows:

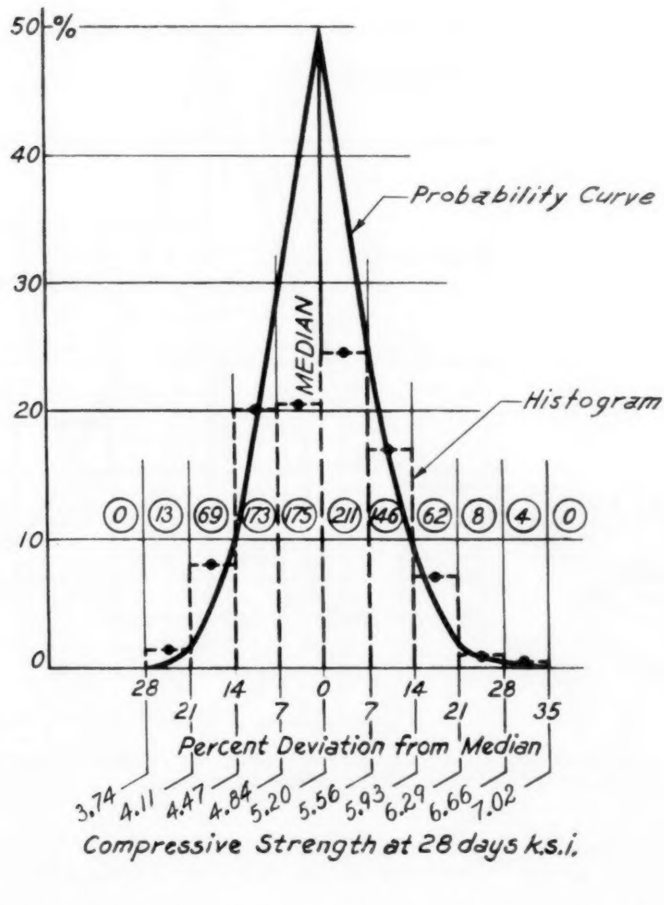
	Difference may be
Ordinarily the unstable upper rather than the stable lower yield strength is reported.	5 to 10%
The high time-rate of strain employed in making tests.	Approximately 10%
Tests are ordinarily made on coupons cut from the web rather than from the thicker metal in the flanges.	5 to 10%
Combined, these three effects may total from	20 to 30%

These values are given as examples only; they may not be typical. However, if their sum is taken as 25%, it may mean that an average yield strength reported as 40 ksi. should be reduced to 30 ksi. in order to pertain to the material under service conditions. The difference on account of the coupons being cut from the web rather than the flanges will vary considerably dependent upon the cross-sectional makeup of the member. Data pertaining to the thicker plates and the flanges of the heavier sections such as the 36 WF 300 beam section and many column sections are rather meager.

In order that the load-effect may be estimated, it is necessary to have statistical data pertaining to the loads and their effects. Such data appear to

4. If the log of the variable is distributed in accordance with the Gauss-Laplace normal distribution, the distribution is said to be "log-normal." (See also Chapter 10 of reference (2).)

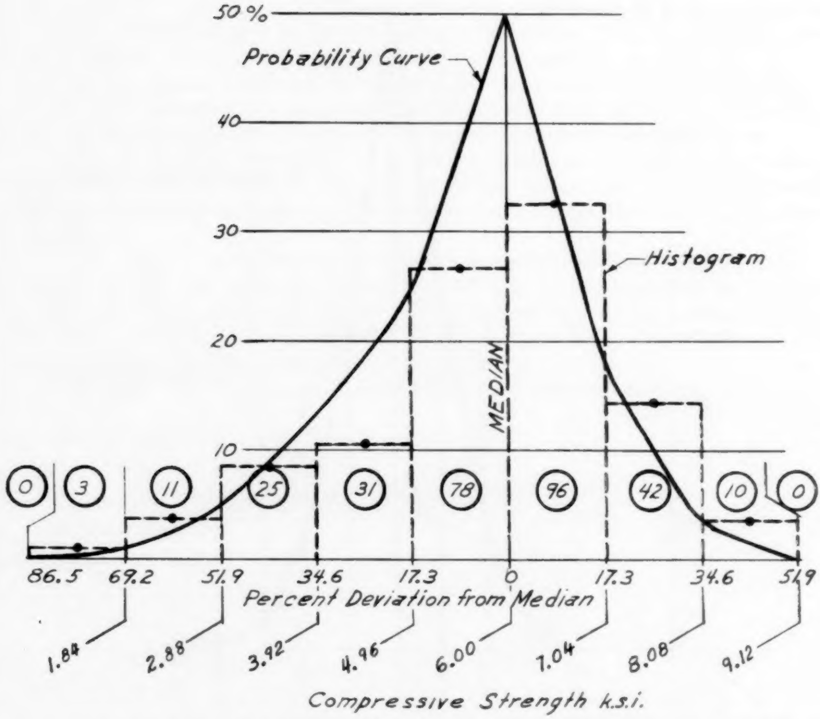
Data from 28 day compression tests on 861 test cylinders representing approximately 33,300 cu. yds.



HISTOGRAM & PROBABILITY CURVE OF COMPRESSIVE STRENGTH OF CONCRETE INDICATING GOOD CONTROL

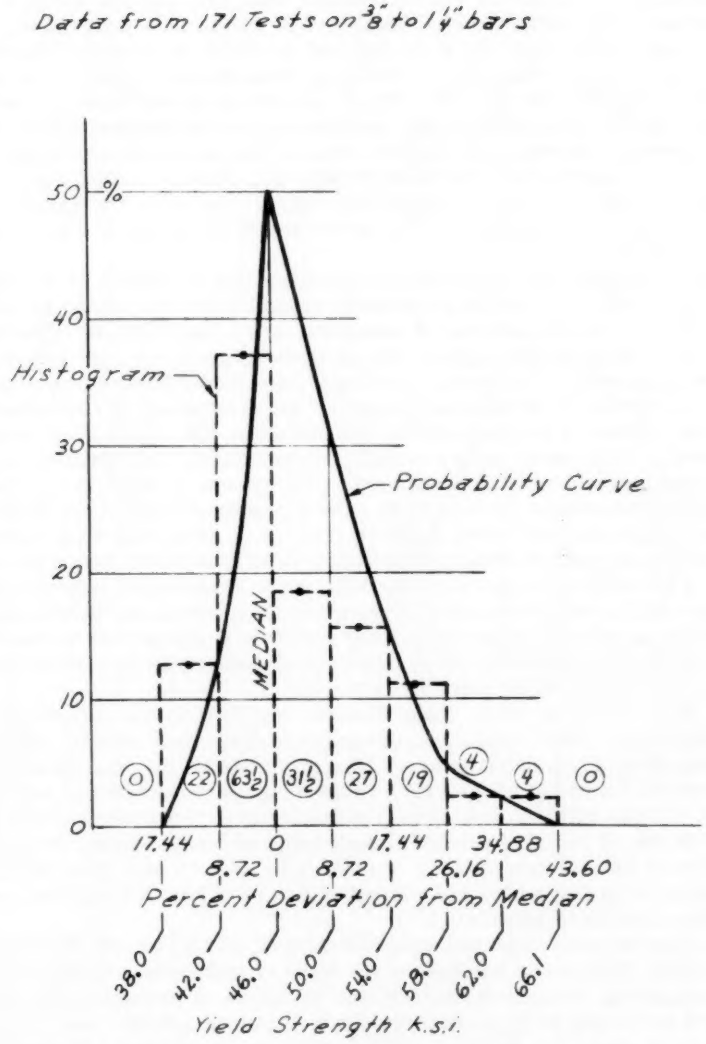
FIG. 3

Data from 28 day compression tests on 296 test cylinders representing approximately 3300 cu. yds.



HISTOGRAM & PROBABILITY CURVE OF COMPRESSIVE STRENGTH OF CONCRETE INDICATING POOR CONTROL.

FIG. 4



HISTOGRAM & PROBABILITY CURVE FOR INTERMEDIATE GRADE NEW BILLET STEEL REINFORCING BARS

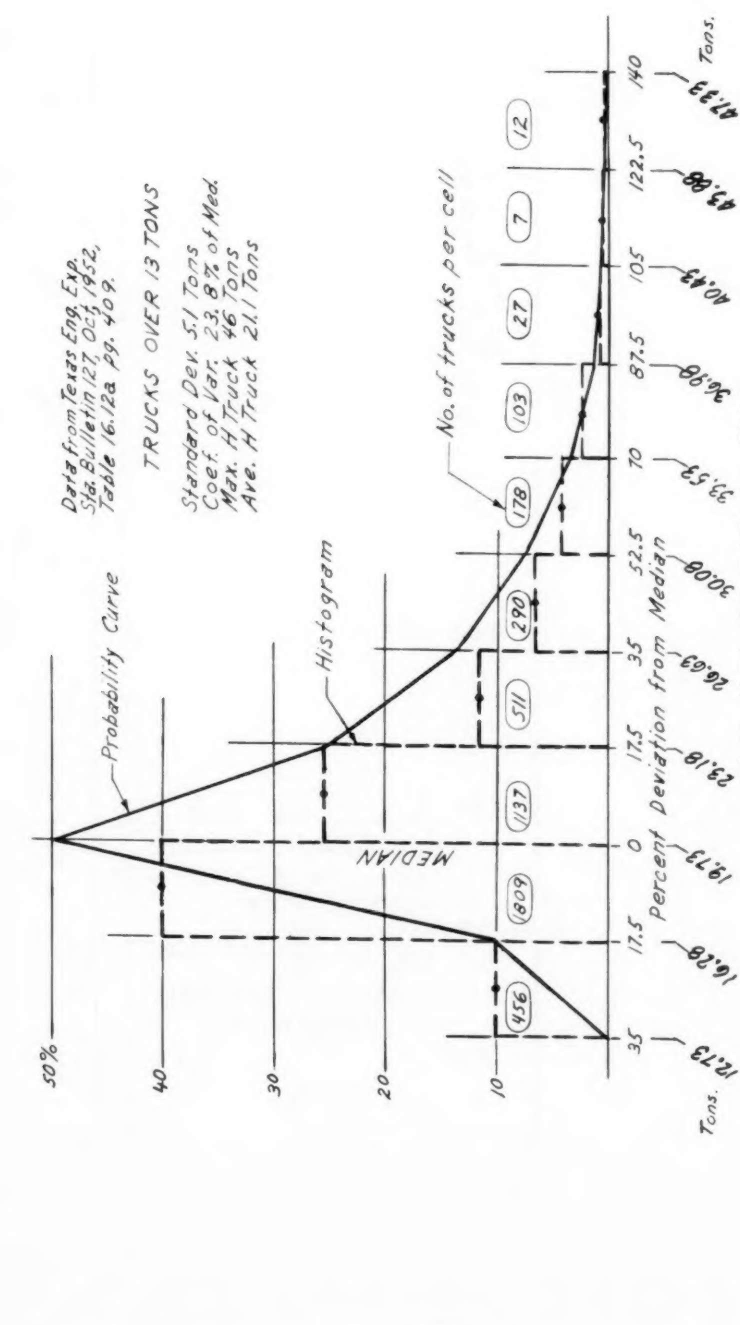
FIG. 5

be quite scarce. Some data regarding loads have been published in Texas Engineering Experiment Station Bulletins 127, 131, 132 and 135 by H. K. Stephenson M. ASCE and K. Cloninger J.M. ASCE; in "Probabilities of Traffic Loads on Bridges," by S. O. Asplund M. ASCE, Proceeding Separate No. 585; and in "Frequency of Maximum Wind Speeds," by H. C. S. Thom, Proceeding Separate No. 539. Fig. 6 has been prepared from tabular data presented in Texas Engineering Experiment Station Bulletin 127 and is here presented to illustrate the general order of the variations which may be expected. The same data has been presented by Freudenthal on log-normal paper and this distribution compared with Poisson's.(1) The values of the maximum loads, 46 tons, and the coefficient of variation, 23.8%, are especially noteworthy.

In some cases such as those of fluid loads and pressures in storage tanks, the maximum load-effects practically speaking are not subject to random variations; the probabilities of occurrence may, therefore, be taken as equal to unity. In many other cases such as those of maximum load-effects on bridges, runways of airports, warehouses and power houses, the collection and interpretation of adequate statistical data pertaining to load-effects are indeed difficult if not impossible. Furthermore, the values may change substantially after the structure is built, or even during construction. Such changes can only be anticipated by the employment of engineering judgment based on experience. It is only in case that adequate data, such as for example those pertinent to water loads on dams made from long-time records—including adequate allowances for floods which may occur during the useful life of the structure—are available that use of a statistical approach to the values of load-effects is warranted. Since the analysis can be no better than the data on which it is based, in many cases it is just as well to estimate the values of design load-effects by means of engineering judgment and consider the probability of occurrence as unity.

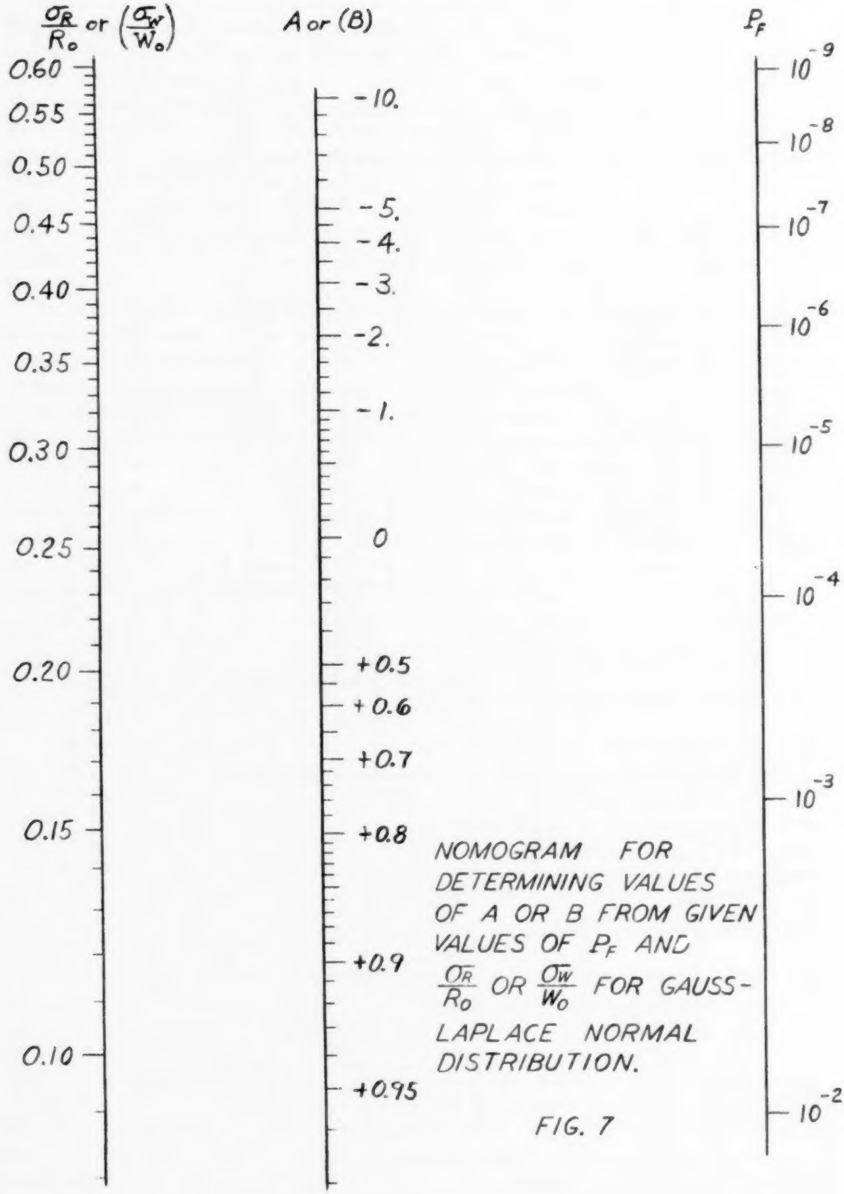
With respect to loads, the profession is primarily interested in the total actually experienced static plus dynamic and other load-effects, and their degree of variation on structures. The only possible way that these values can be determined at all precisely is by comprehensive tests on actual structures such as railroad and highway officials have been conducting over the past decade or so. The results of such tests, of course, must be put in statistical form before they are of much use. In such form they would be of great value as they would make feasible the use of known statistical methods and the calculus of probability.

Having the statistical data mentioned above, estimates can be made as to the values of the mean resistance, the mean of maximum load-effect, the corresponding standard deviations, and the nature of probability density graphs pertaining to the resistance of the structure and the load-effect. Factors of safety (or serviceability) can then be correlated with probability of failure (or unserviceability). The probability density graphs ordinarily can be approximated by the log-normal distribution, or by an "extremal" distribution. These and other distributions are described in Freudenthal's paper mentioned previously. Methods of correlating probabilities and the pertinent factors are also given in that paper and extended in the discussions; the evaluation of the resulting relation for the Gauss-Laplace normal distribution of the load-effect and the resistance is indicated in the form of alignment charts in Figs. 7 and 8 and for the log-normal distribution in Fig. 9. In these figures and the following



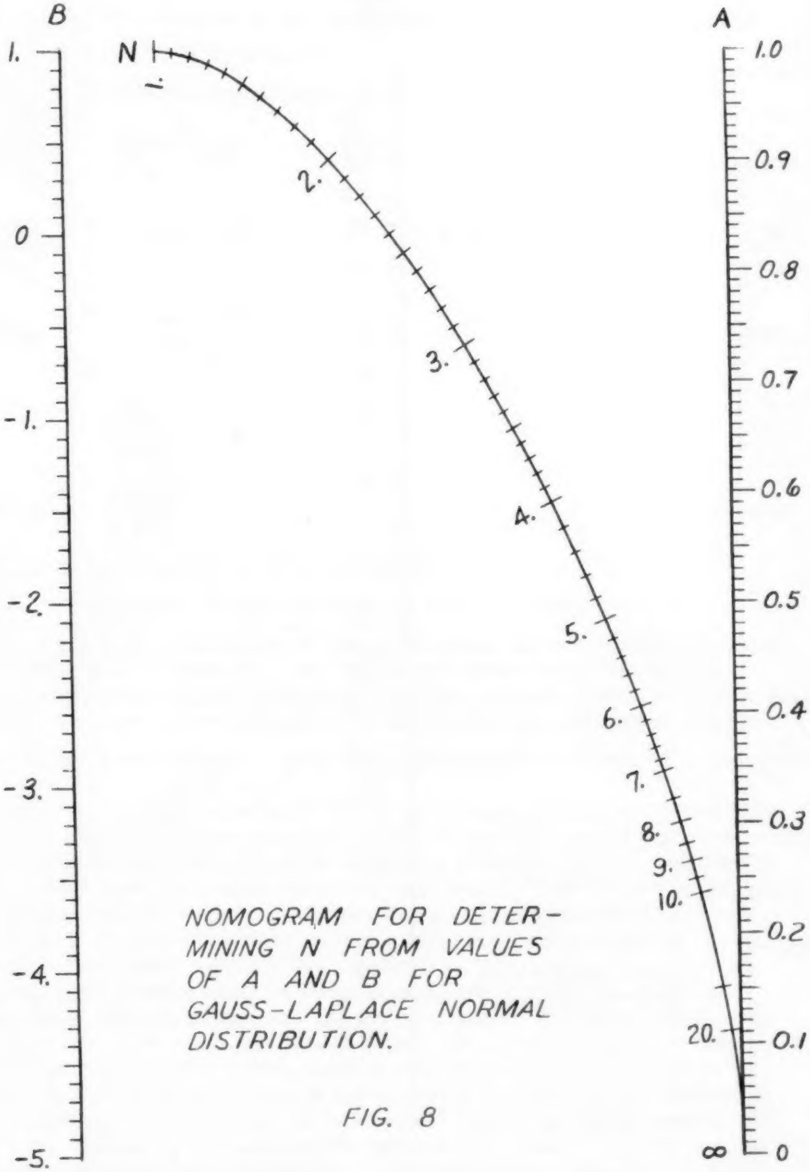
EQUIV. H TRUCK LOADINGS - TONS
4530 HEAVY TRUCKS - ALL TYPES
INDEPENDENT OF SPAN

FIG. 6



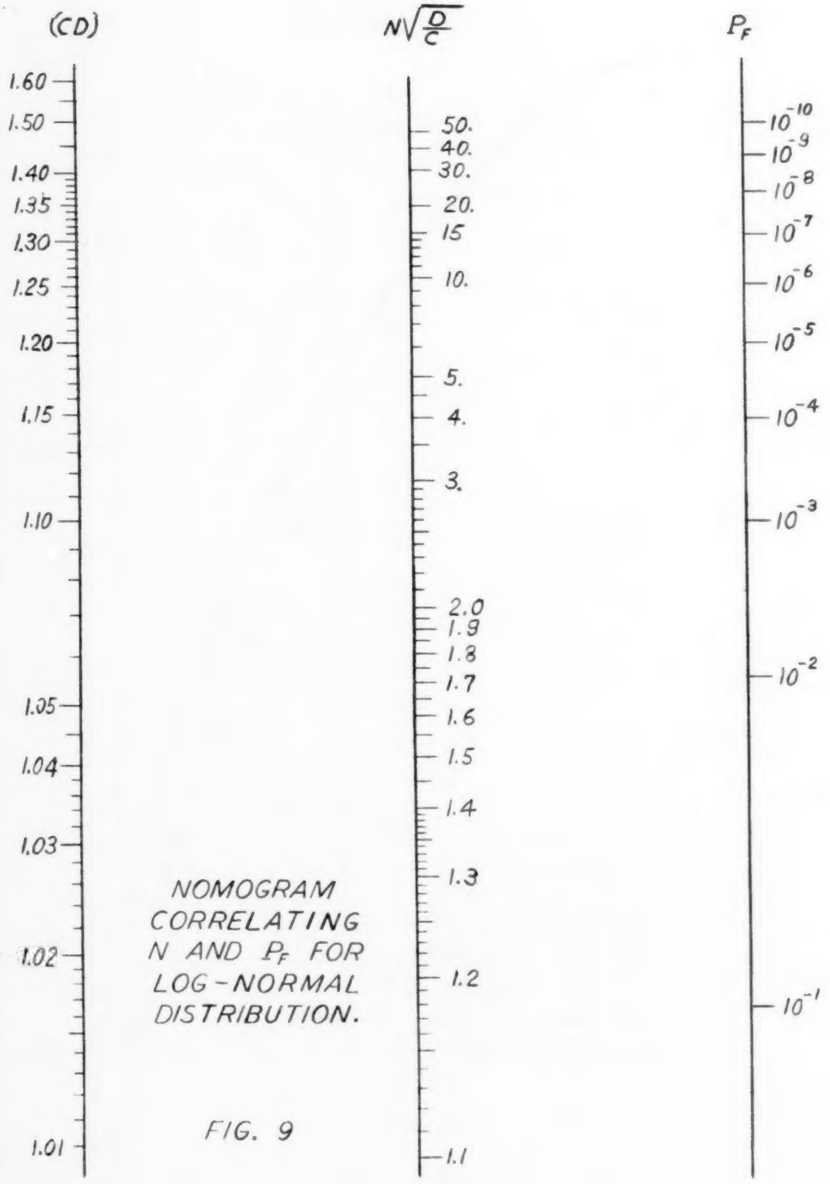
NOMOGRAM FOR DETERMINING VALUES OF A OR B FROM GIVEN VALUES OF P_F AND $\frac{\sigma_R}{R_O}$ OR $\frac{\sigma_W}{W_O}$ FOR GAUSS-LAPLACE NORMAL DISTRIBUTION.

FIG. 7



NOMOGRAM FOR DETERMINING N FROM VALUES OF A AND B FOR GAUSS-LAPLACE NORMAL DISTRIBUTION.

FIG. 8



NOMOGRAM CORRELATING N AND P_F FOR LOG-NORMAL DISTRIBUTION.

FIG. 9

σ_R = standard deviation of the resistance.

σ_W = standard deviation of the design load-effect.

R_o = mean value of resistance.

W_o = mean value of design load-effect.

$$\rho_o = \frac{R_o - W_o}{\sqrt{\sigma_R^2 + \sigma_W^2}} > 0 \tag{1}$$

$$A = 1 - \rho_o^2 \left(\frac{\sigma_R}{R_o}\right)^2 ,0 < A < 1 \tag{2}$$

$$B = 1 - \rho_o^2 \left(\frac{\sigma_W}{W_o}\right)^2 \leqq 1 \tag{3}$$

$$C = \left(\frac{\sigma_R}{R_o}\right)^2 + 1 \tag{4}$$

$$D = \left(\frac{\sigma_W}{W_o}\right)^2 + 1 \tag{5}$$

N = factor of safety (or of serviceability).

P_F = probability of failure (or of being unserviceable).

Tables 2 and 3 show the values of factors of safety (or serviceability) corresponding to $P_F = 10^{-4}$ and $P_F = 10^{-6}$ calculated on the basis that the log-normal distribution applies both to resistances and load-effect. This distribution has been chosen because it is believed to have the largest field of application for our purpose. Given the coefficients of variation $\frac{\sigma R}{R_o}$ and $\frac{\sigma W}{W_o}$ pertaining to resistance and load-effect, correlation between the probability of failure and the required minimum factor of safety to ensure that such probability is not exceeded can be made at a glance from these and similar tables. Such data can also be exhibited in graphical form as in Fig. 10, which emphasizes the not too well known fact that a comparative small decrease (say 10%) in the factor of safety causes a large increase (say 900%) in the probability of failure. It indicates that from an insurance point of view, it pays not to be parsimonious on items such as adequate engineering job supervision, which will increase the factor of safety somewhat and thereby decrease the probability of failure greatly. The difference between one in 10,000, and one in 100,000 structures failing is by no means negligible.

Fig. 11 compares the values of the minimum required factors of safety corresponding to a probability of failure 10^{-6}, based on the log-normal distribution, with those computed on the basis that the Gauss-Laplace normal distribution applies both to resistance and load-effects. Some of the differences will be noted as major. It is only in case both coefficients of variation are small that the agreement is close. Although the most popular and without doubt having wide fields of application, the Gauss-Laplace distribution does not appear to be suitable for the cases discussed here. This is largely

TABLE II Minimum Required Factor of Safety

for

$P_F = 10^{-4}$ and Log-Normal Dist. of R & W

$\frac{\sigma_W}{W_o}$ \ $\frac{\sigma_R}{R_o}$	0	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40
0	-	1.21	1.46	1.76	2.13	2.58	3.11	3.75	4.51
0.05	1.20	1.30	1.52	1.81	2.18	2.62	3.16	3.80	4.56
0.10	1.44	1.51	1.69	1.96	2.32	2.76	3.29	3.94	4.71
0.15	1.72	1.78	1.94	2.19	2.54	2.97	3.51	4.17	4.95
0.20	2.05	2.10	2.25	2.49	2.83	3.27	3.82	4.49	5.29
0.25	2.42	2.47	2.62	2.86	3.20	3.65	4.21	4.90	5.72
0.30	2.85	2.90	3.05	3.30	3.65	4.10	4.68	5.40	6.25
0.35	3.34	3.39	3.54	3.80	4.16	4.64	5.24	5.98	6.87
0.40	3.89	3.94	4.10	4.36	4.74	5.24	5.88	6.65	7.58

TABLE III Minimum Required Factor of Safety

for

$P_F = 10^{-6}$ and Log-Normal Dist. of R & W

$\frac{\sigma_W}{W_o}$ \ $\frac{\sigma_R}{R_o}$	0	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40
0	-	1.27	1.61	2.06	2.61	3.32	4.22	5.33	6.72
0.05	1.27	1.40	1.71	2.13	2.69	3.40	4.29	5.42	6.82
0.10	1.60	1.69	1.96	2.36	2.91	3.63	4.53	5.68	7.10
0.15	2.01	2.09	2.33	2.73	3.28	4.00	4.94	6.12	7.59
0.20	2.51	2.59	2.83	3.22	3.78	4.54	5.51	6.74	8.27
0.25	3.13	3.21	3.45	3.85	4.44	5.23	6.26	7.56	9.18
0.30	3.87	3.95	4.20	4.63	5.26	6.10	7.20	8.58	10.32
0.35	4.74	4.84	5.11	5.57	6.24	7.16	8.34	9.83	11.68
0.40	5.79	5.89	6.18	6.69	7.42	8.41	9.69	11.31	13.32

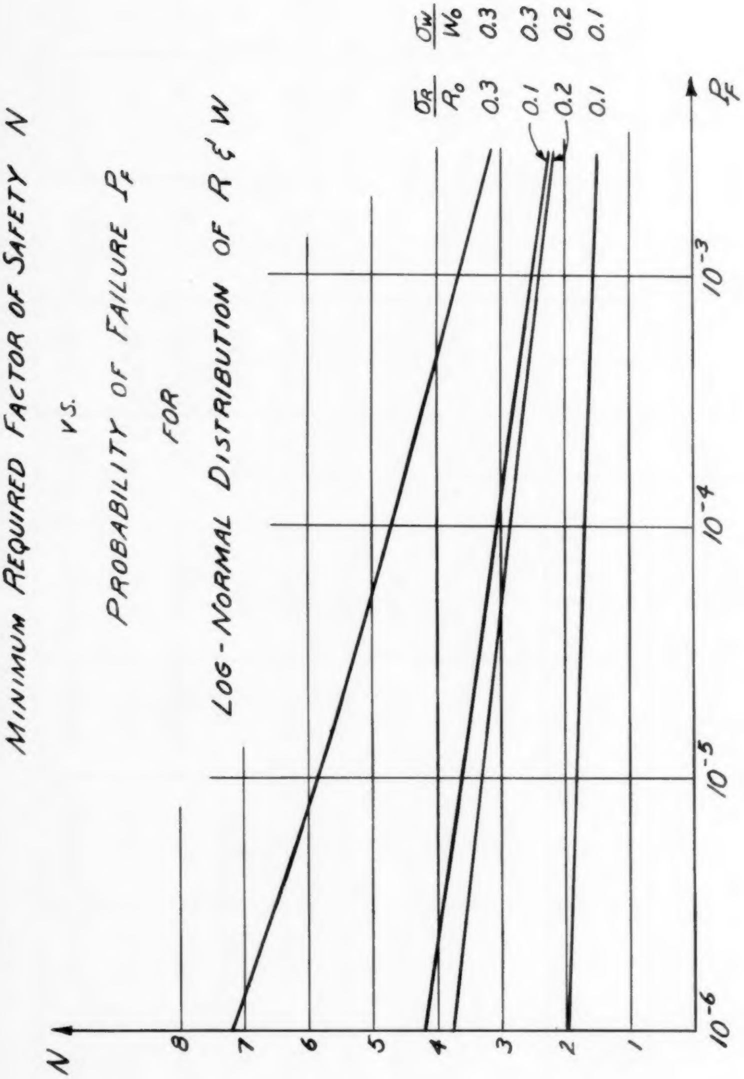

MINIMUM REQUIRED FACTOR OF SAFETY N
VS.
PROBABILITY OF FAILURE P_F
FOR
LOG-NORMAL DISTRIBUTION OF R & W
N
8
7
6
5
4
3
2
1
10^{-6}
10^{-5}
10^{-4}
10^{-3}
P_F
$\frac{\sigma_R}{R_o}$ $\frac{\sigma_W}{W_o}$
0.3 0.3
0.1 0.3
0.2 0.2
0.1 0.1

FIG. 10

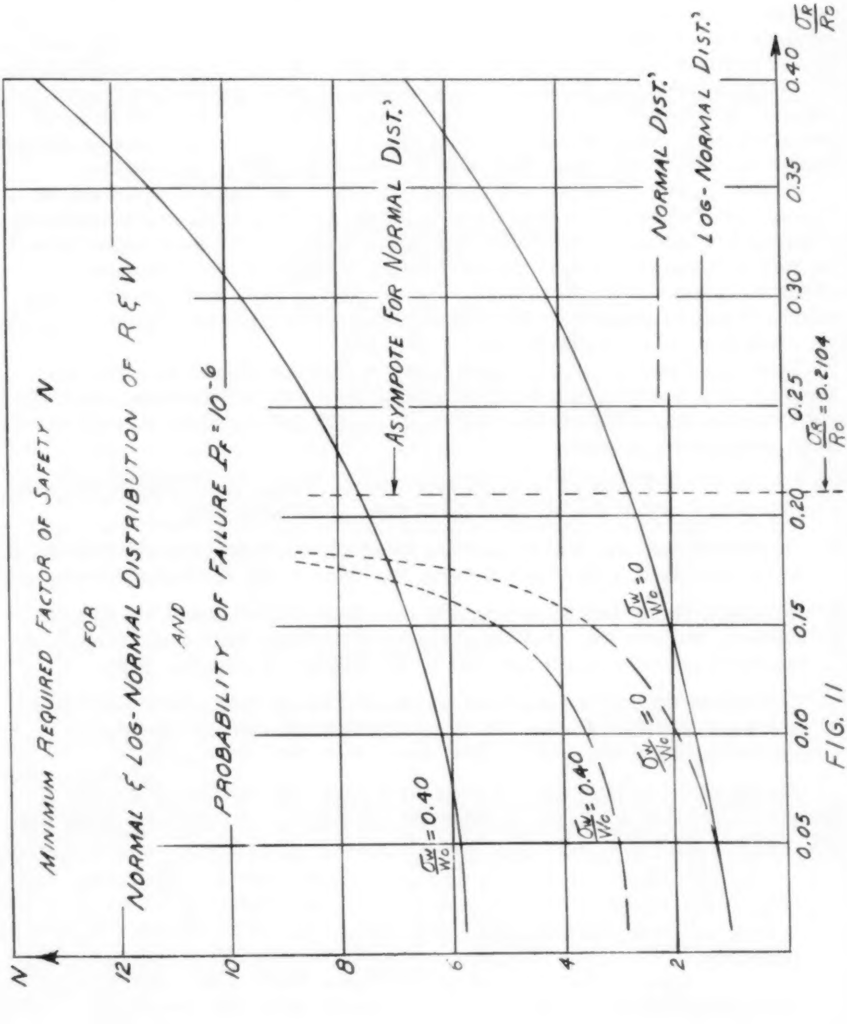

MINIMUM REQUIRED FACTOR OF SAFETY N
FOR
NORMAL & LOG-NORMAL DISTRIBUTION OF R & W
AND
PROBABILITY OF FAILURE $P_F = 10^{-6}$
ASYMPOTE FOR NORMAL DIST.
NORMAL DIST.
LOG-NORMAL DIST.
$\frac{\sigma_W}{W_o} = 0.40$
$\frac{\sigma_W}{W_o} = 0.40$
$\frac{\sigma_W}{W_o} = 0$
$\frac{\sigma_W}{W_o} = 0$
$\frac{\sigma_R}{R_o} = 0.2104$
N
12
10
8
6
4
2
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
$\frac{\sigma_R}{R_o}$

FIG. 11

because the left tail of the distribution curve extends past zero to minus infinity, whereas negative values are obviously impossible with respect to factors of safety. When dealing with such small probabilities as those with which this discussion is concerned, accuracy of the extreme values (represented by the tails) is of prime importance. Examination of data pertaining to materials in case good or even fair control is exercised, and that pertaining to loads on highway bridges shows that they fit the log-normal distribution quite closely and that the normal distribution is not a good fit for the extreme values. If the control is poor as represented by Fig. 4 pertaining to some concrete jobs, which shows a marked skew to the left, the data can be approximated by an extremal distribution such as Gumbel's[4] or Poisson's.

In conclusion, attention is directed to the British "Report on Structural Safety," published in The Structural Engineer for May 1955, and discussions published in the same journal for September 1956. From this British report as well as from the work of this committee, it appears that, although considerable progress has been made, a great deal of work remains to be done before it will be possible to determine factors of safety and related quantities on scientific and economic bases.

Those who have not had an opportunity to acquaint themselves with the language and mathematical methods of statistics and probabilities may find the following listed references helpful. It is suggested that these works be read in the order given.

1. "Facts From Figures" by M. J. Moroney, a Pelican Book 2nd edition published by Penguin Books, Inc., Baltimore, Maryland, 1953.

2. "Introduction to the Theory of Probability and Statistics" by Niels Arley & K. Rander Buch, John Wiley & Sons, Inc., New York, N. Y., 1950.

3. "Strength, Safety and Economical Dimensions of Structures" by Arne I. Johnson, Bulletin No. 12 of the Division of Building Statics and Structural Engineering at the Royal Institute of Technology, Stockholm, 1953.

4. "Statistical Theory of Extreme Values and Some Practical Applications" by Emil J. Gumbel, National Bureau of Standards Applied Mathematics Series No. 33, 1954.

An extensive bibliography is given in "Safety and the Probability of Structural Failure" by A. M. Freudenthal, M. ASCE, Transactions ASCE Vol. 121, 1956.

Journal of the

STRUCTURAL DIVISION

Proceedings of the American Society of Civil Engineers

TESTING POST-TENSIONED SLAB AND BEAM WITHOUT GROUTING[a]

Zachary Sherman,[1] A.M. ASCE
(Proc. Paper 1317)

SYNOPSIS

Two industrial building post-tensioned members were tested in the laboratories at the University of Mississippi in November, 1955. One was a 5' x 30' x 11" roof channel slab, and the other was a 25' long roof girder. The wire units, fabricated by Prescon of Texas, were coated with mastic, and were not grouted in. The end anchorages are steel plate shims.

Load v.s. deflection curves were plotted and cracking and ultimate loads were noted. Stress diagrams were computed and drawn. Factors of safety were also determined.

INTRODUCTION

Up to the present time, not very much testing work has been done in the field of prestressed concrete where non-grouted wires were used. Even less laboratory work has been done on full-size post-tensioned beams and slabs without grouting.

The tests described in this paper were done for the new Chambers Gas Range Company factory of Oxford, Mississippi, during November, 1955. The wire units were fabricated by the Prescon Company of Texas. The jacking ends of the cold drawn wire units are held by prefabricated button heads bearing against a threaded washer. The headed ends are formed under pressures better than 500,000 p.s.i. These wires have a 0.25" diameter, and have a minimum tensile ultimate strength of 240,000 p.s.i. There are four wires to the unit. These units were first coated with a mastic and then double wrapped with Sisalkraft paper, in order to prevent any bonding of the wires with the concrete.

Note: Discussion open until December 1, 1957. Paper 1317 is part of the copyrighted Journal of the Structural Division of the American Society of Civil Engineers, Vol. 83, No. ST 4, July, 1957.

a. Presented at a meeting of the American Society of Civil Engineers, Jackson, Miss., February, 1957.

1. Associate Prof. of Civ. Eng., Univ. of Mississippi, University, Miss.

The concrete beams and slabs were cast at the building site by the contractor. They were first wet-burlap cured and then air-cured for a total of 28 days before being post-tensioned. The steel plate shims for the end anchorages were designed on the basis of 15% losses due to creep and shrinkage. Additional losses of 9% were allowed for seating of the shims and threaded bearing washers.

The 16" x 30" x 25' Post-Tensioned Spandrel Girder was loaded at the third points to the ultimate. The girder failed by crushing of the concrete. The 5' x 30' x 11" Post-Tensioned Channel Slab was loaded at the centerline. Here also the concrete crushed at the ultimate. For both members, the wire stresses at failure were well below the ultimate.

Notations:

A	. . . area of concrete section
$A_{EFFECTIVE}$	. . . area of concrete section effective for transverse bending
α, α_1	. . . slope of prestress wire at reaction
c	. . . distance from C.G.C. to farthest fiber
C.G.C.	. . . center of gravity of concrete section
C'	. . . compressive force at ultimate load
d	. . . depth of beam measured to the center of gravity of the prestress steel
e	. . . eccentricity of prestress steel with respect to C.G.C.
E_s	. . . modulus of elasticity of prestress steel
f_c	. . . unit stress in concrete
f'_c	. . concrete ultimate cylinder strength @ 28 days
f'_{ci}	. . . concrete ultimate cylinder strength at time of prestressing
f'	. . . modulus of rupture of concrete
f_i	. . . initial unit prestress in steel at time of prestressing
f_s	. . . effective unit prestress in steel after deducting losses
f'_s	. . . ultimate unit stress in prestress steel
f'_{sl}	. . . unit stress in prestress steel at ultimate load
F	. . . total effective prestress after deducting losses
I	. . . moment of inertia of Section
$I_{EFFECTIVE}$	. . . moment of inertia of section effective for transverse bending
K'd in a beam	. . . coefficient for depth of compressive area k'd in a beam section at ultimate load

M	. . . moment due to experimental load P and girder or slab weight
M_G	. . . moment due to girder or slab weight
$M_{L.L.}$	. . . moment due to live load
M_T	. . . moment due to total load (in service)
n	. . . modular ratio E_s/E_c
P	. . . experimental concentrated load
$P', P'_{ULTIMATE}$	. . . ultimate experimental concentrated load
Q	. . . static moment area
R	. . . radius of gyration $= \sqrt{I/A}$
S_t	. . . principal diagonal tension stress
t	. . . web thickness of girder
T'	. . . total tension in mild steel at ultimate load
T'_1	. . . total tension in prestressed steel at cross-section, at ultimate load
v	. . . unit shearing stress in concrete
$V_{D.L.}, V_{L.L.}$	. . . end shear due to dead load or live load, respectively
V_c	. . . total shear carried by concrete
y_t, y_b	. . . distance from C.G.C. to top and bottom fibers, respectively
% Reinforcing	. . . area of prestress steel divided by effective concrete area, x 100

Stresses in 16" x 30" x 25' Post-Tensioned Spandrel Girder

(a) Girder Properties:

Plate No. 1 shows the basic girder cross-section at the mid-span. Two wire units of 4-0.25" diameter wires each are shown draped in elevation. Referring to Table No. 1, an initial prestress of 160,000 p.s.i. was applied at the jack, which reduced to an effective prestress of 136,000 p.s.i. (assuming 15% losses). A 0.145% reinforcing indicates that the girder is essentially under-reinforced.[2] The 2-#3 bars top and bottom assist in the distributing of cracks, and increase the ultimate strength of the girder. These non-prestressed mild steel bars help but little in reducing the elastic deflection of the girder. Within the elastic range the mild steel bars carry little or no tensile stress, and up to the cracking load are ineffectual even for handling stresses. After cracking occurs, these bars will tend to minimize opening of large cracks which reduce the effective cross-section of the beam and hasten compressive failures.

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2. "Design of Prestressed Concrete Structures," by T.Y. Lin, John Wiley, p. 133-134 (1955).

A concrete ultimate stress value f'_{ci} = 5000 p.s.i. was used at transfer. This was the same as the ultimate strength value, since the members were tensioned after 28 days.

The girder dead load is 282 p.l.f. The slab's dead load contributes 515 p.l.f., and the slab's live load 375 p.l.f. to the girder, for a total superimposed load on the girder of 890 p.l.f. This girder was made deeper than required by design in order to reuse the formwork for interior girders.

(b) Load v.s. Girder Deflection:

Plate No. 2 shows a plot of experimental girder loading v.s. deflection at the mid-span. The cracking load (*) was observed to be 34,150#, although the actual load is no doubt somewhat lower, and closer to the straight-line portion of the curve. The crack at this load occurred 32" off the centerline, within the middle third.

At a load of 36,850#(ϕ), this crack opened to $\frac{5"}{32}$. It closed upon complete release of the load. The permanent deflection (set) then measured was 0.12" at mid-span.

The girder was then continually loaded to 45,000#, at which load the girder failed by crushing of the concrete.

(c) Girder Stresses

Items (1), (2), and (3), on Plate No. 3, indicate relatively low stresses for the three conditions of loading, namely: (1) Prestress only; (2) Prestress and M_G at Transfer-No Losses; and (3) Prestress and M_T after all Losses.

In item (4) the experimental cracking moment due to dead load and live load was 1,864,000 in-#. Assuming a modulus of rupture, $f' = 0.12f'_c$ = 600 p.s.i., and 15% losses due to creep and shrinkage, the theoretical cracking moment computed was 2,166,000 in.-#. This difference could indicate a lower value for modulus of rupture, or a lower effective prestress, F. In item (5), the modulus of rupture is computed to be 446 p.s.i. based on experimental loading and assuming 15% losses.

For industrial building roof construction, a factor of safety based on the cracking load is more significant than one based on the ultimate load, because over-loads in buildings are not as probable as in bridge design. This is especially true for post-tensioning without grouting, since cracks begin to open "fast" soon after cracking starts. These large cracks no doubt reduce the shearing strength of the girder. In item (6), Plate No. 4, the experimental cracking moment for live load was 1,626,000 in.-#, and the girder moment due to working live load (superimposed load on girder) was 735,000 in.-#, indicating a factor of safety of 2.22.

Exact stresses in a post-tensioned member are difficult to compute at the ultimate load. In item (7), Plate No. 4, an average concrete failure stress of 0.85 f'_c = 4,250 p.s.i. was assumed. The mild steel bottom bars were assumed to be stressed to the yield point at failure. From static equilibrium at the section of failure, the stress in the post-tensioned steel was then computed to be f'_{si} = 215,000 p.s.i. A prestressed beam which is lightly reinforced starts to fail usually by excessive elongation of the steel. When the steel is not bonded, large cracks soon develop which lead to eventual concrete crushing on the compression side.

End Plate bearing pressures were not excessive even at ultimate load. The plates were 3" x 4 1/2" x 5/8" thick. At the initial prestress the bear-pressure was $\frac{42,330}{3 \times 4.5}$ = 3,140 p.s.i. An allowable bearing value of 0.70 f'_c = 3,5000 p.s.i. was the basis of design.

Stresses in 5' x 30' x 11" Post-Tensioned Channel Slab

(a) Slab Properties:

Plate No. 5 shows the basic slab cross-section at the mid-span. Two wire units of 4-0.25" diameter each are shown draped in elevation. The slab is only 1-1/2" thick and is reinforced with a 4" x 4"/8 x 8 wire mesh in the lateral direction. Only 30" of the 60" width of slab was considered effective in bending in the long direction. In Table No. 2, the effective reinforcing noted is 0.33%. For this percentage the slab may also be considered as under-reinforced. Other steel and concrete properties noted are the same as for the girder. The slab dead load is 172 p.l.f., and the roofing and snow load contribute 125 p.l.f., for a total load of 297 p.l.f.

(b) Load v.s. Slab Deflection:

Plate No. 6 shows a plot of experimental slab loading v.s. deflection at the mid-span. The cracking load (*) was observed to be 4,130#. At a load of 4,880# (ϕ), many flexure and diagonal tension cracks were visible. The widest crack opening was 3/32" at that load. The post-tensioned steel was yielding considerably at that point. At a load of 5,380# ( ), the load was released and a permanent deflection (set) of 7/16" was measured.

The slab was then continually loaded to the ultimate load of 5600# (θ), at which point the slab failed by crushing of the concrete at the mid-span. The widest crack opening[3] was 1/2", measured at the mid-span. Upon removal of the ultimate load, the slab maintained a crack width opening of 3/16", and a permanent deflection (set) at the mid-span of 1.20".

(c) Slab Stresses:

On Plate No. 7, the same conditions of loading are noted for the slab as was for the girder. For condition (1), Prestress Only, the concrete tensile stress is shown as 460 p.s.i. Actually the slabs were tensioned out of the forms so that some dead load tended to reduce this stress because of upward camber. Ordinarily, no more than 0.05 f'_{ci}[4] tension should be allowed in the concrete at the initial condition. For loading condition (3), the maximum compressive stress is seen to be quite small. This is no doubt due to the shallow effect of the channel slab. Condition (1) is therefore the critical design condition. At this condition, the compressive stress of 2,140 p.s.i. is well below 0.55 f'_{ci},[4] the maximum permitted for prestress only.

3. "Ultimate Flexural Strength of Prestressed and Conventionally Reinforced Concrete Beams," by J. R. Janney, E. Hognestad, and D. McHenry, Journal of the American Concrete Institute, Jan. 1956, Proceedings Vol. 52, p. 601-620.
4. "Criteria for Prestressed Concrete Bridges,"U.S. Department of Commerce, Bureau of Public Roads, Washington, D.C.: 1954.

In item (4), the experimental cracking moment due to dead load plus live load was 582,000 in-#, and the theoretical cracking moment was 529,700 in-# (assuming 15% losses and f' = 600 p.s.i.) In item (5), the modulus of rupture was computed to be 846 p.s.i. based on experimental loading and assuming 15% losses.

In item (6), Plate No. 8, the experimental cracking moment for live load was 362,000 in-#, and the slab moment due to working live load was 169,000 in-#, indicating a factor of safety of 2.14.

At ultimate load, the post-tensioned steel stress, f'_{sl}, was computed to be 191,000 p.s.i., with the average crushing strength of the concrete being assumed as 4,250 p.s.i. This is illustrated in item (7), Plate No. 8, for static equilibrium at the mid-span section of failure.

Principal Tension Stresses:

At the working load, the principal tensile stress is only 26 p.s.i. This stress occurs at the juncture of the top flange and the web, at the reaction where the external bending moment is zero. Nevertheless, stirrups were added. This is customary in present practice. Figure 1, a and b below, show the fiber stress distribution at the end of the girder due to the eccentric prestress force.

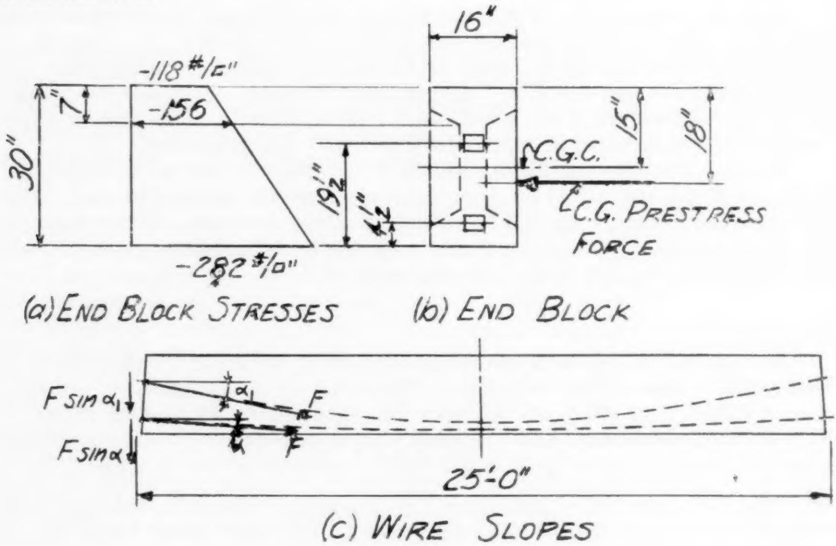

Figure 1

The stress computations for principal diagonal tension are as follows:

$$V_{D.L.} = 282 \times 12.5 = 3{,}530\#$$

$$V_{L.L.} = 890 \times 12.5 = 11{,}100\#$$

$$V_{D.L. + L.L.} = 14{,}630\#$$

$F = 27{,}000\#/\text{wire unit}$, after deducting all losses

$F(\sin\alpha + \sin\alpha_1) = 27{,}000\,(4/150 + 28/150) = 5{,}750\#$

$V_e = V_{D.L.} + V_{L.L.} - F(\sin\alpha + \sin\alpha_1) = 14{,}630 - 5{,}750 = 8{,}880\#$

$Q = 1{,}139\ \text{in.}^3$ — (7" from top of girder)

$I = 29{,}530\ \text{in.}^4$

$$\therefore v = \frac{V_c Q}{It} = \frac{8{,}880 \times 1{,}139}{29{,}530 \times 5} = 68.5\ \text{p.s.i.}$$

$f_c = -156$ p.s.i. — (7" from top of girder)

$$\therefore S_t = \sqrt{\left(\frac{f_c}{2}\right)^2 + v^2} - \frac{f_c}{2} = \sqrt{(-78)^2 = (68.5)^2} - 78$$

$$= 104 - 78 = +26\ \text{p.s.i.}$$

At the cracking load, the principal tension stress was computed to be 112 p.s.i., which is still less than $0.03\,f'_c = 150$ p.s.i.[3], the allowable value without stirrups.

CONCLUSION

Post-tensioned slabs and girders for use in building design may be safely designed on the basis of cracking loads when the wires are not grouted in. A factor of safety of two (2) above working loads should be adequate in most cases. Especially when overloads might be expected, the designer should include some mild steel bars on the tension side to distribute the cracks, prevent large cracks, and reduce deflections beyond the elastic range of loading, thereby increasing the beam's ultimate strength. Large cracks do not necessarily indicate imminent collapse, especially for under-reinforced beams, although the steel may be excessively elongated. Nevertheless, the opening and closing of large cracks cause "working" of the concrete, leading to spalling and gradual crushing on the compression side.

For bridge members, post-tensioning without grouting is quite satisfactory, but some mild steel should be included to help distribute cracks by bond action with the concrete. The economical combinations of non-grouted post-tensioned steel and bonded mild steel can best be determined by more laboratory testing.[4]

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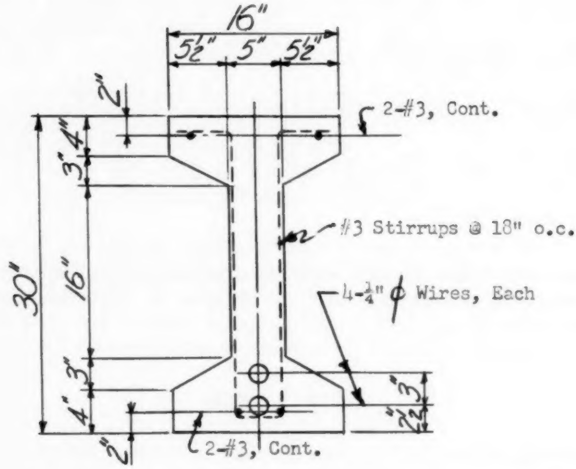
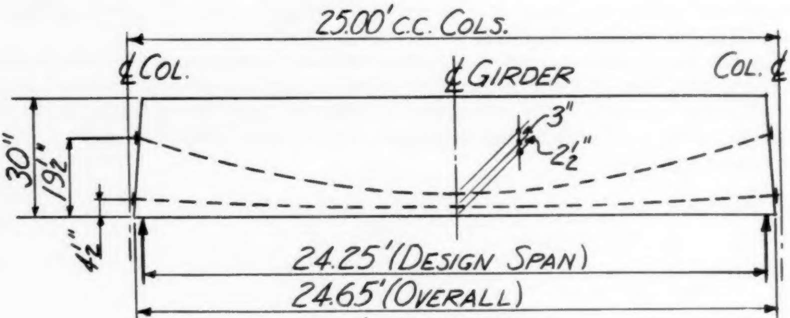
4. "Criteria for Prestressed Concrete Bridges," U.S. Department of Commerce, Bureau of Public Roads, Washington, D.C.: 1954.

16" x 30" x 25' POST-TENSIONED SPANDREL GIRDER



CROSS-SECTION AT MID-SPAN

I = 29,530 in.^{4}
A = 271 in.^{2}
R^2 = 109



ELEVATION

PLATE NO. 1

DATA FOR POST-TENSIONED GIRDER

(A) STEEL:

(1) 8-1/4" ϕ WIRES (2-4 WIRE UNITS-NOT GROUTED IN)

(2) $f_s' = 240,000$ p.s.i.

(3) $f_i = 160,000$ p.s.i.

(4) $f_s = 0.85 \times 160,000 = 136,000$ p.s.i. (ASSUME 15% LOSSES)

(5) $E_s = 29,000,000$ p.s.i.

(6) % REINFORCING = 0.145% (ESSENTIALLY UNDER-REINFORCED)

(B) CONCRETE:

(1) $f_c' = 5,000$ p.s.i. @ 28 days

(2) $f_{ci}' = 5,000$ p.s.i. AT TIME OF TRANSFER

(3) $f_c = 0.40 \times 5,000 = 2,000$ p.s.i.

(4) $n = 6$

(C) LOADINGS:

(1) GIRDER DEAD LOAD = 282 p.f.

(2) SLAB DEAD LOAD = 35$^{\pm}$ p.f. x 15' = 515

(3) SLAB LIVE LOAD = 25 p.f. x 15' = 375

$\left.\begin{matrix} 515 \\ 375 \end{matrix}\right\} = 890$ p.f.

$W_{D.L. + L.L.} = 1,172$ p.f.

TABLE NO. 1

LOAD V.S. GIRDER DEFLECTION

16" X 30" X 25' POST-TENSIONED SPANDREL GIRDER

(NO GROUTING)

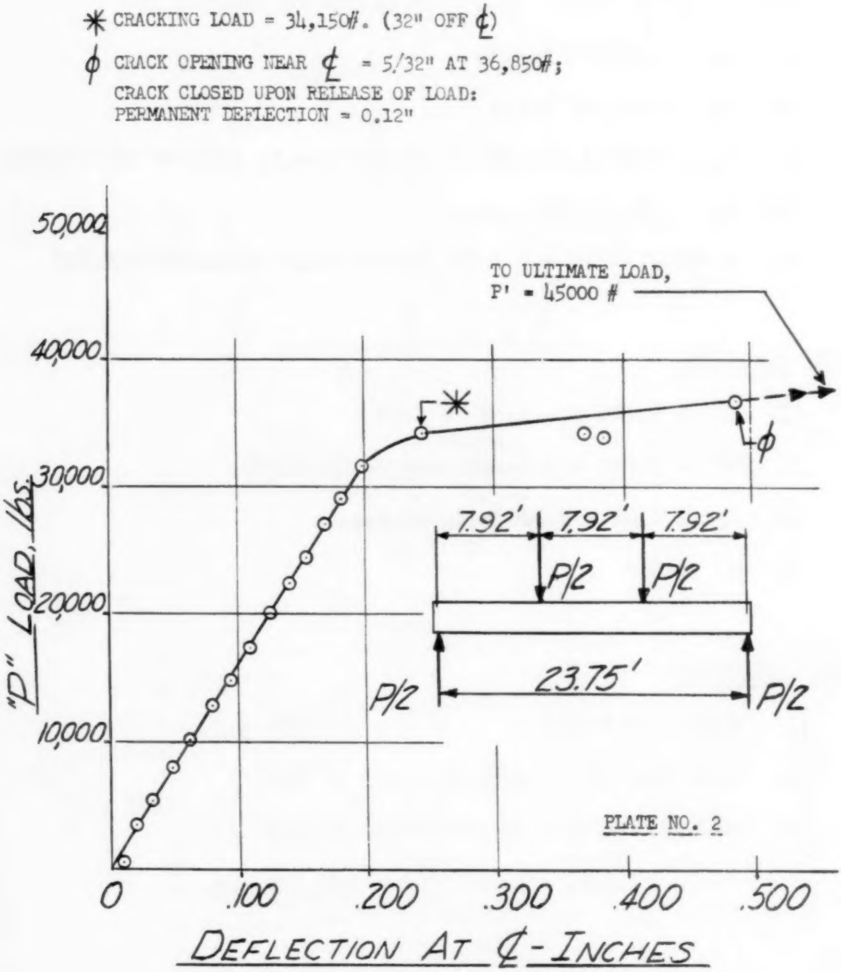
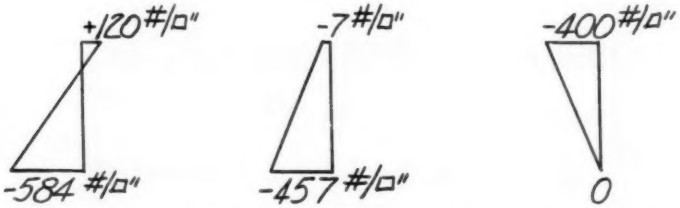


PLATE NO. 2

(D) STRESSES IN 16" x 30" x 25' POST-TENSIONED SPANDREL GIRDER AT THE MID-SPAN (NO GROUTING)



(1) PRESTRESS ONLY

(2) PRESTRESS AND M_G AT TRANSFER-NO LOSSES

(3) PRESTRESS AND M_T AFTER ALL LOSSES

(4) CRACKING MOMENT (EXPERIMENTAL) $= M_{L.L.} + M_G = \frac{34,150 \times 7.92 \times 12}{2} + \frac{282 \times (23.75)^2 \times 12}{8} = 1,626,000 + 238,000 = 1,864,000$ "-#

CRACKING MOMENT (ASSUMING $f' = 0.12\, f'_c = 600$ p.s.i. AND 15% LOSSES) $= Fe + \frac{FI}{Ac} + \frac{f'I}{c} = 54,000 \times 11 + \frac{54,000 \times 29,530}{271 \times 15} + \frac{600 \times 29,530}{15} = 594,000 + 392,000 + 1,180,000 = 2,166,000$ "-#

(5) f' (EXPERIMENTAL, ASSUMING 15% LOSSES) $= -\frac{F}{A} - \frac{Fec}{I} + \frac{Mc}{I} = \frac{54,000}{271} - \frac{54,000 \times 11 \times 15}{29,530} + \frac{34,150 \times 7.92 \times 12 \times 15}{2 \times 29,530} + \frac{238,000 \times 15}{29,530} = +446$ p.s.i.

PLATE NO. 3

(D) STRESSES IN 16" x 30" x 25' POST-TENSIONED SPANDREL GIRDER AT THE MID-SPAN (NO GROUTING)

(6) FACTOR OF SAFETY-WORKING V.S. CRACKING LIVE LOADS:

$$M_{\text{WORKING L.L.}} = \frac{890 \times (24.25)^2 \times 12}{8} = 735{,}000 \text{ "-\#}$$

$$M_{\text{CRACKING L.L.}} = \frac{34{,}150 \times 7.92 \times 12}{2} = 1{,}626{,}000 \text{ "-\#}$$

$$\text{F. S.} = \frac{1{,}626{,}000}{735{,}000} = \underline{\underline{2.22}}$$

(7) ULTIMATE STRESSES:

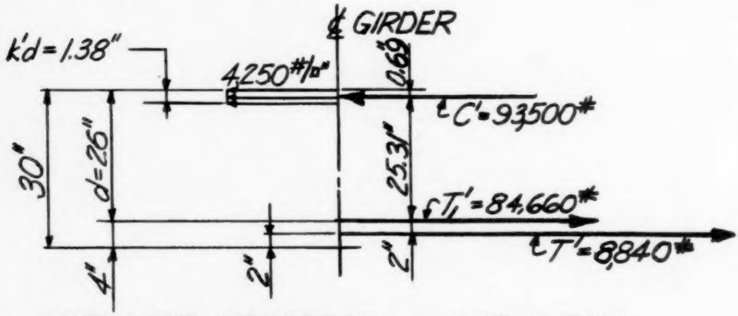
$P'_{\text{ULTIMATE}} = 45{,}000$ #

$T' = 2 \times 0.1105 \times 40{,}000 = 8{,}840$ # (MILD STEEL)

$T'_1 = 84{,}660$ # (PRESTRESS STEEL)

$f'_{s1} = 84{,}660/0.393 = 215{,}000$ p.s.i. ($<240{,}000$ p.s.i.)

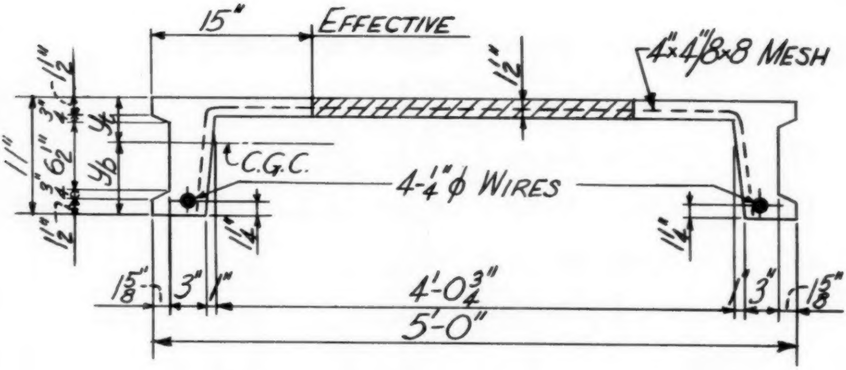
AVERAGE CONCRETE STRESS AT ULTIMATE = $0.85 \times 5{,}000 = 4{,}250$ p.s.i.



ULTIMATE STRESSES AT MID-SPAN

PLATE NO. 4

5' x 30' x 11" POST-TENSIONED CHANNEL SLAB



CROSS-SECTION AT MID-SPAN

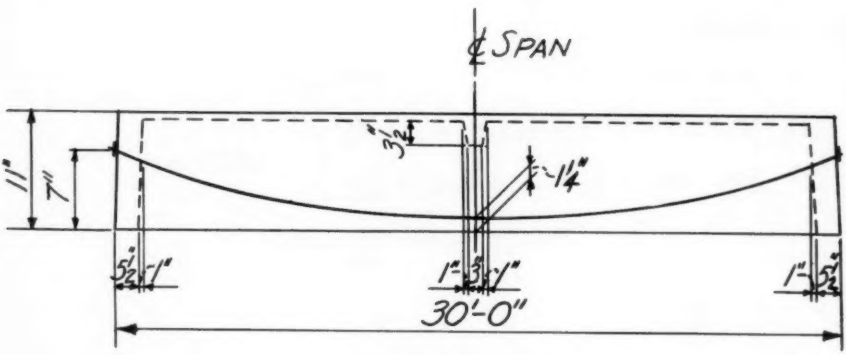
$A_{Effective} = 118.84 \text{ in.}^2$

$I_{Effective} = 1{,}480.8 \text{ in.}^4$

$R^2 = 12.5 \text{ in.}^2$

$y_t = 4.20"$

$y_b = 6.80"$



ELEVATION

PLATE NO. 5

DATA FOR POST-TENSIONED SLAB

(A) STEEL:

(1) 8-1/4"ϕ WIRES (2-4 WIRE UNITS - NOT GROUTED IN)

(2) $f_s' = 240{,}000$ p.s.i.

(3) $f_i = 160{,}000$ p.s.i.

(4) $f_s = 0.85 \times 160{,}000 = 136{,}000$ p.s.i. (ASSUME 15% LOSSES)

(5) $E_s = 29{,}000{,}000$ p.s.i.

(6) EFFECTIVE % REINFORCING = 0.33%

(B) CONCRETE

(1) $f_c' = 5{,}000$ p.s.i. @ 28 days

(2) $f_{ci}' = 5{,}000$ p.s.i. AT TIME OF TRANSFER

(3) $f_c = 0.40 \times 5{,}000 = 2{,}000$ p.s.i.

(4) $n = 6$

(C) LOADINGS:

(1) SLAB DEAD LOAD = 172 p.f.

(2) ROOFING = 5' x 5 p.s.f. = 25

(3) SNOW LOAD = 5' x 20 p.s.f. = 100

$\left.\begin{matrix} 25 \\ 100 \end{matrix}\right\}$ LIVE LOAD

$W_{D.L. + L.L.} = 297$ p.f.

TABLE NO. 2

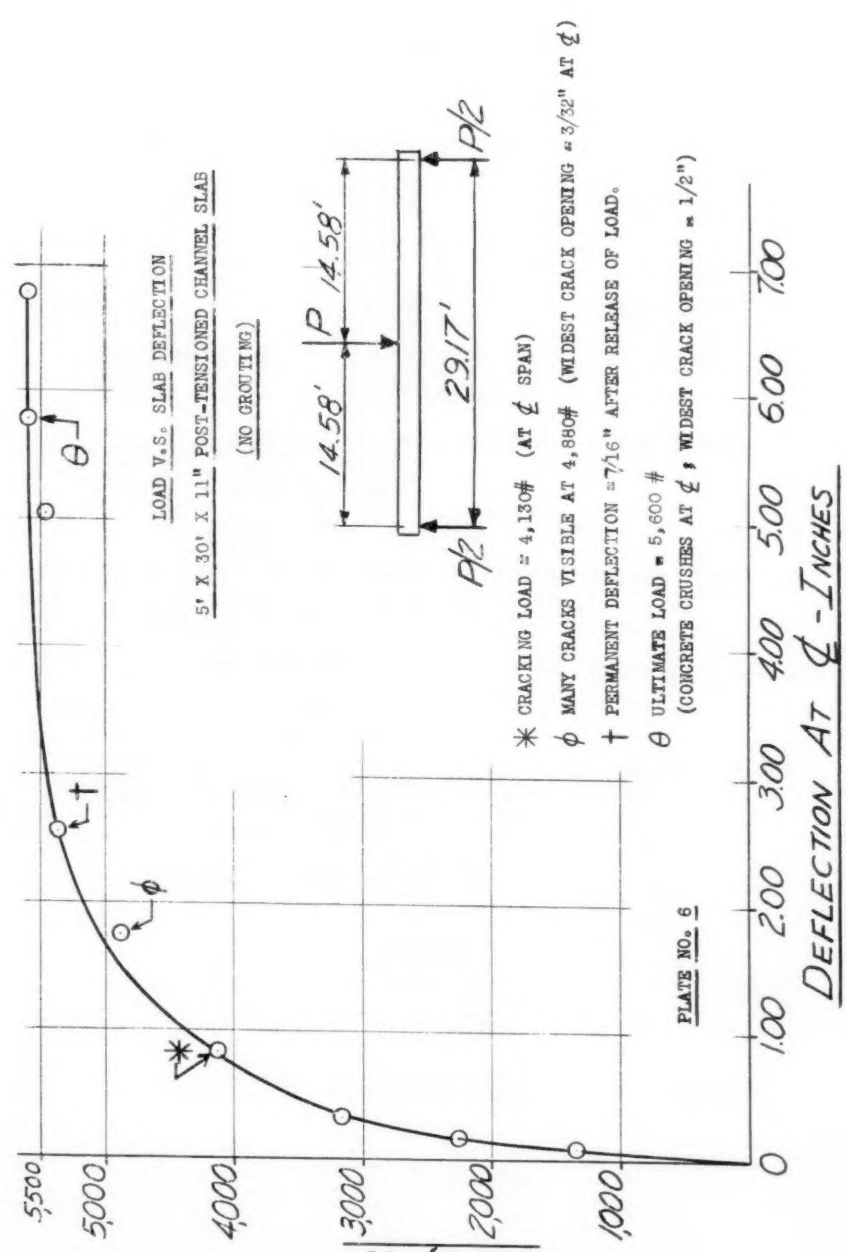
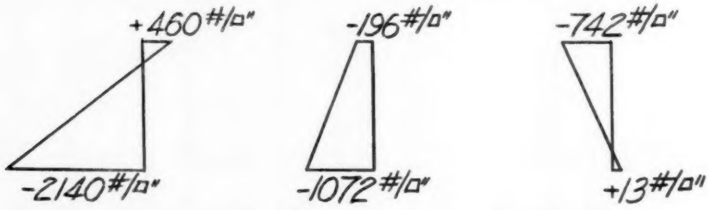

LOAD V.S. SLAB DEFLECTION
5' X 30' X 11" POST-TENSIONED CHANNEL SLAB
(NO GROUTING)
P
P/2
P/2
14.58'
14.58'
29.17'
* CRACKING LOAD = 4,130# (AT ℄ SPAN)
ϕ MANY CRACKS VISIBLE AT 4,880# (WIDEST CRACK OPENING = 3/32" AT ℄)
† PERMANENT DEFLECTION = 7/16" AFTER RELEASE OF LOAD.
θ ULTIMATE LOAD = 5,600 #
(CONCRETE CRUSHES AT ℄ ; WIDEST CRACK OPENING = 1/2")
LOAD, lbs.
5500
5000
4000
3000
2000
1000
0
1.00
2.00
3.00
4.00
5.00
6.00
7.00
DEFLECTION AT ℄ - INCHES

PLATE NO. 6

(D) STRESSES IN 5'x30'x11" POST-TENSIONED CHANNEL SLAB AT THE MID-SPAN

(NO GROUTING)



(1) PRESTRESS ONLY (2) PRESTRESS & M_G AT TRANSFER-NO LOSSES (3) PRESTRESS & M_T AFTER ALL LOSSES

(4) CRACKING MOMENT (EXPERIMENTAL) $= M_{L.L.} + M_G = \frac{4130}{2} \times 14.58 \times 12 + \frac{172 \times (29.17)^2 \times 12}{8} = 362{,}000 + 220{,}000 = \underline{582{,}000}$ "-#

CRACKING MOMENT (ASSUMING 15% LOSSES) (AND $f' = 0.12\ f_c'$) (= 600 p.s.i.) $= Fe + \frac{FI}{Ac} + \frac{f'I}{c} = 54{,}000 \times 5.55 + \frac{54{,}000 \times 1{,}480.8}{118.8 \times 6.80} + \frac{600 \times 1{,}480.8}{6.80}$

$= 300{,}000 + 99{,}000 + 130{,}700$
$= \underline{\underline{529{,}700}}$ "-#

(5) f' (EXPERIMENTAL, ASSUMING 15% LOSSES) $= -\frac{F}{A} - \frac{Fec}{I} + \frac{Mc}{I} = \frac{-54{,}000}{118.8} \frac{-54{,}000 \times 5.55 \times 6.80}{1{,}480.8} + \frac{4{,}130 \times 14.58 \times 12 \times 6.80}{2 \times 1{,}480.8} + \frac{220{,}000 \times 6.80}{1{,}480.8} = + \underline{\underline{846}}$ p.s.i.

PLATE NO. 7

(D) STRESSES IN 5' x30'x11" POST-TENSIONED CHANNEL SLAB AT THE MID-SPAN (NO GROUTING)

(6) FACTOR OF SAFETY, WORKING V.S. CRACKING LIVE LOADS:

$$M_{\text{WORKING L.L.}} = \frac{125x(30)^2x12}{8} = 169{,}000 \text{ "-\#}$$

$$M_{\text{CRACKING L.L.}} = \frac{4{,}130x14.58x12}{2} = 362{,}000 \text{ "-\#}$$

$$\text{F.S.} = \frac{362{,}000}{169{,}000} = 2.14$$

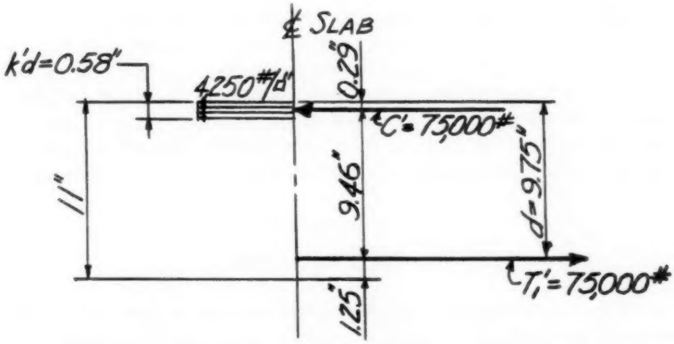
(7) ULTIMATE STRESSES:

$P'_{\text{ULTIMATE}} = \cancel{4{,}130}$ 5600 #

$T_1' = 75{,}000$ #

$f'_{s1} = 75{,}000/0.393 = 191{,}000$ p.s.i. ($<$ 240,000 p.s.i.)

AVERAGE CONCRETE STRESS AT ULTIMATE = 0.85x5,000= 4,250 p.s.i.



ULTIMATE STRESSES AT MID-SPAN

PLATE NO. 8

Journal of the
STRUCTURAL DIVISION
Proceedings of the American Society of Civil Engineers

VIBRATION SUSCEPTIBILITIES OF VARIOUS HIGHWAY BRIDGE TYPES[a]

LeRoy T. Oehler,[1] A.M. ASCE
(Proc. Paper 1318)

SYNOPSIS

This paper reports the results of deflection and vibration measurements on thirty-four spans of fifteen bridges of three types: simple-span, continuous-span, and cantilever-type. Simple-span and cantilever-type bridges included those designed both with and without composite action between the concrete deck and steel beams. The continuous-span type included both steel and reinforced concrete bridges. The same test truck with a constant load was used for testing all bridges. Similarly, the testing procedure was as uniform as possible, considering different methods of fastening deflectometers and variation in bridge site conditions.

The vibration data from these bridges indicate that the cantilever-type structure is much more susceptible to larger amplitudes of vibration than are the other types. Comparison of bridge spans designed with and without composite action indicates that the latter are much more conservatively designed, when actual performance is compared with design values. From these tests, it appears that impact, as measured by increased dynamic deflection over static deflection, is related more directly to riding quality of the roadway survace than to other factors. This report suggests methods of analysis for computing the fundamental frequency of vibration for all bridge types. These methods give reasonable agreement with the experimental values.

INTRODUCTION

This research survey of the deflection and of vibratory tendencies of several types of highway bridges was made to determine the type of highway structure most readily susceptible to excessive vibration, and possibly, the

Note: Discussion open until December 1, 1957. Paper 1318 is part of the copyrighted Journal of the Structural Division of the American Society of Civil Engineers, Vol. 83, No. ST 4, July, 1957.

[a] Presented at a meeting of the American Society of Civil Engineers, Buffalo, N. Y., June, 1957.

1. Physical Research Engr., Research Lab., Michigan State Highway Dept., Michigan State University, East Lansing, Mich.

range in amplitude of vibration for various types under similar loading conditions. From the outset, the study was planned as an extensive rather than an intensive one. Taking cognizance of limitations in time, equipment, and personnel, it appeared advisable to spend only a short time testing each bridge, thus testing as many bridges and bridge types as possible. With a background of vibration data on a significant number of bridges of various types, it seemed likely that improved methods for controlling or limiting vibration, might become apparent.

Previous bridge studies, conducted by the Michigan State Highway Department in 1950, dealt primarily with the lateral distribution of stress and deflection to the individual beams under design loadings for the Fennville Bridge, a six span, rolled beam, concrete deck bridge.(1)* Additional studies in 1952 and 1953 were conducted on the vibration and deflection of this bridge and of the Jackson Bypass Bridge, an eight span, plate girder bridge, consisting of five simple spans and three spans of continuous beam design.(2) These studies were carried out to contribute to the research program of the ASCE Committee on Deflection Limitations of Bridges, and of the Committee on Bridges of the Highway Research Board, of which George M. Foster, Chief Deputy Commissioner, is a member. This bridge study was a project of the Research Laboratory, directed by E. A. Finney. The Laboratory is a part of the Testing and Research Division, under the direction of W. W. McLaughlin.

Research Objectives

The immediate objectives of the test program were:

1. To obtain the natural frequency of vibration for several bridges of each type, including,

(a) Simple-span bridges with concrete deck, designed with and without composite action between beams and concrete deck.

(b) Continuous-span bridges of the rolled beam, plate girder, and reinforced concrete types.

(c) Cantilever-type structures of rolled beam and plate girder types with concrete deck, designed with and without composite action.

2. To find the variation in amplitude of vibration for these bridge types, and their susceptibility to vibration.

3. To compare the flexibility of the various bridge types and to correlate this with susceptibility to vibration.

4. To determine effective axle load fluctuation as the test vehicle passed over the structure.

5. To compare the computed natural frequency of vibration for each type of structure, with the experimentally determined value.

Bridges were selected for testing on the basis of the following criteria: (1) bridges which might be flexible, built in the last ten years, (2) bridges representing as many structural types as possible, (3) bridges over ground

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*Numbers in parentheses, unless otherwise identified, refer to the Bibliography at the end of this paper.

or shallow water, rather than deep water, to simplify installation of instrumentation, (4) bridges close to Lansing, other factors being equal.

In summary, the tested bridges included:

1. Eleven simple-spans of four rolled beam bridges with concrete deck: nine spans with and two without spiral shear developers.

2. Four continuous-span structures: two of reinforced concrete, one rolled beam, and one plate girder.

3. Nine cantilever-type structures: six rolled beam with concrete deck—five with and one without spiral shear developers, three plate girder structures—one without floor beams and two with floor beams and stringers.

The locations of the fifteen bridges, of which thirty-four spans were tested, are shown on a map of Michigan in Figure 1. Physical limitations necessitated selecting only certain spans of these fifteen bridges for testing. Electrical lead wires longer than 150 ft. made it impossible to balance out the capacitance in the electrical bridge circuit. Therefore on symmetrical three span structures, only two spans were tested. In addition, testing of certain spans above deep water made instrumentation so difficult, that it appeared inadvisable to attempt testing these spans. Also, on multiple span structures with identical span lengths, only a few spans of each bridge were tested.

The pattern of instrumentation was kept as similar as possible for each bridge, considering site conditions at various locations. The work program was carried out so that installation of instrumentation on the bridge, testing, and subsequent removal of instrumentation, could be conducted in a single day. The bridge deflectometers for obtaining vibration and deflection data were the same ones used in the 1952-53 study(2) and are shown in Figure 2. Bridge movement could be ovserved visually by reading the dial gage, but a permanent record was also obtained by means of a wire resistance strain gage fastened to an aluminum cantilever beam, which was deflected by movement of the top end of the dial gage stem. Bending in the aluminum cantilever beam was accompanied by change in electrical resistance of the bonded strain gage, which caused an unbalance in the electrical bridge circuit, and resulted in deflection of a light trace on a photosensitive paper strip in a Hathaway 12-channel recording oscillograph. The recording equipment is shown in Figure 3, mounted in the Instrumentation Truck. Calibration of trace deflection with bridge movement was obtained prior to each testing program.

Deflectometers were mounted at mid-span for all spans tested. The lateral position of the deflectometer depended on possible methods of fastening the deflectometer to the bridge, and on the proposed path for the test vehicle. On bridges without a separate superstructure for each roadway, the test vehicle passed along the longitudinal centerline wherever possible, and the deflectometer was placed directly beneath the path of the test vehicle, and on the lower flange of rolled beam and plate girder bridges. For bridges with a separate superstructure for each roadway and a split safety median between roadways, the deflectometer was placed on the concrete median strip and the path of the test vehicle was as close as possible to this median strip.

The speed of the truck and its position on the test span were recorded by pneumatic traffic cables which caused a pip on an inactive oscillograph trace as a wheel passed over the cable.

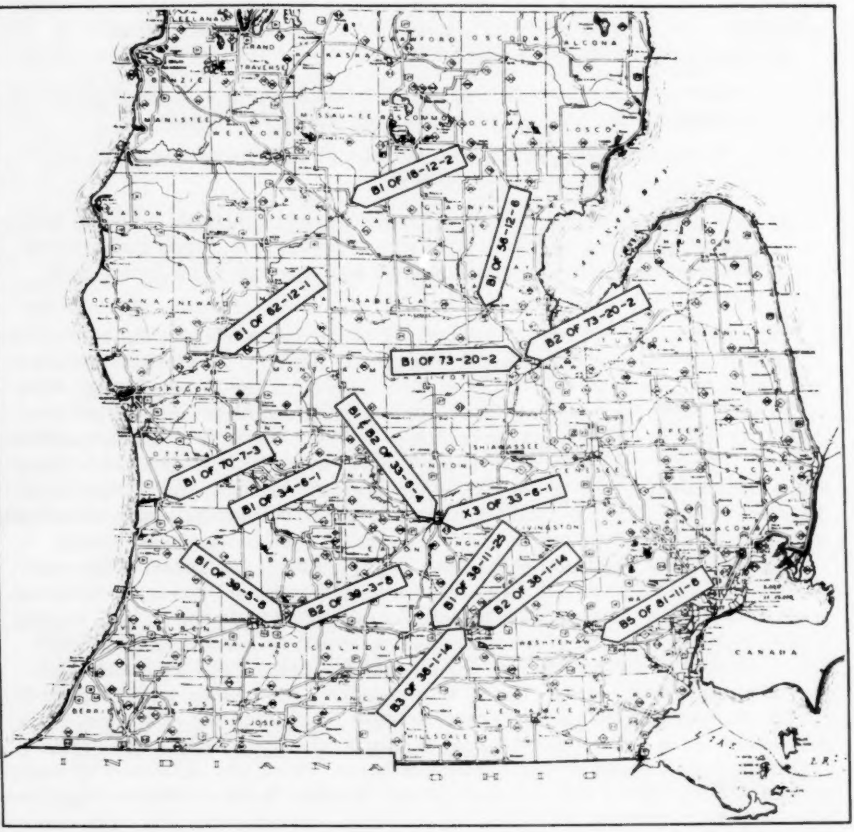
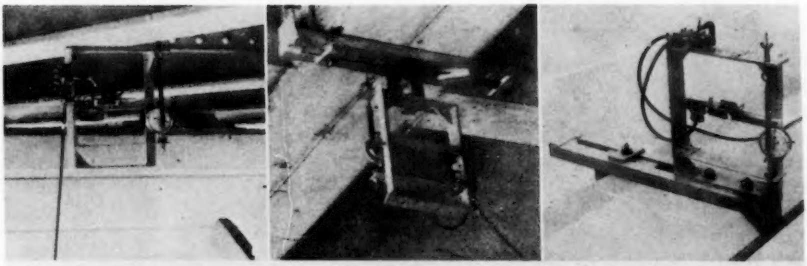


Figure 1. Location of bridges tested



Deflectometer fastened to bottom flange of rolled beam bridge.

Deflectometer fastened to reinforced concrete tee-beam bridge.

Deflectometer fastened to median strip for bridges with separate superstructures for each roadway.

Figure 2. Methods of Fastening Deflectometers

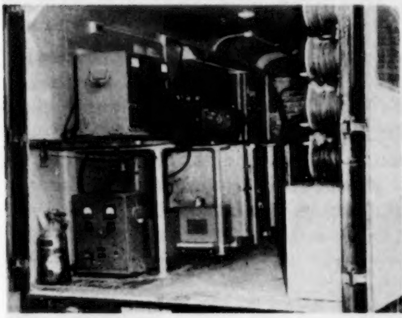


Figure 3. Hathaway 12-channel recording oscillograph in Instrumentation Truck.

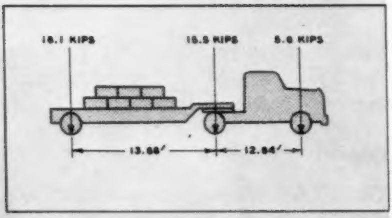


Figure 4. Test vehicle and its loading.

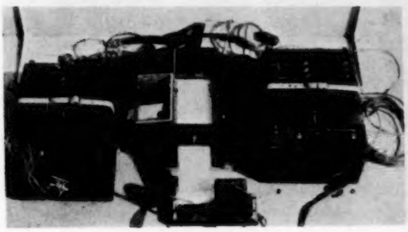


Figure 5. Instrumentation panel for determining wheel load variation on test vehicle.

Test Truck and Its Instrumentation

In order to obtain a comparison between bridges, the same test vehicle with identical axle loads was used for all testing (Figure 4). Prior to loading, bonded strain gages of the A-1 type were placed on the axle between the inner wheels and the springs or frame mounting on both load axles. During the loading process, the load on each pair of dual wheels and the corresponding strain reading on the axle were recorded by means of a static strain indicator. During the testing these strain gages were connected to a Brush two-channel recording oscillograph in order to note the variation in effective wheel load as the truck approached and passed over the bridge. The instrumentation panel for recording dynamic wheel load variations is shown in Figure 5.

Although the second axle of the test truck had a conventional suspension system, the third or trailer axle was not sprung, but was connected directly to the frame. A preliminary study conducted at various truck speeds showed that the variation in effective wheel load on wheels of a given axle was generally similar in pattern and amplitude. Therefore, strain gages on a given axle were wired in series, and effective axle load variation rather than effective wheel load variation was recorded for each load axle.

The test program for each bridge was conducted in as uniform a manner as possible taking cognizance of variations in bridges, traffic, and site conditions. All runs were along a prescribed path with a variation in truck speed from creep to a maximum truck speed of 45 mph. in 5 mph. increments. Three runs were made at each truck speed, but the higher speeds could be obtained only at favorable site locations. For several bridges the maximum speed was limited to 20 or 25 mph. which no doubt reduced the magnitude of the vibration. It was noted however, that those bridges which vibrated more readily, showed a tendency toward vibration even at low speeds.

Test Results

For comparative purposes the results of tests on the thirty-four highway bridge spans will be presented according to bridge types, that is, simple-span, continuous, and cantilever types.

Simple-Span Bridges

The eleven simple spans tested varied in span length from 44.8 to 64.92 ft. Photographs of these bridges are shown in Figures 6 through 10. They were all constructed with superstructures of wide flange rolled beams with concrete decks. Nine of these spans were designed for composite action between rolled beams and a concrete deck, and spiral shear developers were used to assure the composite action. The two tested spans which had been designed without the benefit of composite action were short spans, the longer being 48.65 ft. Table 1 presents a summary of vibration and deflection data for the simple spans as well as pertinent design information.

Deflection

Deflections resulting from the test truck passing at "creep speed" over the tested spans, varied from 0.039 to 0.072 in. In order to compare the observed deflection with the theoretical values, the mid-span deflections without impact were calculated for the test truck, in accordance with the AASHO

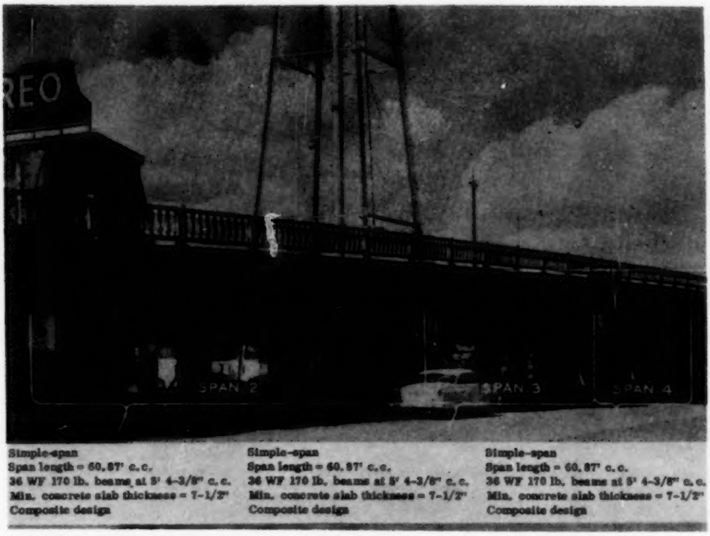


Figure 6. Bridge X3 of 33-6-1. Cedar Street in Lansing.

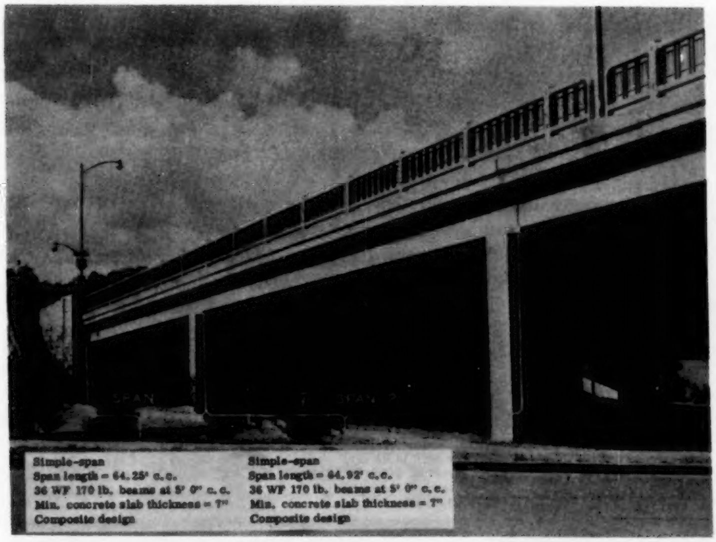


Figure 7. Bridge B1 & B2 of 33-6-4. Main Street in Lansing.

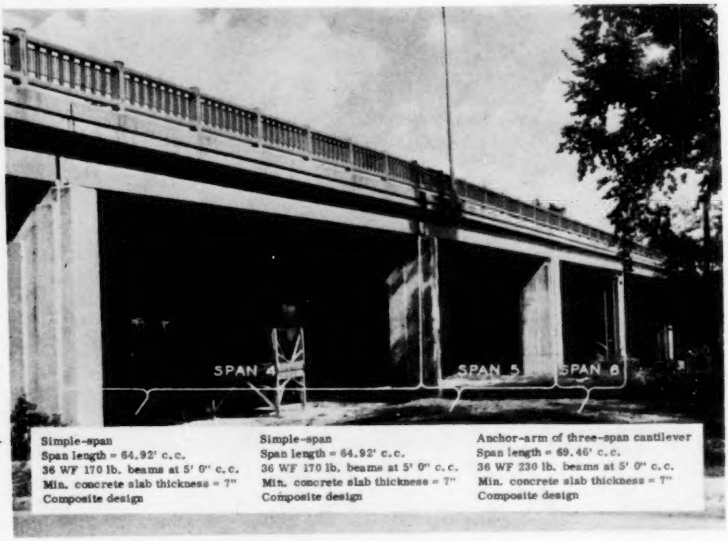


Figure 8. Bridge B1 & B2 of 33-6-4. Main Street in Lansing.

Figure 9. Bridge B1 of 56-12-6. On route M-20 in Midland.

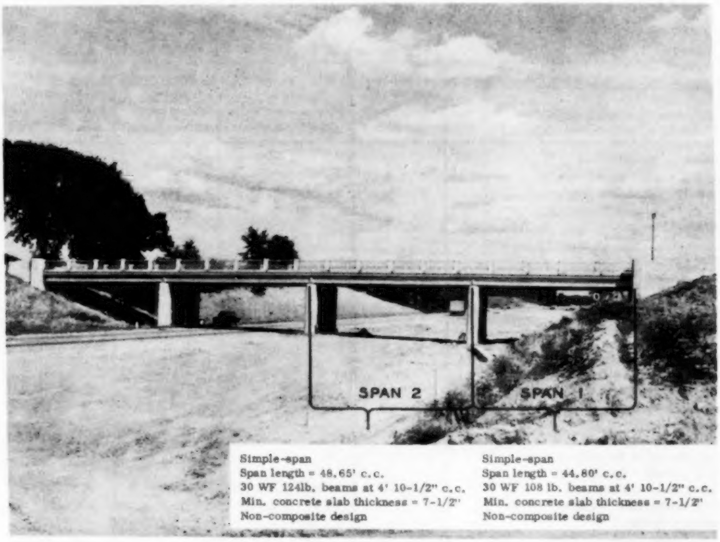


Figure 10. Bridge B2 of 39-3-8. Over route US-12 near Kalamazoo.

Standard Specifications for Highway Bridges, and on the basis of a ratio of modulus of elasticity of steel to concrete of 10. As shown in Table 1, the ratio of observed to theoretical deflection for the composite spans varied from 0.24 to 0.40, with an average of 0.32. The same averaged ratio for non-composite spans was 0.16. Previous tests(2) indicated comparable ratios of 0.28 and 0.14 for composite and non-composite spans respectively. These ratios verify a generally accepted view that by present specifications, non-composite spans are much more conservatively designed than are composite spans. The two non-composite spans tested would have had approximately the same ratio of observed to theoretical deflection as did the composite spans, if the theoretical deflection for these spans were based on a moment of inertia considering 50 percent composite action of slab with steel beam.

Amplitude of Vibration

The maximum amplitudes of bridge vibration for each span, for the test truck on the span, on other spans, and off the bridge, are shown in Table 1. This data indicates that seven of the eleven spans (those from bridges X3 of 33-6-1 and B1 and B2 of 33-6-4) had a tendency to vibrate at greater amplitudes than the other four spans (those from bridges B1 of 56-12-6 and B2 of 39-3-8). This is most apparent when the amplitudes of free vibration, truck on other spans, or off the bridge, are compared. The seven spans with the larger amplitudes of vibration are all over 60 ft. in span length and were designed for composite action. Of the four remaining spans, two were designed for composite action, but are only short spans of 55.3 and 55.9 ft. The two non-composite spans are also short spans, being only 44.8 and 48.6 ft. in length. There was also a marked difference in the maximum duration of free vibration for the previously-mentioned seven spans (average: 11.6 seconds) as compared to the other four spans (average: 2.1 seconds).

TABLE 1

SUMMARY OF DATA ON SIMPLE-SPAN HIGHWAY BRIDGES

Data on Spans	Ratio of Depth to Span Length	Design L. L. Plus Impact Defl. in Inches	Deflection Due to Test Truck: Observed Deflection-In. (Creep Speed)	Deflection Due to Test Truck: Theoretical Deflection-In. (No Impact)
1. Bridge No. X3 of 33-6-1 Span 2: Length = 60.87' c.c. 36 WF 170 lb. at 5' 4-3/8" c.c. Min. concrete slab thickness = 7.5" Composite design	1/20.3	-----	0.072	0.178
2. Bridge No. X3 of 33-6-1 Span 3: Length = 60.87' c.c. 36 WF 170 lb. at 5' 4-3/8" c.c. Min. concrete slab thickness = 7.5" Composite design	1/20.3	-----	0.063	0.178
3. Bridge No. X3 of 33-6-1 Span 4: Length = 60.87' c.c. 36 WF 170 lb. at 5' 4-3/8" c.c. Min. concrete slab thickness = 7.5" Composite design	1/20.3	-----	0.054	0.178
4. Bridge No. B1 & B2 of 33-6-4 Span 1: Length = 64.25' c.c. 36 WF 170 lb. at 5' 0" c.c. Min. concrete slab thickness = 7.0" Composite design	1/21.4	approx. 0.67 or 1/1150 of span	0.070	0.211
5. Bridge No. B1 & B2 of 33-6-4 Span 2: Length = 64.92' 36 WF 170 lb. at 5'0" c.c. Min. concrete slab thickness = 7.0" Composite design	1/21.6	0.67 or 1/1150 of span	0.066	0.211
6. Bridge No. B1 & B2 of 33-6-4 Span 4: Length = 64.92' 36 WF 170 lb. at 5' 0" c.c. Min. concrete slab thickness = 7.0" Composite design	1/21.6	0.67 or 1/1150 of span	0.068	0.211
7. Bridge No. B1 & B2 of 33-6-4 Span 5: Length = 64.92' 36 WF 170 lb. at 5' 0" Min. concrete slab thickness = 7.0" Composite design	1/21.6	0.67 or 1/1150 of span	0.062	0.211
8. Bridge No. B1 of 56-12-6 Span 1: Length = 55.30' c.c. 36 WF 150 lb. at 5' 6-1/2" c.c. Min. concrete slab thickness = 7.0" Composite design	1/18.4	approx. 0.47 or 1/1410 of span	0.039	0.165
9. Bridge No. B1 of 56-12-6 Span 2: Length = 55.90' c.c. 36 WF 150 lb. at 5' 6-1/2" c.c. Min. concrete slab thickness = 7.0" Composite design	1/18.6	0.47 or 1/1410 of span	0.053	0.171
10. Bridge No. B2 of 39-3-8 Span 1: Length = 44.80' c.c. 30 WF 108 lb. at 4' 10-1/2" c.c. Min. concrete slab thickness = 7.5" Non-composite design	1/17.9	-----	0.050	0.324
11. Bridge No. B2 of 39-3-8 Span 2: Length = 48.65' c.c. 30 WF 124 lb. at 4' 10-1/2" c.c. Min. concrete slab thickness = 7.5" Non-composite design	1/19.5	-----	0.057	0.360

(1) The percent difference was not calculated, since the computed frequency was based on simply-supported conditions, but on end spans the concrete backwall was poured around the end of the beam giving at least partial restraint. Previous studies have also shown that the end spans are much stiffer due to this partial restraint.

(2) Insufficient samples of free vibration were obtained to determine the experimental natural frequency with any accuracy.

Table 1 (contd.)

Ratio: Observed / Theoretical	Max. Amplitude of Vibration in Inches: Truck On Span	Truck On Other Spans	Truck Off Bridge	Max. Duration of Vibration in Seconds After Truck was off Span	Fundamental Freq. of Vib. in c.p.s.: Observed	Computed Theoretical	% Difference Theoretical to Observed
0.40	0.009	0.005	0.005	14.3	6.42	6.26	2.5
0.35	0.007	0.005	0.004	13.7	6.34	6.26	1.3
0.30	0.007	0.004	0.004	14.3	6.23	6.26	0.5
0.33	0.009	0.005	0.001	6.5	7.77	6.34	(1)
0.31	0.013	0.004	0.002	10.0	6.35	6.24	1.7
0.32	0.022	0.006	0.004	14.6	6.07	6.24	2.8
0.29	0.012	0.008	0.002	8.2	(2)	6.24	----
0.24	0.005	0.001	-----	0	(2)	7.45	-----
0.31	0.010	0.001	0.001	2.4	7.25	7.31	0.8
0.15	0.008	0.001	-----	2.0	8.00	8.10	1.4
0.16	0.005	0.001	-----	4.0	7.85	7.79	0.8

Frequency of Vibration

The natural frequency of vibration as observed was obtained by averaging the frequency of free vibration on runs where the vibration was uniform and sustained. The spans may again be placed in two groups with respect to the natural frequency of vibration: six in the group with lower values, and five in the higher value group. All six in the group with natural frequencies less than 6.5 cps. were also subject to higher amplitudes of vibration. In the group with the natural frequency of vibration more than 7.0 cps., were the four spans which demonstrated lower amplitudes of vibration, and one which vibrated at greater amplitudes. A partial explanation for this exception may be the unusual roughness on this bridge deck, which caused a much greater variation in effective axle loads than customary. This unusual variation in effective axle loads of the test truck may have contributed to increased amplitude of vibration for this span.

The theoretical fundamental frequency of vibration was calculated for all simple spans by a method suggested in a previous report[2]. An effective superstructure cross-section was selected, composed of one or two steel beams directly beneath the path of the test truck, and of the accompanying portion of concrete deck slab above the one or two steel beams. For spans designed for composite action the entire concrete slab in the selected portion of the superstructure cross-section was considered with the steel beam in computing the effective moment of inertia. However, for non-composite spans only 50 percent of the concrete slab was considered effective. The ratio of modulus of elasticity of steel to concrete was taken as 6. The fundamental frequency of vibration was then computed on the basis of a simple beam with a uniform load.

With the exception of Span 1 of Bridge B1 and B2 of 33-6-4, the largest difference between experimental and calculated frequency of vibration was 2.8 percent, and the average error was 1.5 percent. The large difference between experimental and calculated value for Span 1 of B1 and B2 of 33-6-4 is due to the fact that the calculated value was for a simply supported beam, while this span was an end span with one end of the steel beams encased in the concrete backwall above the abutment, resulting in considerable restraint at this support. This stiffening effect was previously noted in testing a six span rolled beam bridge where all spans had the same geometric design and a nominal length of 60 ft. One of the interior spans was of composite design, but all others were non-composite. The end spans, with one end of the steel beams encased in the concrete backwall, demonstrated a higher natural frequency of vibration than all other spans, slightly higher than even the composite span.

Continuous-Span Bridges

Seven spans from four continuous-span bridges were tested. One bridge consisted of three spans and was constructed of rolled beams with concrete deck, another was a three span plate girder with concrete deck, and the remaining two were three and four span reinforced concrete tee-beam bridges. None of the steel bridges were designed for composite action. The continuous-span bridges are shown in Figures 11 through 14.

Deflection

The observed deflections for the seven continuous-spans tested varied from 0.014 in. for the end span of a reinforced concrete bridge, to 0.072 in. for the

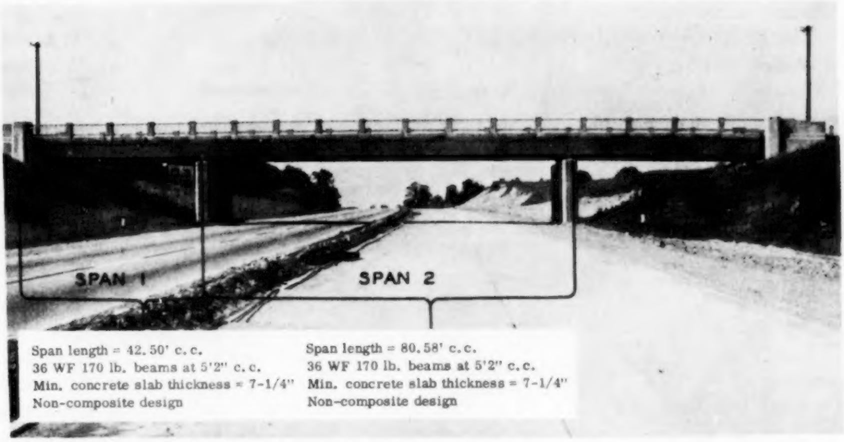


Figure 11. Bridge B2 of 38-1-14. Over US-12 east of Jackson. Three-span continuous, rolled beam bridge.

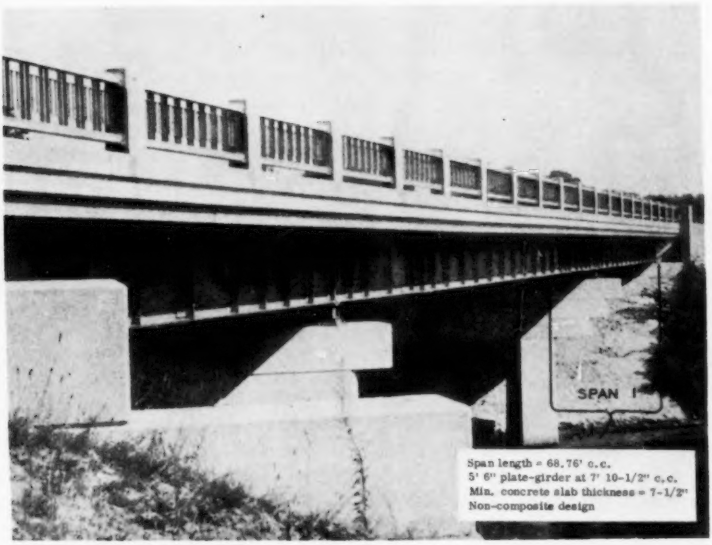


Figure 12. Bridge B1 of 70-7-3. On route US-31 north of Holland. Three-span continuous plate girder bridge.

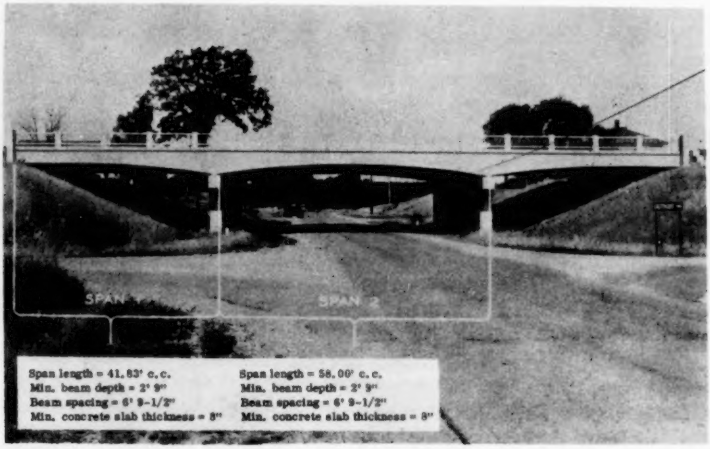


Figure 13. Bridge B1 of 38-11-25. On US-12 near Parma. Reinforced concrete tee-beam bridge.

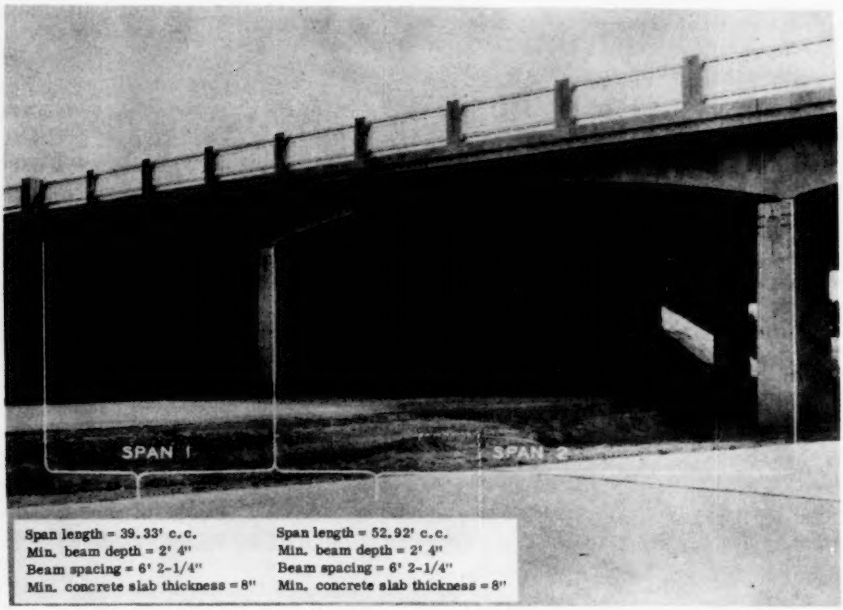


Figure 14. Bridge B5 of 81-11-8. On route US-23 south of Ann Arbor. Four-span reinforced concrete tee-beam bridge.

center span of the rolled beam bridge (Table 2). The maximum deflection of 0.072 in. for the continuous-spans matches the same value for the maximum deflection for the simple-spans. The ratio of observed to theoretical deflection for the steel spans varied from 0.17 to 0.35 with an average of 0.24. The reinforced concrete tee-beam bridges, as might be expected, had much smaller deflection than the steel bridges, with an average of 45 percent less deflection for end spans and 63 percent less deflection for center spans.

Amplitude of Vibration

The reinforced concrete bridges vibrated with smaller amplitudes than the steel bridges. For the continuous-spans, the maximum amplitude of vibration for truck on span, truck on other spans, and truck off the bridge, was 0.010, 0.005, and 0.003 in. respectively. Each maximum was obtained on Span 2 of the rolled beam bridge. Unfortunately it was not possible to instrument the center span of the plate girder bridge due to site difficulties, but a comparison of results from the end spans of rolled beam and plate girder bridges indicates that the center span of the plate girder bridge would probably have vibrated with somewhat greater amplitudes than the center span of the rolled beam bridge. For the bridges tested and reported here, the simple-span bridges of composite design vibrated in general with larger amplitudes of vibration than did the non-composite continuous-span bridges.

The maximum duration of vibration after the test truck passed off the span is also given in Table 2. The averaged maximum duration of vibration for the steel spans was 12.3 seconds as compared to 7.2 seconds for the reinforced concrete bridges.

Frequency of Vibration

The range in the observed fundamental frequency of vibration for the continuous-span bridges was from 5.13 to 8.11 cps. with the steel bridges having an average of 5.21 cps. and the reinforced concrete bridges an average of 7.66 cps.

Since the continuous-span steel bridges were uniform in cross-section the simplest means of computing the natural frequency of vibration appeared to be the numerical method presented by A. S. Veletsos and N. W. Newmark.[3] By this method the natural frequencies may be calculated for undamped flexural vibration of continuous beams on rigid supports and for rigid jointed plane frameworks without sidesway. However, this method as presented was restricted to span members having a uniform cross-section and mass per unitlength. The additional complications of applying this method to the reinforced concrete bridges with variations of weight and moment of inertia throughout the span length, suggested that a different approach was desirable.

For the reinforced concrete bridges, the fundamental frequency of vibration was calculated by the method of Influence Coefficients.

"For free vibrations, the system vibrating at one of its principal modes with frequency 'ω' is loaded with inertia forces '$-m_i\ddot{x}_i = m_i\omega^2 x_i$' of each mass, where '$x_i$' is the deflection of the mass 'm_i' at position 'i'. The equations for the deflection can hence be written as

$$x_1 = a_{11}(m_1\omega^2 x_1) + a_{12}(m_2\omega^2 x_2) + a_{13}(m_3\omega^2 x_3) + \dots$$

$$x_2 = a_{21}(m_1\omega^2 x_1) + a_{22}(m_2\omega^2 x_2) + a_{23}(m_3\omega^2 x_3) + \dots$$

$$x_3 = a_{31}(m_1\omega^2 x_1) + a_{32}(m_2\omega^2 x_2) + a_{33}(m\,\omega^2 x_3) + \dots$$

$$\vdots$$

TABLE 2

SUMMARY OF DATA ON CONTINUOUS-SPAN HIGHWAY BRIDGES

Data on Spans	Ratio of Depth to Span Length[1]	Design L. L. Plus Impact Defl. in Inches	Deflection Due to Test Truck		
			Observed Deflection-In. (Creep Speed)	Theoretical Deflection-In. (No Impact)	Ratio:
1. Bridge No. B2 of 38-1-14 3-span continuous, rolled-beam bridge Span 1: Length = 42.50' c.c. 36 WF 170 lb. at 5' 2" c.c. Min. concrete slab thickness = 7.25" Non-composite design	approx. 1/10	(2)	0.027	0.077	
2. Bridge No. B2 of 38-1-14 3-span continuous, rolled-beam bridge Span 2: Length = 80.58' c.c. 36 WF 170 lb. at 5' 2" Min. concrete slab thickness = 7.25" Non-composite design	approx. 1/16	1.05 or 1/922 of span	0.072	0.426	
3. Bridge No. B1 of 70-7-3 3-span continuous, plate-girder bridge Span 1: Length = 68.76' c.c. 5'6" plate girder at 7'10-1/2" c.c. Min. concrete slab thickness = 7.50" Non-composite design	approx. 1/11	(2)	0.027	0.142	
4. Bridge No. B1 of 38-11-25 Reinforced concrete, haunched tee-beam, continuous 3-span structure Span 1: Length = 41.83' c.c. Min. beam depth = 2'9" Beams at 6' 9-1/2" c.c. Min. concrete slab thickness = 8"	approx. 1/11	(2)	0.014	(2)	
5. Bridge No. B1 of 38-11-25 Reinforced concrete, haunched tee-beam, continuous 3-span structure Span 2: Length = 58.00' c.c. Min. beam depth = 2'9" Beams at 6' 9-1/2" c.c. Min. concrete slab thickness = 8"	approx. 1/13	(2)	0.027	(2)	
6. Bridge No. B5 of 81-11-8 Reinforced concrete, haunched tee-beam, continuous 4-span structure Span 1: Length = 39.33' c.c. Min. beam depth = 2'4" Beam at 6' 2-1/4" c.c. Min. concrete slab thickness = 8"	approx. 1/12	(2)	0.016	(2)	
7. Bridge No. B5 of 81-11-8 Reinforced concrete, haunched tee-beam, continuous 4-span structure Span 2: Length = 52.92' c.c. Min. beam depth = 2'4" Beams at 6' 2-1/4" c.c. Min. concrete slab thickness = 8"	approx. 1/14	(2)	0.026	(2)	

1 According to AASHO Specification the span length for continuous spans shall be considered as the distance between dead load points of contraflexure.

2 No data on this item was available.

Table 2 (contd.)

Observed / Theoretical	Max. Amplitude of Vibration in Inches: Truck On Span	Truck On Other Spans	Truck Off Bridge	Max. Duration of Vibration in Seconds After Truck was off Span	Fundamental Freq. of Vib. in c.p.s.: Observed	Computed Theoretical	% Difference Theoretical to Observed
0.35	0.003	0.002	0.001	10.4	5.23	5.28	1.0
0.17	0.010	0.005	0.003	13.4	5.27	5.28	0.2
0.19	0.005	0.004	0.002	13.2	5.13	5.25	2.3
---	0.003	0.002	0.001	4.9	8.08	7.95	1.6
---	0.004	0.003	0.002	4.1	8.11	7.95	2.0
---	0.003	0.002	0.001	11.6	7.16	7.10	0.8
---	0.005	0.002	0.002	8.2	7.30	7.10	2.8

. . . 'a_{ii}' being the deflection at 'i' due to a unit load at 'i', . . . and similarly 'a_{ij}' defined as the deflection at 'i' due to a unit load at 'j'".(4)

The deflection coefficients for the continuous beam may be calculated by any suitable means.

The previous equation can be modified to the following by matrix notation:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \cdot \\ \cdot \\ \cdot \end{bmatrix} = \omega^2 \begin{bmatrix} a_{11}m_1 & a_{12}m_2 & a_{13}m_3 \cdots \\ a_{21}m_1 & a_{22}m_2 & a_{23}m_3 \cdots \\ 2\ \ a_{31}m_1 & a_{32}m_2 & a_{33}m_3 \cdots \\ \cdot & & \\ \cdot & & \\ \cdot & & \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \cdot \\ \cdot \\ \cdot \end{bmatrix}$$

The iteration procedure was used assuming a set of deflections x_1, x_2, x_3, etc., for the right column in the last equation and performing the proper operation. The resulting column was then normalized, by reducing one of the amplitudes to unity by dividing each term of the column by the particular amplitude. The procedure was repeated with the normalized column until the amplitudes stabilized to a definite pattern, and then the fundamental frequency was found directly.

In the computations, the inertia forces were lumped at three points for each span. The moment of inertia of the gross tee-beam cross-section was used and the modulus of elasticity of the concrete was assumed to be 5×10^6 psi.

The maximum difference between the computed fundamental frequency of vibration and the experimental value for the continuous-span bridges was 2.8 percent, while the average difference was 1.5 percent.

Cantilever-Type Bridges

Sixteen spans from nine cantilever-type bridges were tested. Six of the nine bridges were constructed with rolled beams and concrete deck, and five of these were designed for composite action between slab and beam. Anchor arm span lengths for these six bridges varied from 50 to 75 ft., while the suspended span lengths ranged from 47 to 69 ft. Two of the bridges were plate girder spans constructed with floor beams and stringers, and the remaining bridge was a combination of plate girder and rolled beam with anchor arm spans being plate girder and suspended spans rolled beam. Photographs of these bridges are shown in Figures 8, 9, and 15 through 21.

Deflection

The observed deflections for the sixteen spans varied from 0.049 to 0.201 in. It should be noted that this maximum of 0.201 in. is much higher than the 0.072 in. maximum which occurred on simple and continuous-span bridges. A comparison of the ratio of observed to theoretical deflection for anchor arm spans with composite design varied from 0.23 to 0.60, with an average of 0.43. For suspended spans designed for composite action this same ratio varied from 0.27 to 0.64, with an average of 0.45. In contrast, the average ratio for non-composite anchor arm spans was 0.20, and for non-composite suspended spans, 0.24.

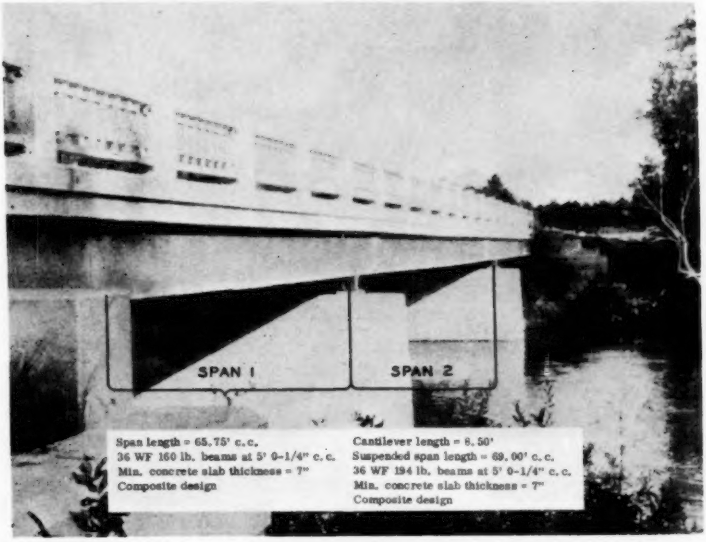


Figure 15. Bridge B1 of 18-12-2. On route M-61 near Temple. Rolled beam cantilever-type bridge.

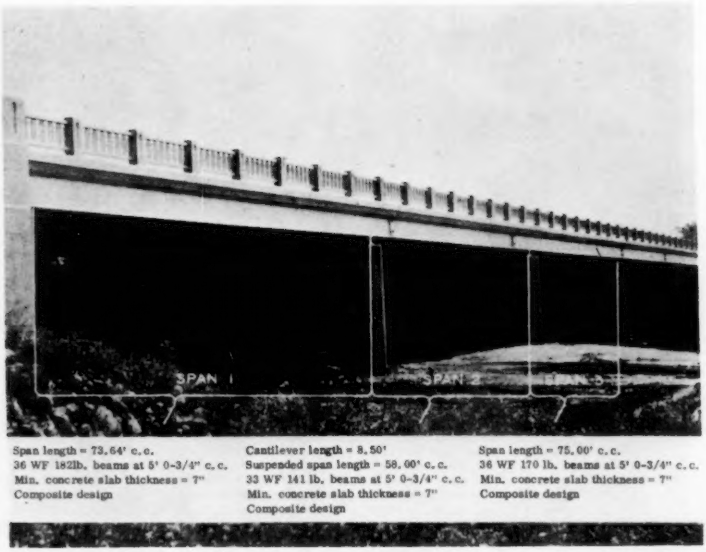


Figure 16. Bridge B1 of 73-20-2. On routes M-46 and M-47 west of Saginaw. Five-span rolled beam cantilever-type bridge.

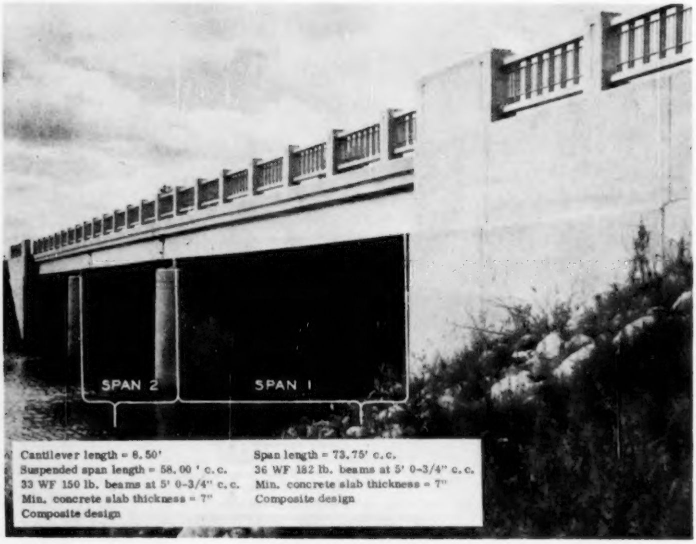


Figure 17. Bridge B2 of 73-20-2. On routes M-46 and M-47 west of Saginaw. Rolled beam cantilever-type bridge.

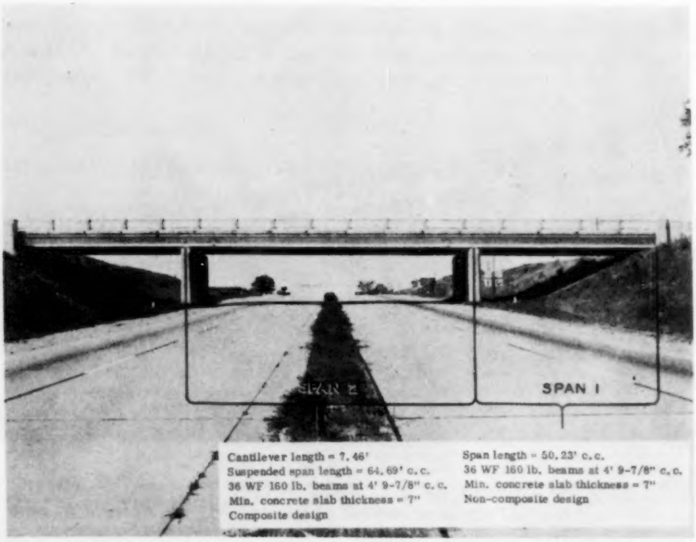


Figure 18. Bridge B3 of 38-1-14. On route US-127 over US-12 north of Jackson. Rolled beam cantilever-type bridge.

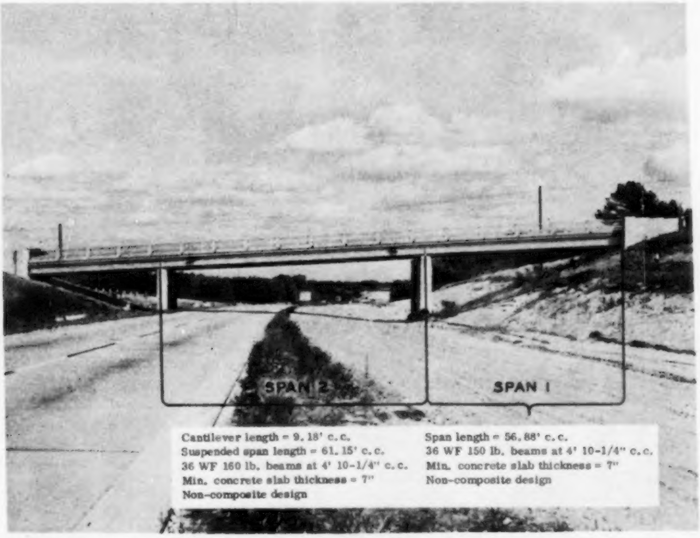


Figure 19. Bridge B1 of 39-5-8. Over route US-12 near Kalamazoo. Rolled beam cantilever-type bridge.

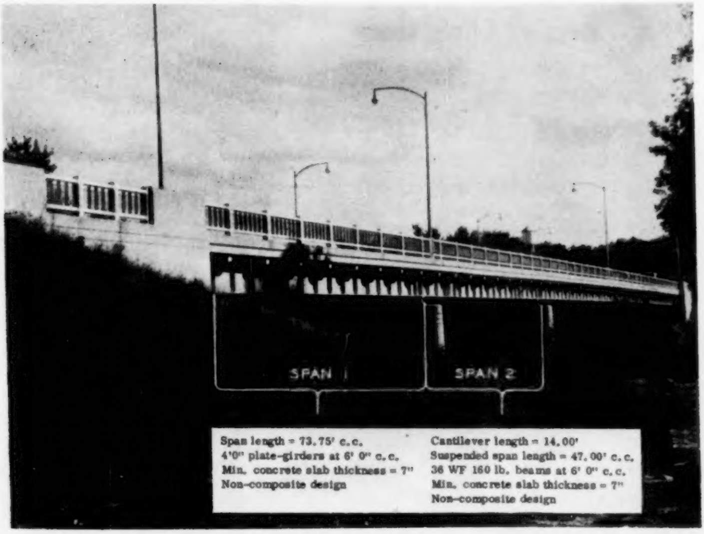


Figure 20. Bridge B1 of 34-6-1. On route M-66 south of Ionia. Five-span plate girder and rolled beam cantilever-type bridge.

Figure 21. Bridge B1 of 62-12-1. On routes M-37 and M-82 in Newaygo. Five-span plate girder cantilever-type bridge with floor beams and stringers.

As similarly shown for simple-spans, the non-composite spans were designed much more conservatively for deflection than the composite spans.

Amplitude of Vibration

The maximum amplitudes of vibration for truck on span, truck on other spans, and truck off the bridge, were 0.037, 0.028, and 0.012 in. respectively, for the cantilever-type bridges (Table 3). Eight of the nine cantilever-type bridges appeared susceptible to larger amplitudes of vibration. These amplitudes were generally much larger than for the other types tested. The larger amplitudes of vibration after the truck had passed off the bridge are of particular significance, for this vibration was also of longer duration. On the three span structures, the suspended spans were more flexible than the anchor arm spans and vibrated with larger amplitudes. When the maximum amplitudes of vibration for the three conditions shown in Table 3 are studied, it appears that four bridges demonstrated the most prominent vibration. These bridges are B1 of 18-12-2, B1 of 73-20-2, B2 of 73-20-2, and B1 and B2 of 33-6-4. It should be noted that all of these bridges were constructed of rolled beams and designed with composite action. The only cantilever-type bridge which did not demonstrate a tendency for larger amplitudes of vibration was a non-composite plate girder bridge with floor beams and stringers. However, it was not possible to test the suspended span on this bridge because

of site difficulties, and it is expected that the suspended span would vibrate at larger amplitudes than the anchor arm span.

Frequency of Vibration

The natural frequency of vibration for the cantilever-type bridges was lower than for the other types, and ranged from 4.35 to 5.56 cps. For all three span bridges the anchor arm and suspended spans vibrated at the same frequency during sustained vibrations. However, on Bridge B1 of 73-20-2, a seven span structure, the amplitudes of vibration were larger, but the vibration pattern was irregular and not as long in duration as might be expected. Also the spans did not appear to vibrate together in harmony, and the interaction of the different frequencies probably reduced the duration of vibration.

On the first attempt at computing the theoretical fundamental frequency of vibration for these structures, the anchor arm together with the cantilever portion of the structure was analyzed as a structural unit independent of the suspended span. This led to different computed natural frequencies for the anchor arm span and the suspended span of the same bridge, and these values were markedly different from the experimental natural frequency. In particular, the experimental frequency was much lower than the computed frequency for the suspended span, when computed as a simple beam between points of suspension.

For Bridge B1 of 18-12-2, the theoretical frequency was next computed by the method used for the reinforced concrete bridges, discussed previously in this report. The only difference in procedure was that the influence coefficients were computed on the basis of frictionless hinges at the points of suspension for the suspended span. This gave a computed fundamental frequency of 4.37 cps. as compared to an experimental frequency of 4.53 cps., a difference of 3.8 percent. As these computations are laborious and time consuming, it was not possible to complete them for the other cantilever-type structures for this report, but they will be computed as soon as possible.

One very simple method of estimating the fundamental frequency of vibration for these cantilever-type structures, was to treat the center span portion of the structure as a simple beam, with a corrected effective span length greater than the suspended span length. This rough estimate gave a maximum error of 4.4 percent and an average error of 2.7 percent when compared to the experimental frequency of vibration.

Dynamic Axle Load Variation

The two load axles of the test truck were instrumented with strain gages to determine the dynamic axle load variation of the test truck as it passed over the bridge. It was determined experimentally that the natural frequency of vibration for the test truck was 3.24 cps., and the solid damping factor, approximately 0.056 lb./in. per second. Due to instrumentation difficulties the axle load variation was obtained on only twelve of the fifteen bridges tested. The maximum dynamic axle load variations for the tractor and trailer axles are shown in Table 4 for various positions of these axles, on the bridge approach, on other spans, on the test span, and off the bridge. In addition, this table gives the maximum percent impact for each span tested as obtained by using the increased dynamic deflection over static deflection as a measure of impact.

TABLE 3

SUMMARY OF DATA ON CANTILEVER-TYPE HIGHWAY BRIDGES

	Data on Spans	Ratio of Depth to Span Length	Design L. L. Plus Impact Defl. in Inches	Deflection Due to Test Truck			Max. Amplitude of Vibration in Inches			Max. Duration of Vibration in Seconds After Truck was off Span	Fundamental Freq. of Vib. in c.p.s.		
				Observed Deflection-In. (Creep Speed)	Theoretical Deflection-In. (No Impact)	Ratio: Observed / Theoretical	Truck On Span	Truck On Other Spans	Truck Off Bridge		Observed	Computed Theoretical	% Difference Theoretical to Observed
1.	Bridge No. B1 of 18-12-2 3-span, rolled-beam, cantilever structure. Span 1: Length = 65.75' c.c. 36 WF 160 lb. at 5' 0-1/4" c.c. Min. concrete slab thickness = 7" Composite design	approx. 1/18.6	0.675 or 1/1170 of span	0.052	0.227	0.23	0.012	0.009	0.008	19.2	4.54	4.37	3.8
2.	Bridge No. B1 of 18-12-2 3-span, rolled-beam, cantilever structure. Span 2: Cantilever = 8.50' Susp. span length = 69.00' c.c. 36 WF 194 lb. at 5' 0-1/4" c.c. Min. concrete slab thickness = 7" Composite design	1/23	(Cantilever) 0.295 or 1/346 of length. (Susp.) 0.665 or 1/250 of span.	0.070	0.259	0.27	0.017	0.017	0.012	21.2	4.53	4.37	3.5
3.	Bridge No. B1 of 73-20-2 7-span, rolled-beam, cantilever structure. Span 1: Length = 73.64' c.c. 36 WF 182 lb. at 5' 0-3/4" c.c. Min. concrete slab thickness = 7" Composite design	approx. 1/20.8	(1)	0.180	0.304	0.59	0.030	0.015	0.010	8.3	5.56		
4.	Bridge No. B1 of 73-20-2 7-span, rolled-beam, cantilever structure. Span 2: Cantilever = 8.50' Susp. span length = 58.00' c.c. 33 WF 141 lb. at 5' 0-3/4" c.c. Min. concrete slab thickness = 7" Composite design	1/19.3	(Cantilever) 0.344 or 1/296 of length (Susp.) 0.557 or 1/1250 of span	0.153	0.239	0.64	0.030	0.013	0.006	12.3	4.90		
5.	Bridge No. B1 of 73-20-2 7-span, rolled-beam, cantilever structure. Span 3: Length = 75.00' c.c. 36 WF 170 lb. at 5' 0-3/4" c.c. Min. concrete slab thickness = 7" Composite design	approx. 1/17.5	0.855 or 1/1040 of span	0.201	0.336	0.60	0.028	0.028	0.08	16.5	4.42		
6.	Bridge No. B2 of 73-20-2 3-span, rolled-beam, cantilever structure. Span 1: Length = 73.75' c.c. 36 WF 102 lb. at 5' 0-3/4" c.c. Min. concrete slab thickness = 7" Composite design	approx. 1/20.9	0.855 or 1/1035 of span	0.129	0.322	0.40	0.030	0.014	0.010	16.4	5.27		
7.	Bridge No. B2 of 73-20-2 3-span, rolled-beam, cantilever structure. Span 2: Cantilever = 8.50' Susp. span length = 58.00' c.c. 33 WF 150 lb. at 5' 0-3/4" c.c. Min. concrete slab thickness = 7" Composite design	1/21.1	(Cantilever) 0.344 or 1/296 of length (Susp.) 0.557 or 1/1250 of span	0.150	0.250	0.60	0.037	0.017	0.008	13.0	5.15		
8.	Bridge No. B3 of 38-1-14 3-span, rolled-beam, cantilever structure. Span 1: Length = 50.23' c.c. 36 WF 160 lb. at 4' 9-7/8" c.c. Min. concrete slab thickness = 7" Non-composite design	approx. 1/14.2	0.546 or 1/1100 of span	0.053	0.218	0.24	0.003	0.002	0.001	6.0	5.00		

Table 3 (contd.)

9.	Bridge No. B3 of 38-1-14 3-span, rolled-beam, cantilever structure. Span 2: Cantilever = 7.46' Susp. span length = 64.69' c.c. 36 WF 160 lb. at 4' 9-7/8" c.c. Min. concrete slab thickness = 7" Composite design	1/21.6	(Cantilever) 0.250 or 1/360 of length (Susp.) 0.529 or 1/1470 of span	0.068	0.242	0.28	0.009	0.005	0.006	10.8	4.90
10.	Bridge No. B1 & B2 of 33-6-4 3-span, rolled-beam, cantilever structure. Span 6: Length = 69.46' c.c. 36 WF 230 lb. at 5' 0" c.c. Min. concrete slab thickness = 7" Composite design	approx. 1/19.7	0.52 or 1/1600 of span	0.093	0.269	0.35	0.023	0.019	0.008	19.4	4.58
11.	Bridge No. B1 of 39-5-8 3-span, rolled-beam, cantilever structure. Span 1: Length = 56.88' c.c. 36 WF 150 lb. at 4' 10-1/4" c.c. Min. concrete slab thickness = 7" Non-composite design	approx. 1/16.1	(1)	0.065	0.355	0.18	0.002	0.001	0.001	7.8	4.71
12.	Bridge No. B1 of 39-5-8 3-span, rolled-beam, cantilever structure. Span 2: Cantilever = 9.18' Susp. span length = 61.15' 36 WF 160 lb. at 4' 10-1/4" c.c. Min. concrete slab thickness = 7" Non-composite design	1/20.4	(1)	0.131	0.558	0.24	0.022	0.007	0.003	14.0	4.78
13.	Bridge No. B1 of 34-6-1 5-span, plate-girder and rolled-beam, cantilever structure. Span 1: Length = 73.75' c.c. 48" plate-girder at 6' 0" c.c. Min. concrete slab thickness = 7" Non-composite design	approx. 1/15.6	(1)	0.062	0.338	0.18	0.015	0.008	0.003	26.2	4.54
14.	Bridge No. B1 of 34-6-1 5-span, plate-girder and rolled-beam, cantilever structure. Span 2: Cantilever length = 14.00' Susp. span length = 47.00' c.c. 36 WF 160 lb. at 6' 0" c.c. Min. concrete slab thickness = 7" Non-composite design	1/15.6	(1)	0.082	0.337	0.24	0.016	0.012	0.005	26.2	4.50
15.	Bridge No. B1 of 62-12-1 5-span, plate-girder, cantilever structure with floor-beams and stringers. Span 1: Length = 97.25' c.c. 6' 9-1/2" min., plate-girder - 2 per roadway Min. concrete slab thickness = 7" Non-composite design	approx. 1/12.2	(1)	0.083	-----	-----	0.014	0.012	0.006	12.0	4.35
16.	Bridge No. B1 of 56-12-6 3-span, plate-girder, cantilever structure with floor-beams and stringers. Span 3: Length = 73.90' c.c. 5' 6" min., plate-girder - 2 per roadway Min. concrete slab thickness = 7" Non-composite design	approx. 1/11.4	0.268 or 1/3300 of span	0.049	-----	-----	0.007	0.004	-----	0.7	-----

(1) Data on a few bridges is lacking, since design computations were destroyed in a fire.

TABLE 4

SUMMARY OF DATA ON MAXIMUM IMPACT AND
DYNAMIC AXLE-LOAD VARIATION FOR ALL SPANS

Bridge and Span Number		Max. Percent Impact based on:[1] $\frac{Dd - Ds}{Ds} \times 100$	Maximum Axle-Load Tractor Axle - 15.5 Kips (Sprung) Bridge Approach	Other Spans	On Span
1. Simple-Span Bridges					
X3 of 33-6-1	Span 2	5.6	-----	± 0.9	± 0.8
X3 of 33-6-1	Span 3	5.7	-----	± 0.9	± 0.8
X3 of 33-6-1	Span 4	8.9	-----	± 0.9	± 0.9
B1 & B2 of 33-6-4	Span 1	14.8	-----	± 3.3	± 2.5
B1 & B2 of 33-6-4	Span 2	14.0	-----	± 3.3	± 2.5
B1 & B2 of 33-6-4	Span 4	45.5	-----	± 5.4	± 6.8
B1 & B2 of 33-6-4	Span 5	16.4	-----	± 6.8	± 2.0
B1 of 56-12-6	Span 1	10.7			(No Data)
B1 of 56-12-6	Span 2	15.7			(No Data)
B2 of 39-3-8	Span 1	0.0	± 1.9	± 5.6	± 2.7
B2 of 39-3-8	Span 2	2.4	± 1.9	± 5.6	± 2.8
Average simple-span bridges		12.7	± 1.9	± 3.6	± 2.4
2. Steel Continuous-Span Bridges					
B2 of 38-1-14	Span 1	8.3	± 2.5	± 2.5	± 2.0
B2 of 38-1-14	Span 2	10.2	± 2.5	± 2.5	± 1.5
B1 of 70-7-3	Span 1	13.0	± 2.5	± 2.3	± 2.4
Average steel continuous-span bridges.		10.8	± 2.5	± 2.4	± 2.0
3. Concrete Continuous-Span Bridges					
B1 of 38-11-25	Span 1	8.3	± 2.8	± 2.6	± 2.6
B1 of 38-11-25	Span 2	14.3	± 2.8	± 2.6	± 2.6
B5 of 81-11-8	Span 1	9.7	± 1.4	± 2.2	± 1.4
B5 of 81-11-8	Span 2	12.0	± 1.4	± 2.2	± 2.0
Average concrete continuous-span bridges.		11.1	± 2.1	± 2.4	± 2.2
4. Cantilever-Type Bridges					
B1 of 18-12-2	Span 1	23.9	± 3.0	± 2.9	± 2.6
B1 of 18-12-2	Span 2	23.8	± 3.0	± 2.9	± 2.3
B1 of 73-20-2	Span 1	11.5	± 2.1	± 2.2	± 1.5
B1 of 73-20-2	Span 2	8.5	± 2.1	± 2.2	± 1.7
B1 of 73-20-2	Span 3	3.5	± 2.1	± 2.2	± 1.6
B2 of 73-20-2	Span 1	18.3	± 1.7	± 2.5	± 2.5
B2 of 73-20-2	Span 2	15.8	± 1.7	± 2.5	± 2.5
B3 of 38-1-14	Span 1	8.9			(No Data)
B3 of 38-1-14	Span 2	7.1			(No Data)
B1 & B2 of 33-6-4	Span 6	28.2	-----	± 6.8	± 4.4
B1 of 39-5-8	Span 1	24.1	± 2.7	± 1.9	± 4.0
B1 of 39-5-8	Span 2	22.1	± 2.7	± 4.0	± 1.9
B1 of 34-6-1	Span 1	27.1			(No Data)
B1 of 34-6-1	Span 2	16.1			(No Data)
B1 of 62-12-1	Span 1	11.8	± 2.6	± 1.5	± 2.4
B1 of 56-12-6	Span 3	12.5			(No Data)
Average cantilever-type bridges		16.4	± 2.4	± 2.9	± 2.5

[1] Dd = Dynamic deflection
Ds = Static deflection

Table 4 (contd.)

Variation in Kips and Percent (Test Truck at Various Positions as Noted)						
		Trailer Axle - 18.1 Kips (Unsprung)				
Off Bridge	Max. Percent	Bridge Approach	Other Spans	On Span	Off Bridge	Max. Percent
-----	6.0	-----	± 4.0	± 1.5	-----	21.9
-----	6.0	-----	± 4.0	± 2.6	-----	21.9
-----	6.0	-----	± 4.0	± 2.7	-----	21.9
± 2.0	21.6	-----	± 6.4	± 4.7	± 3.6	35.2
± 2.0	21.6	-----	± 6.4	± 5.0	± 3.6	35.2
± 5.4	44.1	-----	± 9.6	±10.6	± 8.6	58.5
± 5.4	44.1	-----	±10.6	± 4.6	± 8.6	58.5
				(No Data)		
				(No Data)		
± 4.5	36.0	± 2.5	± 7.6	± 4.1	± 8.5	47.0
± 4.5	36.0	± 2.5	± 7.6	± 3.5	± 8.5	47.0
± 4.0		± 2.5	± 6.7	± 4.4	± 6.9	
-----	16.3	± 2.3	± 2.4	± 2.2	-----	13.6
-----	16.3	± 2.3	± 2.4	± 1.4	-----	13.6
± 2.0	16.3	± 4.9	± 5.7	± 4.0	± 8.8	31.7
± 2.0		± 3.2	± 3.5	± 2.5	± 3.8	
± 3.6	23.0	± 2.7	± 2.6	± 2.8	± 3.2	17.7
± 3.6	23.0	± 2.7	± 2.8	± 2.6	± 3.2	17.7
± 1.4	14.0	± 2.4	± 2.1	± 2.1	± 2.7	15.0
± 1.4	14.0	± 2.4	± 2.1	± 2.1	± 2.7	15.0
± 2.5		± 2.6	± 2.4	± 2.4	± 3.0	
± 3.2	20.4	± 4.2	± 4.1	± 3.7	± 3.8	23.2
± 3.2	20.4	± 4.2	± 4.1	± 3.4	± 3.8	23.2
-----	14.1	± 2.2	± 2.4	± 1.8	-----	13.6
-----	14.1	± 2.2	± 2.4	± 2.4	-----	13.6
-----	14.1	± 2.2	± 2.4	± 1.7	-----	13.6
± 1.8	16.0	± 3.2	± 2.5	± 3.6	± 3.6	19.8
± 1.8	16.0	± 3.2	± 3.6	± 2.4	± 3.6	19.8
				(No Data)		
				(No Data)		
± 5.4	44.1	-----	±10.6	± 7.8	± 8.6	58.5
± 3.8	25.6	± 2.6	± 1.6	± 3.5	± 3.6	19.8
± 3.8	25.6	± 2.6	± 3.5	± 1.6	± 3.6	19.8
				(No Data)		
				(No Data)		
± 2.2	15.3	± 2.6	± 1.7	± 2.3	± 2.2	14.6
				(No Data)		
± 3.2		± 2.9	± 3.5	± 3.1	± 4.2	

The axle load variation was seldom more than ± 4 kips, except in the case of data from two bridges. One of these bridges (B1 and B2 of 33-6-4) has had a very rough riding surface since construction. The maximum axle load variation on this bridge was ± 6.8 kips on the tractor axle, and ± 10.6 kips on the trailer axle. This departure from the static load is ± 44 and ± 58 percent, respectively. The other bridge, (B2 of 39-3-8), had a bituminous surface for the bridge approach and a bump had formed adjacent to the north end of the bridge, causing a maximum axle load variation of ± 8.5 kips. This bridge consisted of four simple spans and the test spans were at the opposite end of the bridge. The vertical oscillation of the test truck resulting from passing over this bump, had dissipated by the time it reached the test spans.

The bridge span with the largest percent impact was also the span on which the largest axle load variation occurred (B1 and B2 of 33-6-4, Span 4). In general the percent impact for the various spans appears to be reasonably consistent with the maximum percent axle load variation recorded while the truck was on the span (Figure 22). However, there are six points which fall farther away from the general pattern. Two of these points, representing low ratios of impact to axle load variation, are from Bridge B2 of 39-3-8 which had relatively stiff simple spans. On the other hand, the four points representing high ratios of impact to axle load variation were from two of the more flexible cantilever-type bridges.

The axle load variation increased in proportion to test truck speed. In Figure 23 the data from the tractor axle on Bridge B2 of 73-20-2 was used to illustrate this correlation.

An incidental, but interesting observation is shown in Figure 24, where three test truck runs were made on Bridge B2 of 73-20-2, at approximately the same speed. Both spans of this bridge responded in almost identical pattern and amplitude for all three of these test runs. For two of the three test runs the dynamic axle load variation was also recorded and these two traces are also similar to each other, although not as uniform in pattern as the bridge oscillations.

Comparison of Bridge Types

In comparing the vibration behavior of various bridges, the same grouping as before, of simple-span, continuous-span, and cantilever-type will be employed. However a differentiation will be made between composite and non-composite structures, and in the case of continuous-span bridges, a distinction between those constructed of steel and of reinforced concrete. No distinction will be made between rolled beam and plate girder bridges of the various types.

Figures 25 through 29 show the subject data averaged for each bridge grouping. Figure 25 compares these bridge types on the basis of observed deflection and it illustrates the fact that the cantilever-type with composite design had the largest average deflection, followed by the same type without composite design. The reinforced concrete bridges, as might be expected, deflected least. In Figure 26 the bridge types are compared with respect to the ratio of observed to theoretical deflection. This graph shows that the non-composite spans had a 50 percent lower ratio, compared to the composite spans, for both simple-span and cantilever-type bridges. Composite and non-composite spans of the cantilever-type bridges had higher ratios than the corresponding simple-span bridges.

A comparison of the amplitudes of vibration in Figure 27 illustrates the much greater susceptibility of the cantilever-type bridges, especially those

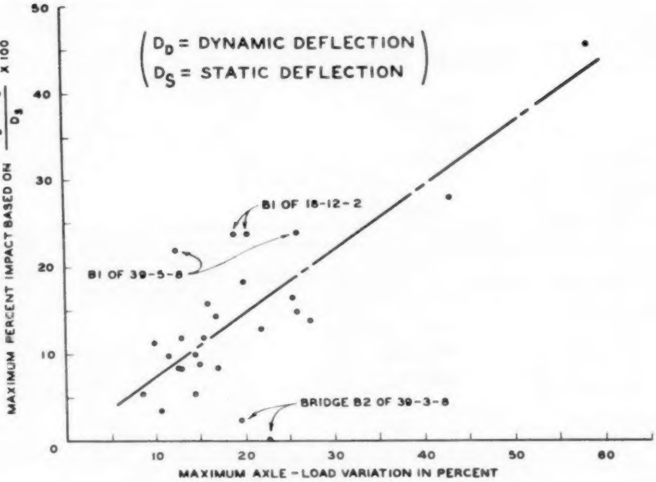


Figure 22. Maximum impact compared with maximum axle-load variation for all types of highway spans.

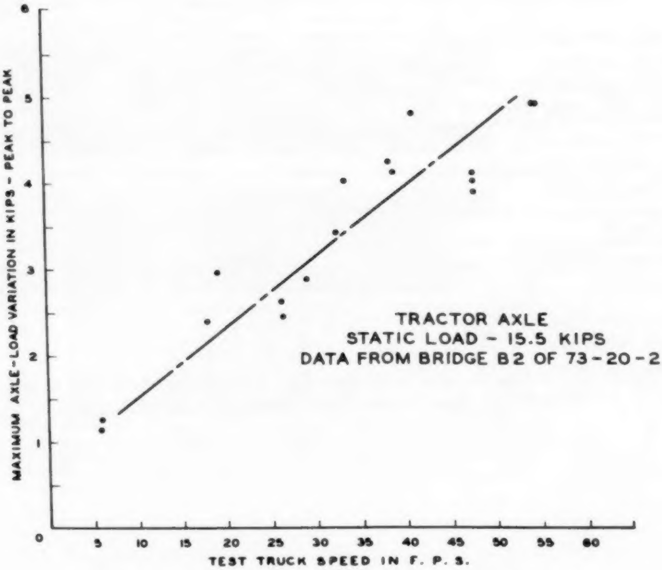


Figure 23. Maximum axle-load variation compared with test truck speed.

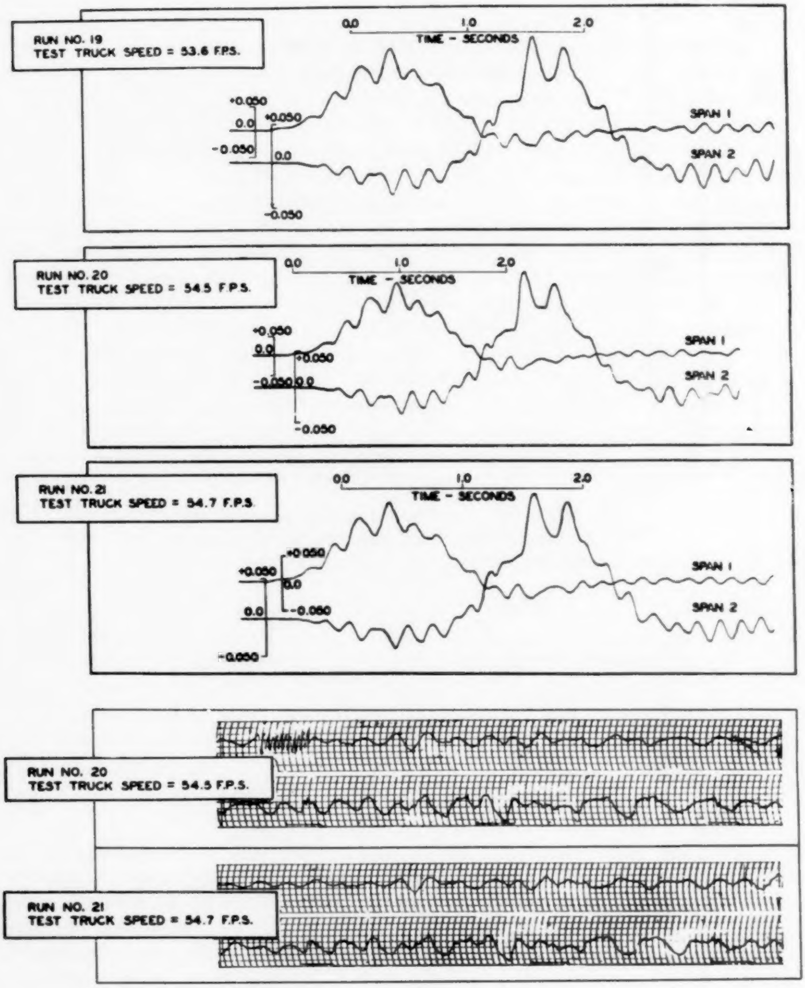


Figure 24. Bridge oscillation and axle-load variation for three runs of test truck on Bridge B2 of 73-20-2 at approximately the same speed.

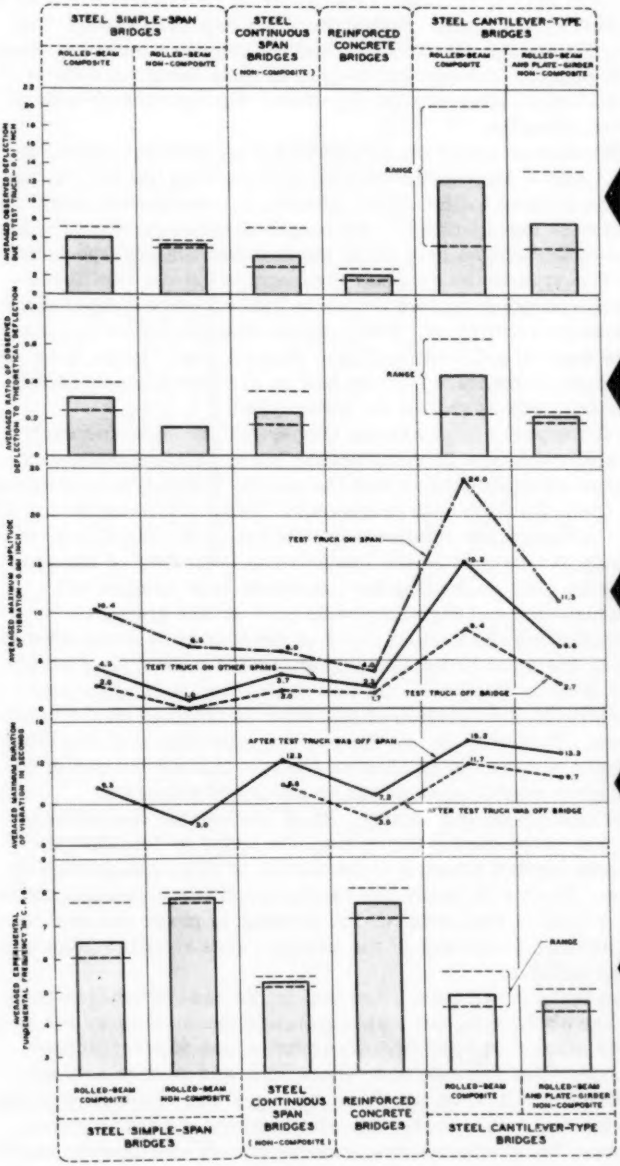


Fig. 25. Observed deflection for various types of highway bridges.

Fig. 26. Ratio of observed deflection to theoretical deflection for various types of highway bridges.

Fig. 27. Maximum observed amplitude of vibration aferaged for various types of highway bridges.

Fig. 28. Maximum duration of vibration after truck was off span or bridge - averaged for various types of highway bridges.

Fig. 29. Experimental fundamental frequency - averaged for various types of highway bridges.

designed for composite action, to larger amplitudes of vibration. It should be noted that the cantilever-type spans tested were not especially long. The maximum anchor arm span length was 75 ft. and the longest suspended span length was 69 ft. In general, the cantilever-type bridges designed without composite action had longer span lengths, but these did not vibrate with as large an amplitude of vibration.

The maximum duration of vibration, averaged for each bridge group, is shown in Figure 28. Again, the cantilever-type bridges lead the others, but steel continuous-span bridges follow rather closely non-composite cantilever-type bridges. A possible explanation for the longer duration of vibration for the steel continuous-span bridges may lie in the fact that the average solid damping factor for this type is the same as the average for the cantilever-type bridge. The solid damping factors shown in Table 5 were computed from examples of sustained free vibration. The average damping factor for the simple span bridges was not sufficiently higher than the other types to be conclusive, but the reinforced concrete bridges had an average damping factor approximately twice as large as that of the other types.

Figure 29 gives the experimental natural frequency, averaged for each bridge type. The cantilever-type bridges had the lowest natural frequency, while the steel continuous-span bridges had the second lowest. A comparison of the amplitude of free vibration with fundamental frequency is shown for all spans in Figure 30. Although the relationship between amplitude of free vibration has a tendency to increase as the fundamental frequency of the span is reduced. This figure also shows that the cantilever type bridges are grouped in the low frequency and high amplitude part of this graph.

Another way of evaluating the various types of bridges is by comparing the oscillograph traces of the most prominent vibrations for each type of structure. This has been done in Figure 31. In this evaluation, it is again clear that the greatest amplitude and duration of vibration is obtained on the cantilever-type structures. Figure 32 shows the oscillograph traces of two other cantilever-type bridges in order to emphasize the fact that all but one of the bridges of this type were readily susceptible to sustained vibration.

On one particular cantilever-type bridge, B1 of 39-5-8, the flexibility of the suspended span was noted during the testing. In order to demonstrate this flexibility one man started jumping at the center of this span producing perceptible vibration. Figure 33 shows the oscillograph trace resulting from three men weighing a total of less than 500 lb. jumping in phase and at a frequency close to the natural frequency of the bridge. This resulted in an amplitude of vibration of 0.010 in.

In previous bridge tests it had been noted that in the case of bridges built with common piers and abutments, but with separate superstructures for each roadway, a noticeable amount of vibration occurred on one superstructure when a truck passed over the opposite one. Since this was particularly noticeable on Bridge B1 of 62-12-1, an oscillograph trace was obtained showing vibration caused by the test truck on the opposite roadway superstructure. This is shown in Figure 34, where the maximum amplitude of vibration under this condition was 0.005 in. It appears that the only way this vibration can be transmitted from one superstructure to the other is through the common piers and abutments. Since this effect had been noted previous to the bridge tests reported here, traffic was stopped temporarily on both roadways during the test program, so that recorded vibrations would be caused solely by passage of the test truck.

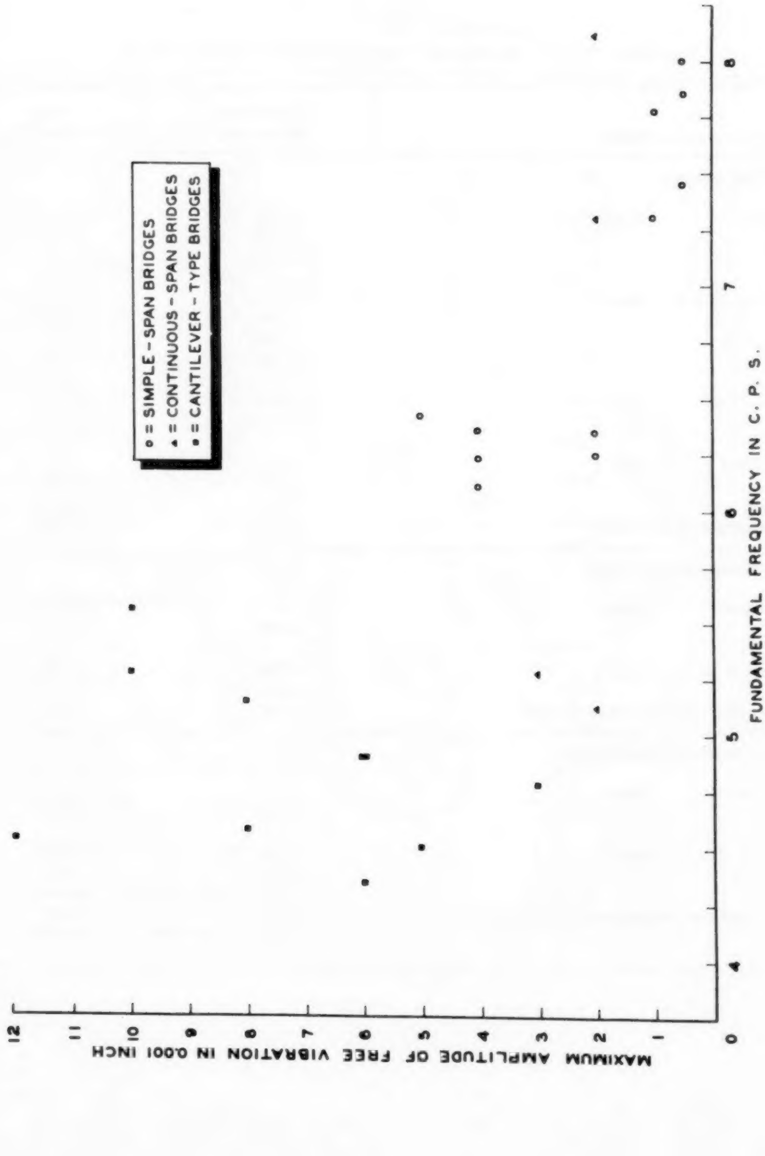


Figure 30. Maximum amplitude of free vibration compared with fundamental frequency for all bridge types.

TABLE 5

SUMMARY OF DATA ON DAMPED FREE VIBRATION FOR ALL SPANS

Bridge and Span Number		Logarithmic Decrement	Solid Damping Factor
1. Simple-Span Bridges			
X3 of 33-6-1	Span 2	0.065	0.021
" " "	" 3	0.050	0.016
" " "	" 4	0.096	0.030
B1 & B2 of 33-6-4	Span 1	-----	-----
" " "	" 2	-----	-----
" " "	" 4	0.069	0.022
" " "	" 5	0.091	0.028
B1 of 56-12-6	Span 1	-----	-----
" " "	" 2	-----	-----
B2 of 39-3-8	Span 1	-----	-----
" " "	" 2	0.076	0.024
Average simple-span bridges		0.074	0.024
2. Steel Continuous-Span Bridges			
B2 of 38-1-14	Span 1	-----	-----
" " "	" 2	0.044	0.014
B1 of 70-7-3	Span 1	0.079	0.025
Average steel continuous-span bridges		0.062	0.020
3. Concrete Continuous-Span Bridges			
B1 of 38-11-25	Span 1	-----	-----
" " "	" 2	0.115	0.036
B5 of 81-11-8	Span 1	0.122	0.039
" " "	" 2	0.142	0.043
Average concrete continuous-span bridges		0.126	0.039

Table 5 (contd.)

4. Cantilever-Span Bridges			
(A) Anchor-Arm Spans			
B1 of 18-12-2	Span 1	0.038	0.012
B1 of 73-20-2	Span 1	0.084	0.027
" " "	" 3	0.071	0.023
B2 of 73-20-2	Span 1	0.073	0.023
B3 of 38-1-14	Span 1	-----	-----
B1 & B2 of 33-6-4	Span 6	0.040	0.013
B1 of 39-5-8	Span 1	-----	-----
B1 of 34-6-1	Span 1	0.057	0.018
B1 of 62-12-1	Span 1	0.075	0.024
B1 of 66-12-6	Span 3	-----	-----
Average anchor-arm spans		0.063	0.020
(B) Suspended Spans			
B1 of 18-12-2	Span 2	0.044	0.014
B1 of 73-20-2	Span 2	0.074	0.024
B2 of 73-20-2	Span 2	0.057	0.018
B3 of 38-1-14	Span 2	0.075	0.024
B1 of 39-5-8	Span 2	0.070	0.022
B1 of 34-6-1	Span 2	0.044	0.014
Average suspended spans		0.061	0.019
Average cantilever-span bridges		0.062	0.020

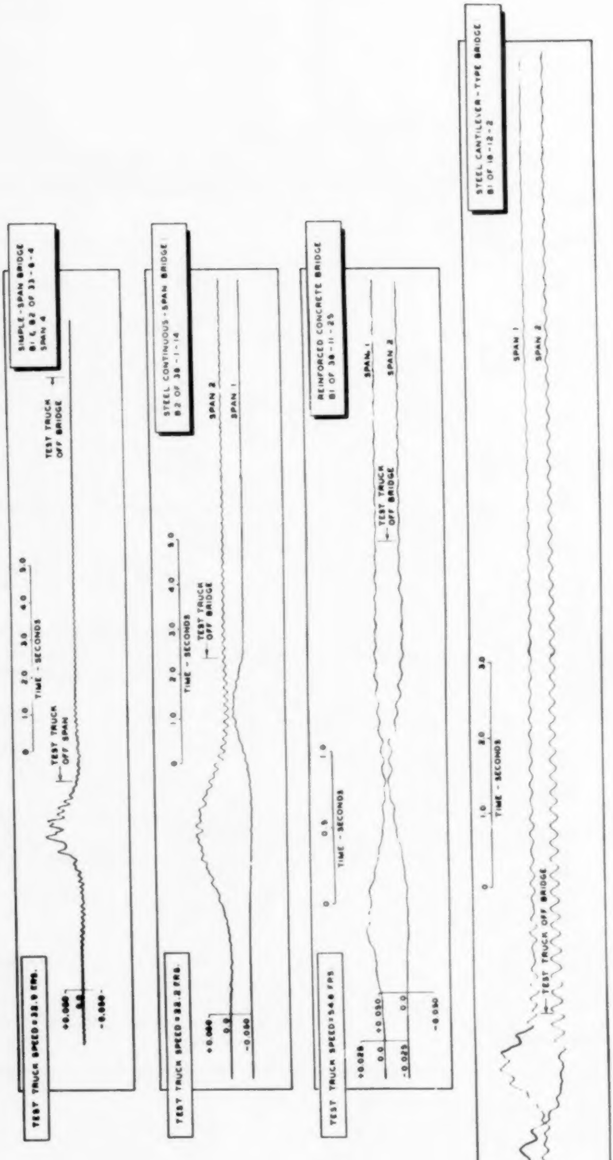


Figure 31. Comparison of oscillograph traces of most pronounced vibration for each bridge type.

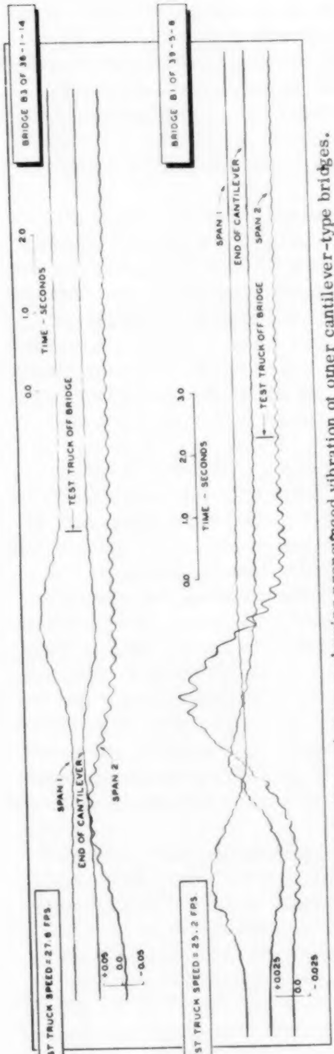


Figure 32. Oscillograph traces showing pronounced vibration of other cantilever-type bridges.

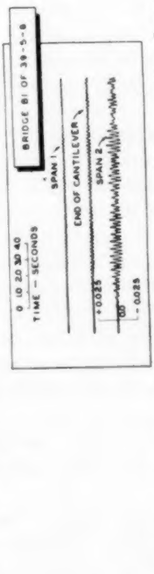


Figure 33. Oscillograph trace showing effect of three men jumping at center of suspended span of Bridge B1 of 39-5-8

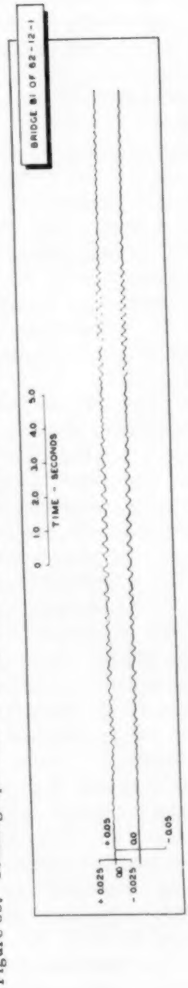


Figure 34. Oscillograph trace showing effect of truck passing over opposite roadway on a bridge with separate superstructure for each roadway.

Psychological Reaction to Vibration

These bridges were not analyzed for susceptibility to vibration on the basis of any fear that such vibration might lead to harmful structural effects. The primary concern in limiting vibration rises from the possibility that vibration may have discomforting psychological effects on pedestrians or motorists. In current and previous testing, the existing magnitudes of bridge vibration apparently would not appreciably affect a motorist in a vehicle traveling across the bridge. However, to a pedestrian walking across the bridge or to a motorist seated in a stalled vehicle on the bridge, the magnitude of the bridge vibration experienced in these tests might have a discomforting effect.

Individual sensitivity to such vibration varies and therefore it is difficult to set an exact limit as to what amplitude of vibration for a given frequency is perceptible, unpleasant, or intolerable. Janeway[5] has recommended certain safe limits for amplitude of vibration at various frequencies of vibration (Figure 35). These limits were based on data from subjects standing, or sitting on a hard seat. From one to six cycles per second, the recommended amplitude limits are based on the equation $af^3 = 2$, where 'a' is the amplitude, and 'f' is the frequency of vibration. From six to twenty cycles per second, the recommended amplitude limits are based on the equation of $af^2 = 1/3$.

On this same Figure the various bridge vibration amplitudes and frequencies have been plotted in order to compare them with the Janeway limits. The amplitude of vibration is shown with the test truck on the span, and off the bridge. For amplitudes of vibration with the test truck on the span, seven cantilever-type spans and seven simple-spans had points falling above the limits. However, the amplitude of vibration with the span loaded was never more than one or two cycles at the magnitude shown; therefore, it is felt that this vibration would not cause discomfort to someone on the bridge. However, several of the bridges tested (B1 of 73-20-2, Span 1; B2 of 73-20-2, Span 1; and B1 and B2 of 33-6-4, Span 4) had amplitudes of vibration with the test truck off the bridge which closely approached the limit line. This free vibration was sustained for a considerable number of cycles on some bridges. Further, data from other tests[2] indicates that in certain instances, normal truck traffic would produce larger amplitudes of vibration than those produced by the test truck.

Personal reaction of personnel engaged in these tests appears somewhat counter to the comparison of bridge vibration with recommended safe limits shown in Figure 35. The vibration of the simple-span and steel continuous-span bridges was perceptible, but was not sufficiently extensive to become discomforting. However, greater amplitude of vibration, although at a lower frequency, was somewhat discomforting on several of the cantilever-type bridges (B1 of 62-12-1, B1 of 73-20-2, B2 of 73-20-2 and Span 6 of B1 and B2 of 33-6-4). For the first three bridges, the instrumentation truck was parked on the roadway superstructure opposite from the one being tested. One person was seated on a hard stool in the instrumentation truck during the test program which lasted several hours. He was subjected to the bridge oscillation from the test truck on the other roadway, plus that due to the passage of normal truck traffic on the same roadway, which occurred between test truck runs. Due to this bridge oscillation he felt mild discomfort leading to a headache.

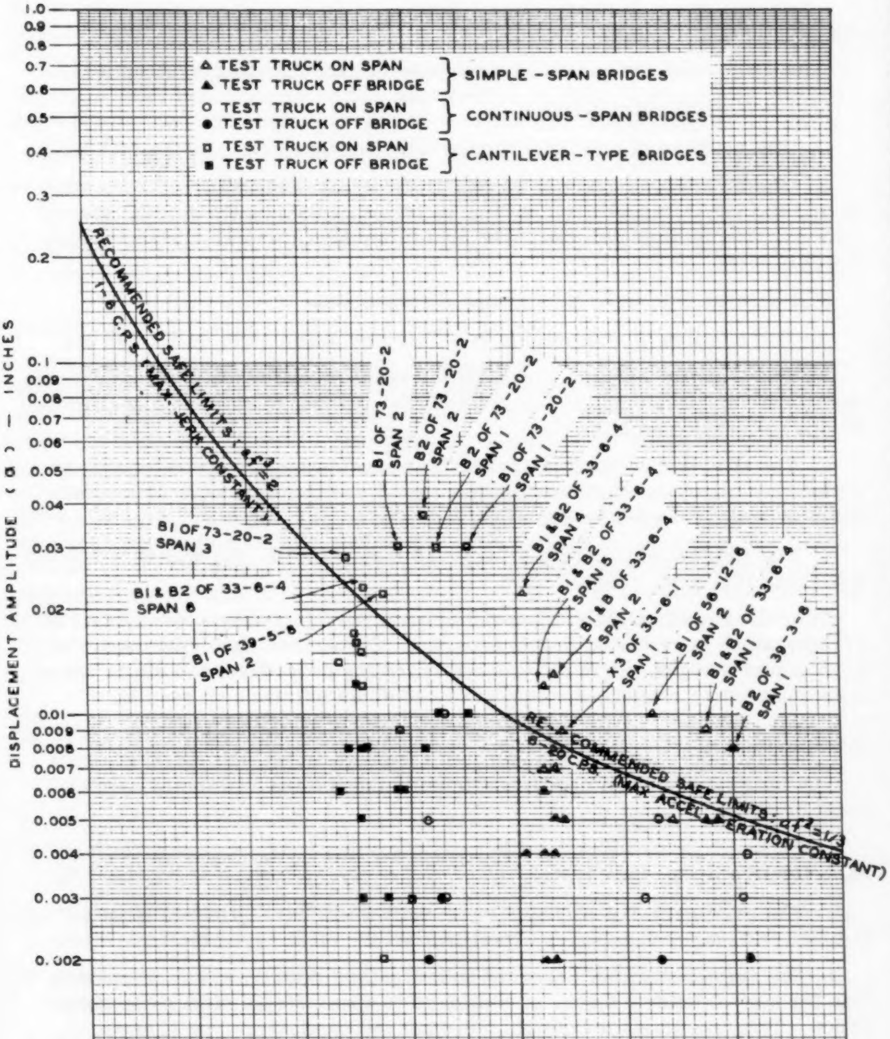


Figure 35. Observed amplitude and frequency of bridge vibration compared with recommended safe limits by Janeway[5].

CONCLUSIONS

Assumptions and Limitations

The theoretical computations were based on the following assumptions:

For deflection calculations—

1. The modulus of elasticity of concrete was assumed to be 3×10^6 psi.

2. The effective moment of inertia of the spans designed for composite action and non-composite action was as recommended by the AASHO Specifications. The lateral distribution of load to stringers was also based on this specification.

For natural frequency of vibration calculations—

3. The modulus of elasticity of concrete was assumed to be 5×10^6 psi.

4. The effective moment of inertia for spans designed for composite action was based on considering 100 percent of the concrete deck above the beam or beams in question as effective.

5. The effective moment of inertia for spans designed for non-composite action was based on considering 50 percent of the concrete deck above the beam or beams in question as effective.

6. For reinforced concrete bridges the moment of inertia was based on the full gross tee-beam cross section.

The following findings are based solely on the thirty-four spans of fifteen bridges which were tested in this program. Cognizance should be taken that selection of bridges, slight differences in testing procedure, and slight variations in roadway roughness, could have had some influence on results.

General Findings

The following findings appear sufficiently conclusive to warrant careful consideration:

1. On the basis of deflection the cantilever-type bridges were definitely the most flexible bridge type tested.

2. The cantilever-type bridges were much more susceptible to large amplitudes and longer duration of vibration than the other types tested.

3. The average ratio of observed to theoretical deflection was 50 percent less for non-composite spans as compared to spans designed with composite action. This was true for both simple-span and cantilever-type bridges.

The following points are indicated from the present data, but more extensive testing or refinement in instrumentation may modify some of these concepts:

4. As a general rule, the bridges with the lower fundamental frequency of vibration were more susceptible to larger amplitudes of vibration.

5. Personal reaction bordering on discomfort was experienced only from vibration of the cantilever-type bridges.

6. In comparing bridges of a given type, designed with and without composite action, those designed with composite action were more susceptible to larger amplitudes of vibration.

7. The maximum percent impact as measured by the increase in dynamic deflection over the static deflection appeared to be related reasonably well to the maximum dynamic axle load variation of the test truck while on the span.

8. The magnitude of the dynamic axle load variation of the test truck increased with test truck speed.

9. It appears possible to compute the fundamental frequency of any of the bridge types tested, with sufficient accuracy, by using the methods and assumptions suggested in this report.

Discussion of Findings Pertinent to Bridge Design

The results of these bridge tests as discussed previously, indicate that there is an inequality in present design methods between bridges designed with and without, composite action. This was shown by the 100 percent higher ratio of actual deflection to theoretical deflection for the spans designed for composite action compared to those designed without it.

These tests also indicate that the present deflection limitations, when applied in design in simple span, continuous and cantilever-type bridges, do not appear to result in equitable stiffness for the various bridge types. If modifications in the deflection limitations are proposed, or if other means of controlling susceptibility to larger amplitudes of vibration are contemplated, then a thorough study of the cantilever-type highway bridge is imperative.

BIBLIOGRAPHY

1. "Test on Rolled-Beam Bridge Using H20-S16 Loading", by G. M. Foster, Highway Research Board, Research Report 14-B (Publication 253), pp. 10-38.

2. "Vibration and Deflection of Rolled-Beam and Plate-Girder Bridges", by G. M. Foster and L. T. Oehler, Highway Research Board, Bulletin 124 (Publication 411), pp. 79-110.

3. "Natural Frequencies of Continouus Flexural Members", by A. S. Veletsos and N. M. Newmark, ASCE Proceedings Separate No. 735, Vol. 81, 1955.

4. "Mechanical Vibrations", by W. T. Thomson, Prentice-Hall, Inc., New York, 1953, pp. 169-170.

5. "Vertical Vibration Limits for Passenger Comfort", by R. N. Janeway from "Ride and Vibration Data", Special Publications Department (SP-6), Society of Automotive Engineers, Inc., 1950, p. 23.

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APPLICATION AND DEVELOPMENT OF AASHO SPECIFICATIONS TO BRIDGE DESIGN *

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(Proc. Paper 1320)

SYNOPSIS

The accelerated program of highway and bridge construction has focused the attention of engineers on the Standard Specifications for Highway Bridges of the American Association of State Highway Officials under which the bridges will be designed and built. In this paper, the application of these specifications is explained, paying special attention to the history and development of the various provisions. A section is included covering their application to bridges on the National System of Interstate and Defense Highways. An extensive bibliography is added for reference.

INTRODUCTION

Until about thirty years ago, highway bridges in the United States were designed for uniform loads of from 80 to 100 pounds per square foot of floor with an alternate loading consisting of a road roller or a traction engine. These loads were applied with no allowance for impact. In the September 3, 1914 issue of Engineering News, there appeared an article by Manville and Gastmeyer,(1) calling attention to the serious overstressing of highway bridges by motor trucks. This is probably the earliest printed article dealing with the effects of motor trucks on bridges.

The subject of truck loadings received considerable attention in the years immediately following the first World War. The American Association of

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State Highway Officials published its first "Standard Specifications for Highway Bridges and Incidental Structures" under date of June 1, 1925. The idea of placing the live loads in definite traffic lanes was a novel feature of this edition.

The AASHO Specifications are now in the Sixth Edition (1953). Several revisions have been tentatively adopted since 1953 and these will be incorporated in the Seventh Edition, which is now in preparation. It is expected that the new edition will make its appearance late in 1957. The more important of these revisions will be printed in full in the Appendix. Brief references to the less important revisions will be made at the proper places in the text of this paper.

The Sixth Edition (1953) embraces four divisions, as follows:

Division 1 - General Provisions
2 - Construction
3 - Design
4 - Materials

These specifications apply to bridges of ordinary span. Supplementary specifications may be required for bridges of unusual type or for those with span lengths greater than 300 or 400 feet. This paper will place special emphasis on the provisions of Division 3 relating to the design of bridges and structures.

Some special provisions not included in the AASHO Specifications have been adopted for bridges on the Interstate System. These will be discussed in "Section 13 - Bridges on the Interstate System."

Section 1 - General Features of Design

The provisions of Division 3, Section 1 are general in nature and they require no special explanation. The roadway clearances, both horizontal and vertical, are absolute minimum values. They must not be encroached upon. Advantage should be taken of opportunities to increase the clearances whenever this can be done at little or no additional expense.

Section 2 - Loads

The loads to be carried by bridges are treated in Section 2. The dead load consists of the weight of the structure itself. In the final designs, the dead load stresses should be determined as accurately as possible. The preliminary estimates of dead loads upon which preliminary designs were based are usually not sufficiently accurate until they have been revised in the light of the final designs.

Two systems of live loads are provided in Articles 3.2.5 and 3.2.7, the H loadings and the H-S loadings. Both systems use a standard truck or an equivalent lane load, whichever produces the greater stress. The H loadings were incorporated in the original 1925 Edition. The standard truck is a two-axle vehicle as shown in Figure 4 of the specifications. When combinations of tractors and semitrailers came into general use on the highways, studies were made which led to the adoption of the H-S loadings in the 1941 Edition. The standard H-S truck is of the tractor-semitrailer type shown in Figure 6 of the specifications. The H and the H-S loadings use the same lane

loading, shown in Figure 5 of the specifications. While there is a need for both systems in bridge design, the H-S loadings make better provision for combination vehicles which are so prevalent on the highways at the present time.

There appear to be only a few points in regard to the live loads which need to be emphasized. Both the standard trucks and the lane loading are assumed to occupy a width of 10.0 feet. The actual number of traffic lanes on a structure is dependent upon the roadway width, as specified in Article 3.2.6. Thus, a bridge roadway of 52 feet will provide four traffic lanes, each with a width of 13 feet. Now the standard trucks and the lane loadings may be placed in any position within their respective traffic lanes such that they will produce the greatest stress. Attention is also called to the additional provision for negative moments in continuous spans made in Article 3.2.8. In this case, the lane loading is to be provided with two equal concentrated loads, placed so as to produce the greatest negative moments. The Overload Provision of Article 3.2.4 must also not be overlooked. It is applicable only to structures designed for the lighter live loads. By placing a standard truck of twice the prescribed weight in any traffic lane and allowing a 50 percent overstress, provision is made in stringers, floorbeams and hangers for the safe passage of heavy loads which otherwise might cause damage.

In bridges having more than two traffic lanes, it is improbable that all the lanes will be fully loaded simultaneously. A reduction in load intensity for this condition is provided in Article 3.2.9. Care must be used in the application of this reduction factor. For example, in a four lane structure it frequently happens that maximum stresses in a member occur with only three lanes loaded at 90 percent of full load intensity rather than four lanes at 75 percent.

Provision is made in Article 3.2.11 for loads on sidewalks, curbs, safety curbs and railings. Bridge railings, and particularly their connections, should be able to withstand impact from collisions with vehicles. The loads specified should therefore be regarded as minimum values. It is highly desirable that railings offer as little obstruction to the view from the roadway as possible.

Impact is to be added to live load stresses as provided in Article 3.2.12. The impact formula used by the AASHO is a modification of the one developed by C. C. Schneider for railroad loading in his bridge specifications for the Pencoyd Iron Company in 1887. Schneider's formula is $I = 300/(L+300)$. For short loaded lengths, the impact effect added to the live load stress by Schneider's formula approaches 100 percent. However, through tests of structures and through research, it has been found that the allowance for impact may be considerably reduced with safety. The current AREA Specifications reflect these reductions. The formula used in the AASHO Specifications appeared originally in the "Conference Specifications for Steel Highway Bridges," dated 1929, compiled by a joint committee composed of representatives of the AASHO and AREA. At the present time, the Bureau of Public Roads is cooperating with the Division of Highways of the State of Illinois in sponsoring an investigation of impact in highway bridges at the University of Illinois. Furthermore, several structures on the AASHO Test Road in Illinois are to be tested under the action of heavy trucks traveling at various speeds. The information gained from these sources will be useful in judging the adequacy of the AASHO specification for impact.

Longitudinal forces are generated in superstructures by the acceleration and deceleration (braking) of the live load. These forces may seriously affect the foundation pressures of high piers and of other substructure units. Article 3.2.13 prescribes the amount of the force and specifies that it shall be assumed to act 4 feet above the bridge floor. Although longitudinal forces are assumed to be transferred to the substructure units mainly through the fixed bearings, a small amount is assumed to be transferred through expansion bearings by virtue of sliding friction.

The entire wind load specification of Article 3.2.14 will be replaced in the 1957 Edition by an entirely new specification. It is printed in full in the Appendix. During 1951 and 1952, wind tunnel tests were conducted at the David Taylor Model Basin on several types of bridge models. The results were published in Highway Research Board Special Report No. 10, "Investigation of Wind Forces on Highway Bridges," dated 1953. The new specification is based on these tests.

Thermal forces are provided for in Article 3.2.15. The coefficient of linear expansion of steel is 0.0000065 per degree Fahrenheit and for concrete, it is 0.000006. In structures which are free to expand and contract, provision must be made for changes in length over the prescribed temperature range. But in structures such as suspension bridges, arches and rigid frames, changes in span length are prevented. Temperature changes in these structures produce changes in shape which induce temperature stresses. These must be thoroughly investigated.

Articles 3.2.16, 3.2.17 and 3.2.18 deal with forces of stream current, ice, drift, buoyancy and earth pressure. The provisions are definite and need no explanation. But in Article 3.2.1, centrifugal forces and earthquake stresses are referred to without specifying how they are to be evaluated.

Centrifugal forces are generated by live loads on structures on curves. These forces may be assumed to act at the roadway surfaces. The live loads are considered as solid bodies, neglecting the action of springs. Then the centrifugal force is:

$$F = \frac{W\,v^2}{g\ r}$$

where F = the centrifugal force, in pounds
W = the weight of the live load on the structure, in pounds
v = the safe speed, in feet per second
g = the acceleration due to gravity, in feet per second per second
r = the radius of curvature, in feet.

It is convenient to recall that a speed of 60 miles per hour is equal to one of 88 feet per second.

The provisions for earthquake forces in regions subject to seismic disturbances are dependent upon several factors. Among these are the geology of the region, the character of the foundations and foundation materials and the type of structure to be built. The flexibility of the superstructure in either a longitudinal or transverse direction is of considerable importance.

Before proceeding with any calculations of earthquake stresses, it is necessary to assume an hypothetical earthquake. The earthquake forces are assumed to be those resulting from a ground motion with an assumed horizontal acceleration, an assumed amplitude and an assumed vibration period. If the horizontal motion of the ground is assumed, for the sake of simplicity, to

be a simple harmonic one, the relationship between acceleration, amplitude and period may be expressed as follows:

$$a = \frac{4\pi^2 r}{T^2},$$

where a = the acceleration in feet per second per second
r = the amplitude in feet
T = the period of vibration in seconds

If the assumed amplitude is 0.10 feet and the assumed period of vibration is 1.5 seconds,

$$a = \frac{4x9.87x0.10}{2.25} = 1.75 \text{ feet per second per second}$$

Since the acceleration due to gravity "g" is 32.2 feet per second per second, the above acceleration is 1.75/32.2 or 0.054 g. In earthquake regions, structures are usually designed for a horizontal acceleration of from 0.05 g to 0.10 g, depending upon the circumstances. A study of the seismological records of a region is helpful in determining the proper allowance. The flexibility of towers reduces the effects of the acceleration. For this reason, a lower acceleration may usually be used for a suspension bridge tower in a longitudinal direction than in a transverse direction. Vertical accelerations from earthquake disturbances may usually be neglected since they cause vertical forces for which structures are designed with a factor of safety.

Section 3 - Distribution of Loads

It is interesting to note that wheel load distribution to stringers and floorbeams was satisfactorily covered in the original 1925 Edition. This subject was later investigated at the University of Illinois.[2,3] The distribution to stringers depends upon the ratio of the stiffness of the stringer to that of the slab, and an exact determination is a complex problem. Because of this, empirical methods are specified in Article 3.3.1 (b). There will be slight revisions in the Seventh Edition. Considering only concrete slabs supported on steel I-Beams, the following fractional parts of a wheel load are applicable to interior stringers:

One traffic lane loaded, S/7.0 for values of S not over 10 feet

Two or more traffic lanes loaded, S/5.5 for values of S not over 14 feet

The load carried on outside stringers is taken as the reaction of the wheel loads assuming the flooring between stringers to act as a simple beam, but for floors supported on four or more stringers, the fraction of the wheel load is not to be less than:

S/5.5 where S is 6 feet or less

S/(4.0+2.5S) where S is more than 6 feet and less than 14 feet

The distribution of loads and design of concrete slabs is covered in Article 3.3.2. The basic investigation of load distribution was made by

Westergaard.(4) Simplified formulas based on the Westergaard theory were developed by Erps, Googins and Parker.(5) The behavior of concrete slabs supported on steel beams was thoroughly investigated at the University of Illinois.(6) Article 3.3.2 covers slabs with main reinforcement perpendicular to traffic and slabs with main reinforcement parallel to traffic. Provision is made for a single axle load and for dual axles spaced 4.0 feet apart on centers. For the purposes of Article 3.3.2, legal axle loads are considered for the design of slabs. The single axle is of 24,000 lb. and each of the dual axles is of 16,000 lb. A wheel load is equal to one-half axle load in all cases.

Section 4 - Unit Stresses, Pile Loads and Bearing Power of Soils

In Article 3.4.1, nine groups are specified, which represent combinations of loads and forces to which structures may be subjected. In the forthcoming Seventh Edition of the Specifications, groups II and III will be revised as follows:

	Percent of Unit Stress
Group II = D + E + B + SF + W	125
Group III = Group I + LF + 30% + WL	125

WL is the wind load on the live load and is to be taken equal to 100 lb. per linear foot.

The revision will also provide that no increase in allowable unit stresses will be permitted for members or connections which carry wind loads only.

In 1934, the ASTM approved some important modifications in its Standard Specifications for Steel for Bridges (ASTM A7-34). Among other revisions, a minimum yield point of 33,000 psi. was guaranteed, with a tensile strength of 60,000 to 72,000 psi. The steel for rivets was covered by ASTM A141-33. In February 1934, Committee 15 of AREA submitted a complete new "Specifications for Steel Railway Bridges" to the Association, based on a unit stress of 18,000 psi. in tension.(7) This specification was adopted in 1935. The AASHO, in its Second (1935) Edition also adopted A7-34 steel, with a basic unit working stress of 18,000 psi. in tension. Very few changes in working stresses have been made since 1935. The most important change is that field rivets driven by pneumatically or electrically operated hammers are now regarded as the equivalent of shop rivets. Unit stresses for A7 structural steel are covered in Article 3.4.2. Since the safety factor is usually based on the minimum yield point, the basic safety factor is 33,000/18,000 or 1.83 for A7 steel.

Unit stresses for A242 low-alloy steels are specified in Article 3.4.7. Here again, the unit stresses are based on the guaranteed minimum yield points, using a safety factor of 1.83. Since rivets of A141 steel are usually used in structures built of low-alloy steel, the shearing and bearing values of these rivets must be taken from Article 3.4.2. In the Seventh Edition, two items will be added to the table of unit stresses. These are as follows:

Yield Point, Minimum	50,000	45,000	40,000
Stress in extreme fiber of pins	40,000	36,000	32,000
Shear in pins	20,000	18,000	16,000

Article 3.4.8 covers the unit stresses in structural nickel and structural silicon steel.

Unit stresses in conventional reinforced concrete and in steel reinforcement bars are covered in Articles 3.4.11 and 3.4.12. The basic unit stress in compression is 40 percent of the ultimate strength of concrete as determined by tests of 6-inch by 12-inch cylinders at 28 days. In the Seventh Edition, the coefficient of shrinkage of concrete will be changed to 0.0002. The column headed "Values if $f'_c = 3000$" is to be deleted. Under "Shear," certain maximum limits are being placed, as follows:

Following $0.02f'_c$, add "maximum 75 psi."
Following $0.03f'_c$, add "maximum 90 psi."
For beams with web reinforcement, the total external shear is $V = 0.075f'_c bjd$

Horizontal shear in shear keys between slab and stem of T-beams and box girders is to be increased from $0.10f'_c$ to $0.15f'_c$

The entire Subarticle 3.4.11 (g) referring to concrete columns is to be deleted. Concrete columns will be covered in Article 3.7.10, which has been entirely rewritten.

At the present time, there are no complete specifications covering the subject of prestressed concrete design and construction. To meet the need for a guide until such time as an American standard code has been prepared and adopted, the Bureau of Public Roads has prepared its "Criteria for Prestressed Concrete Bridges."(8)

Under Article 3.4.17 - Bearing Value of Piling, a sentence under Case A (2), top of page 199, is to be deleted. It reads as follows:

"A pile shall be considered fully supported laterally except that portion which is or may be, as a result of scour, in air, water, muck, peat, thin mud or other very plastic or fluid material."

Section 5 - Substructures and Retaining Walls

The provisions of this section need no explanation. However, some important revisions will be made in the Seventh Edition and these are printed in full in the Appendix. The revised articles are as follows:

Article 3.5.1 (h) - Concrete Piles (Cast in Place)
Article 3.4.2 (d) - Spread Footings
Article 3.5.2 (e) - Internal Stresses in Spread Footings

Section 6 - Structural Steel Design

The provisions of Articles 3.6.1 to 3.6.4 inclusive are general, and they require very little explanation. Article 3.6.1 appeared originally in the "Conference Specifications" of 1929, while the other three were incorporated in the original 1925 Edition of the AASHO Specifications. The use of three through trusses in one span has several objectional features. Such structures are not pleasing in appearance and usually are uneconomical.

Article 3.6.5 relating to reversals of stress is ambiguous. The first sentence of the last paragraph should be deleted. The paragraph would then read:

"The dead load stresses considered as effective in counteracting the live load stresses shall be those produced by the lightest dead load of the structure as completed or with future alterations."

Perhaps an illustration will be useful in clarifying the meaning of the article. In a certain member of a truss, the dead load stress is 150.0 kips tension. Suppose that during the passage of the live load, the live load plus impact stress varies from 250.0 kips tension to 200.0 kips compression. Then:

D. L.	+150.0	
L. L. + I.	+250.0	-200.0
	+400.0	- 50.0
50 percent of smaller stress	25.0	- 25.0
	+425.0	- 75.0

The member itself must therefore be designed for a tensile stress of 425.0 kips, with the additional provision that it be sufficiently rigid to carry 75.0 kips compression. Either the slenderness ratio must be limited to 120 or a counter must be provided. The connections must be designed for 400.0 + 50.0 = 450.0 kips.

The permissible average unit stresses in steel columns which are subjected to combined compression and bending may be determined by Article 3.6.6 and Appendix B of the specifications. Appendix B is based on the work of D. H. Young.(9) The formulas in this appendix are general, in that the end eccentricities which cause the bending may be equal or unequal. They may be on the same side or on opposite sides of the column axis. In short columns, the maximum stress may occur at the end where the eccentricity is the larger. For this condition, formula (B) applies. This is the usual formula for combined compression and bending, and it is very simple to use. However, in longer columns, the maximum stress occurs at some intermediate section and for this condition, the general formula (A) applies. The first step, therefore, is to test the value of L/r as indicated near the top of page 289. If L/r is equal to or less than the prescribed value, the maximum stress occurs at the end of the column and formula (B) is to be used. But if L/r is greater than this value, formula (A) is applicable.

In using Appendix B, it may be helpful to recall that "$\cos^{-1}$ alpha" means "the angle whose cosine is alpha." The angle "alpha" is expressed in radians, one radian being equal to $180/\pi$ or 57.296 degrees. Also, the quantity with the exponent 1/2 may be written as the square root of the quantity.

When the end eccentricities are equal and on the same side of the column axis, formula (A) reduces to the better known secant formula. Sometimes columns are required to sustain transverse loads in addition to their direct loads. Formula (C) has been added to provide for this case. The end eccentricities being taken equal and of the same sign, formula (C) is expressed as a secant formula. In the numerator, the bending stress from the transverse loads is subtracted from the allowable extreme fiber stress. This, of course, is only approximately true.

In the text of Appendix B, "L" is taken to be the "free length" of a column, as defined. However, in the graphs on pages 290 to 293 inclusive, "l" denotes the total length of the column. This was done to facilitate the use of the graphs. The horizontal portions of the curves result from the use of formula (B), while the curved portions are derived from formula (A).

Secondary stresses are treated in Article 3.6.7. The truss distortions caused by secondary stresses lie in the plane of the truss. The reference to the "width measured parallel to the plane of distortion" is therefore the depth of the member, measured in the plane of the truss. This has frequently caused misunderstanding.

In their investigation of shearing stresses in rolled beams, Lyse and Godfrey(10) observed some cases of end twisting at stresses below the shearing yield point. Similar beams were stronger when their ends were stiffened. On the basis of these tests, they recommended the use of end stiffeners whenever shearing stresses in rolled beams exceed 75 percent of the allowable. This recommendation has been incorporated in Article 3.6.8.

The wisdom of the provision in Article 3.6.9 that the area of any portion of a compression member may be neglected in determining the radius of gyration and the effective area is questionable. At least, the determination of the properties of a section in this manner is unrealistic. The provision has been in force since the 1941 Edition.

Both Articles 3.6.10 - Deflection and 3.6.11 - Depth Ratios aim to reduce vibration and impact effects in structures. Article 3.6.10 is particularly applicable to bridges designed for the heavier live loads, while Article 3.6.11 insures that even the lighter structures will have sufficient rigidity to enable them to carry occasional overloads without damage. The deflection provision of Article 3.6.10 appeared first in the 1941 Edition. The depth ratio provision was incorporated in the "Conference Specifications" of 1929.

Throughout Section 6, Structural Steel Design, frequent use is made of the theory of elastic stability to determine the permissible minimum ratios of thickness to width or depth of plates and angles subjected to compression, to shear and to combinations of compression and shear. In using this theory, the writings of Timoshenko,(11) of Moisseiff and Lienhard,(12) and of Bleich(13) were most helpful.

Since the modulus of elasticity of all grades of structural steel is 29,000,000 psi., the rigidities of plates and angles made of the stronger alloy steels must be greater than for those made of A-7 carbon steel. The alloy steel plates must be able to withstand considerably greater stresses.

In Article 3.6.15, minimum permissible ratios of thickness to unsupported width are given for web plates and cover plates of compression members. These plates are assumed to carry uniform compressive stress. The ratios provided are such that these plates will be stable against local buckling up to the guaranteed minimum yield point of the material. The values given for the cover plates meet this requirement in every respect. However, since most of the stresses in compression members usually must pass through the web plates into the gusset plates, the allowable proportions for the web plates are somewhat more conservative than those for the cover plates.

The web plates of solid rib arches are subjected to combinations of direct compression and bending. Shearing stresses are very small. An analysis of several solid rib arches of constant depth indicated that at sections of maximum stress, the intensity of direct stress rarely exceeded 40 percent of the total stress. For the purpose of Article 3.6.16, it was therefore assumed that the direct stress intensity may approach 50 percent of the total stress at the toe of the flange. It was also assumed that the stress at the toe of the flange was 90 percent of that on the extreme fiber. The web plate was therefore developed to 90 percent of the guaranteed minimum yield point. The

ratios of web plate thickness to depth as given in Article 3.6.16 were determined in this manner. They are intended to apply to ribs of constant or of nearly constant depth.

Article 3.6.17 covers the requirements as to thickness of the outstanding legs of angles in compression. These angles were assumed to be stressed to the guaranteed minimum yield point without the occurrence of local buckling.

Important revisions are being made in Articles 3.6.20 - Pitch of Rivets, 3.6.22 - Maximum Pitch, and 3.6.23 - Stitch Rivets. These revisions appear in the Appendix.

In Article 3.6.26 - Rivets in Tension, the AASHO has adopted the recommendation of Higgins and Munse(14) in regard to the value of rivets in tension. The article will specify that the allowable unit stress for rivets in combined shear and tension shall not exceed the value obtained from the formula:

$$s^2+(3/4\ t)^2 = 13{,}500^2,$$

where "s" and "t" are the rivet unit stresses in shear and in tension, respectively.

Lacing bars and perforated cover plates of compression members are treated in Article 3.6.36. Hardesty(15) made a thorough study of shearing stresses in columns and the formulas presented in this article were developed by him. Stang and Greenspan(16) tested the behavior of plates with perforated holes and investigated their stresses. The perforated plate specification is based on these tests.

The method of computing the net sections of riveted tension members as specified in Article 3.6.38 was developed by Cochrane.(17) It was incorporated in the original 1925 Edition in slightly modified form, and has been used continuously since that time.

The revised Article 3.6.43 will provide for the use of high-tensile-strength bolts. The specifications for the bolts, Article 2.10.18-A - Connections Using High-Tensile-Strength Bolts will be included in the Appendix.

Article 3.6.74 deals with the flange sections of plate girders. In regard to partial length cover plates, it is specified that these are to extend one foot beyond the theoretical ends. This frequently causes abrupt changes in flange stress and serious overstresses in rivets at these ends. Although not covered in the specifications, it is recommended that these partial length cover plates be stopped at sections where the stress is only 75 percent of that at the theoretical ends and that a sufficient number of rivets be placed at these ends to develop the initial stresses in the cover plates.

The thicknesses of web plates of plate girders are specified in Article 3.6.75. The permissible ratios of thickness to depth are such that the web plates will not buckle locally until the flange stresses have reached the yield point.

Web plates reinforced only with transverse stiffeners are covered in the first paragraph of Article 3.6.75. The ratios of thickness to depth meet the requirements of the bending stresses such as occur in simply supported spans. These minimum ratios should not be used over the supports of continuous structures, where combinations of maximum bending stress and maximum shearing stress occur simultaneously. In such cases, the minimum ratios should be increased 25 percent or a longitudinal stiffener should be used between the points of inflection, as explained in the second paragraph of the article.

The AASHO Specifications permit the use of plate girder webs reinforced with transverse stiffeners and one longitudinal stiffener properly located with respect to the compression flange. Web plates reinforced in this manner are most frequently used near the intermediate supports of continuous structures, where severe combinations of shearing stress and bending stress occur simultaneously. The ratios of thickness to depth given in the second paragraph of Article 3.6.75 provide for such combinations. If bending stresses only were to be taken care of, as in simple spans, the limiting ratios would theoretically be 1/340 D for carbon steel, 1/290 D for silicon steel, 1/280 D for low-alloy steel and 1/260 D for nickel steel.

The longitudinal stiffener divides the web plate into two parts. The problem is to locate the stiffener so that both parts have equal stability against buckling. This location is found by trial. For riveted girders whose flanges are fully stressed up to the guaranteed minimum yield point, the proper location is found to be 1/5 D from the toe of the compression flange, as specified in Article 3.6.81. The design of the longitudinal stiffener is also covered in this same article. The treatment given by Moisseiff and Lienhard(12) is the most adaptable for this purpose. The formula given in Article 3.6.81 is a simplification of Moisseiff's Equation (19). It will be noticed that Moisseiff's Equation (19) must be solved by trial and error, since it involves the ratio of the area of the longitudinal stiffener to the area of the web plate. For the case of one longitudinal stiffener, this ratio varies from about 0.02 to about 0.04, depending upon the efficiency of the cross section of the stiffener. Since it is on the safe side to estimate the value of the ratio liberally, a value of 0.05 was selected. Making this substitution in Moisseiff's Equation (19),

$$I_e = Dt^3(2.4 \frac{d^2}{D^2} - 0.13 - 0.13 \frac{d^4}{D^4})$$

The value of the term involving the fourth powers of d and D is very small and it is on the safe side to drop it. The formula of Article 3.6.81 then becomes:

$$I_e = Dt^3(2.4 \frac{d^2}{D^2} - 0.13)$$

The sole purpose of the longitudinal stiffener is to stiffen the web plate. It is proper, therefore, to place it on one side of the web plate. Note that I_e is the minimum moment of inertia about the edge in contact with the web plate. The formula yields results slightly more conservative than those from Moisseiff's Equation (19).

Transverse intermediate stiffeners for plate girder webs are covered by Article 3.6.80. The formula for the spacing of stiffeners is of the type developed by Moisseiff and Lienhard(12) in their Equation (35a). For the AASHO Specifications, however, the factor of safety was increased from 1.5 to 1.83 and account was also taken of the fact that in plate girder webs, the actual shearing stresses always exceed the average shearing stress. Usually the excess varies from 10 to 20 percent. Provision has therefore been made for an excess of 20 percent over the average. When these changes are introduced into Moisseiff's Equation (33), the formula reduces to that given in Article 3.6.80 of the AASHO Specifications.

The additional requirements that the spacing of the transverse stiffeners not exceed 6 feet, nor the unsupported depth of the web are of importance, especially in girders reinforced with transverse stiffeners and a longitudinal stiffener.

In addition to reinforcing the web plates to enable them to carry shearing stresses, the transverse stiffeners serve another important purpose. In the theory upon which the lateral buckling of beams is based, one of the fundamental assumptions is that the deformation of the beam when bent and twisted is such that the cross section does not change its shape. If this assumption is to be valid in the case of built-up plate girders, with slender web plates, the transverse stiffeners must be depended upon to support the flanges and to make the entire cross section act as a unit. With this in mind, it is believed necessary that transverse stiffeners be made in pairs and that they be fitted at both ends to the flanges.

In the last paragraph of Article 3.6.80, a formula is given which expresses the minimum moment of inertia of a transverse stiffener so that it will properly stiffen the web plate. This formula is based on early work of Timoshenko.[18] The ratio of rigidity of stiffener to web plate is "J." The formula in the specifications which expresses the minimum value of "J" is a close approximation of Timoshenko's values. Bleich[19] reports a more recent investigation, the results of which are more exact. The required values of "J" according to Bleich may be expressed approximately as follows:

$$J = 25.0\frac{D^2}{d^2} - 20.0$$

This revision should appear in the Seventh Edition of the AASHO Specifications.

The requirements of the specifications as to bracing, Articles 3.6.65 to 3.6.72 inclusive, should be met in all respects. Where diaphragms are used, they should be made as deep as the beams or girders permit. The effect of diaphragms has been investigated at the University of Illinois.[23] Shallow diaphragms were found to be ineffective, except as spacers. Deep diaphragms with rigid connections to the girders aid materially in the lateral distribution of the loads.

Welded Highway Bridge Design

Both the AASHO Specifications for Highway Bridges and the AREA Specifications for Iron and Steel Structures require that where welding is to be used, the American Welding Society Standard Specifications for Welded Highway and Railway Bridges be used as supplemental to the general specifications.

Welding design is based upon the appropriate general specifications for the application and distribution of loads and for general design rules not otherwise specified for welding.

The fundamental difference between designing for riveting and for welding is the metallurgical effect of welding upon the base metal. Notch effect caused by minute cracks and abrupt changes and discontinuities in the stress path, which gave rise to early fatigue failures in riveted members, is more pronounced and serious in welded members and connections. For welding, this difficulty is aggravated by the presence of hardening and by detrimental elements in the steel, such as carbon, sulphur, and phosphorus. Low temperatures in service, large numbers of repeated cycles of loading, particularly those of wide range of stress, and rigidity of the joints are other important factors.

For these reasons, it became necessary for the American Welding Society's Conference Committee on Welded Highway and Railway Bridges to depart from the applicable general specifications for the establishment of working unit stresses. In the 1956 Edition of the American Welding Society Specifications, Paragraphs 207 to 212 inclusive, and Table 1, unit stresses are specified under two headings. The first is for the base metal in axially stressed members adjacent to splices or connected by fillet or plug welds and the second is for the required effective weld areas in all members and parts. The unit stresses specified in the applicable general specifications, AASHO or AREA, are to be used in the determination of the required areas of main material in members connected by butt welding. The formulas of Table 1 for the required areas of members and for the required weld areas are varied according to the loading producing maximum stress, thereby recognizing both the range of stress in the stress cycle and the number of load applications. The formulas are thus grouped to represent 2,000,000 cycles of stress, 600,000 cycles and 100,000 cycles, the more conservative formulas applying to the greater number of cycles. The formulas further recognize the fact that fillet welds are inferior to butt welds in fatigue. Several examples illustrating the application of the formulas of Table 1 in the design of members and their connections are included in Appendix A of the Welding Society Specifications, pages 51 to 53 inclusive. However, with reference to examples 1 and 3, when the live load and the dead load are of opposite sign, the AASHO Specifications, Article 3.6.5, consider 100 percent of the dead load stress effective in counteracting the live load stress, as do the AREA Specifications. In example 1, both the AASHO and the AREA Specifications would require an area of 50.3 sq. in., while in example 3, both specifications would require an area of 41.7 sq. in.

The background for these formulas should not be overlooked. The first edition of the A. W. S. Bridge Specifications appeared in 1936 after two years of work by the committee. At that time tests on welds, particularly fatigue tests, were quite limited and the results were too scattering to draw final conclusions, but on the other hand, they were sufficiently consistent and significant to warrant the specifying of rules which recognized the above basic features. These early tests which gave the committee its start were those of Woehler which were performed on the rotating beam type of specimen in which there was complete reversal of stress, and more recent tests of axially loaded bars under pulsating or nonreversing type of loading. By the time of the 1947 edition of the specifications, advantage was taken of the findings of the Welding Research Council in connection with fatigue testing of welded joints. The committee accordingly revised the formulas for working unit stress to recognize the three controlling factors, as follows: The maximum unit stress, the spread between maximum and minimum, and the number of applications of such cyclical loading. When the second and third factors (spread and number of cycles) are both large for a given member of a bridge, the maximum or fatigue failure stress was recognized to be below the yield point as found in static testing and these stresses consequently control the design. When either the second or third factors (spread or number of cycles) is small, the fatigue stress is above the static yield point and need not be used to control the design.

Welding design for bridges, under the A. W. S. Bridge Specifications, presumes that all welding operators and materials and processes used shall be qualified and that thorough inspection is furnished. Ambiguous welded

connections should be avoided and as much as possible of the welding be done in the shop with preference to flat position welding and horizontal fillet welding. Rivets and welds in a single joint do not share in carrying the stress because the rivets or bolts cannot carry load until the welds have deflected or failed. Overlooking these basic principles has been responsible for welding difficulties and defects which could have been avoided.

The welding specifications are based upon the assumptions that the unit stress in shear in the base metal does not exceed 13,000 psi., that the basic unit stress in groove welds is the same as for the base metal joined and that the basic unit stresses in fillet, plug, and slot welds do not exceed 69 percent of the basic unit stress in tension for the base metal.

The welding specifications permit the use of the same basic unit stresses as prescribed for the base metal when the parts joined are over one inch thick, have the welds ground smooth and flush and receive radiographic inspection. It is believed that all important stress-carrying butt welds should be radiographed or otherwise inspected, regardless of thickness of the parts joined and whether or not the welds are ground smooth and flush with the base material. The acceptability of welds examined by radiography should be judged by the engineer, following the requirements of the A.S.M.E. "Boiler Construction Code, Section VIII, Unfired Pressure Vessels."

The stress on fillet welds is always considered as shear on the throat of the weld, regardless of the direction of applied stress. Plug and slot welds are not ascribed any value in the resistance of stresses other than shear.

Welding design presupposes the making of welds that are as free as practicable of internal stresses. For this reason, the specifications caution the designer to see that such stresses are minimized. Rigid connections should be avoided or else provision should be made for weld shrinkage. Eccentric connections should be avoided or else the stresses should be balanced about the gravity axis of the member. Butt welds should not be supplemented by fillet welded splice straps. Sharp notches or changes in lines of stress should be avoided. When butt welds are used to join material of different thicknesses, there should be a smooth transition between the offset surfaces by chamfering the end of the thicker part on a slope of not over 1 in 2-1/2 or a 5 to 12 bevel.

Welding details should be carefully worked out to avoid or minimize insofar as practicable, restraint against ductile behavior and secondary stresses due to bending. Thus an undue concentration of welding and the use of intersecting welds should be avoided and parts should be free to deflect normally without bending other parts. For noncontinuous beams, seated connections with a flexible or guiding device to prevent end twisting, are recommended.

The welding of the individual elements of built-up members designed to carry axial loads, either tension or compression, may be done in accordance with Paragraph 229 - Stitch Welds of the American Welding Society Specifications. Paragraph 229 is not applicable, however, to the welding of the elements of beams and girders, which carry transverse loads and therefore will be subjected to bending stresses. Also, the welding of eccentrically loaded columns is outside the scope of Paragraph 229. Stresses in the welds of beams, girders and eccentrically loaded columns, should be computed and the welds designed accordingly. The thickness of plates should be sufficient to prevent buckling. The tendency of plates to curve as the result of the shrinkage of the fillet welds should be recognized.

The welding of one stiffener bar to one side of a girder web only, is not a satisfactory type of construction and it is doubtful whether the single item of appearance for the outside face of the outer girders should override the difficulties involved. The buckling of the girder web in each stiffener panel due to shrinkage of the fillet welds is the first difficulty. The lack of proper stiffening of the girder flanges is considered detrimental. It is good that the welding specifications frown upon this type of construction, but compliance with its intention is largely lacking.

It is important that a weldable quality of base metal be used at all times. The new ASTM-A-373 steel would not have been promolgated had the A-7 steel been considered satisfactory from the standpoint of weldability. A certain proportion of the A-7 products will meet the A-373 requirements, but it should be remembered that the latter specification carries a minimum for manganese above the 1/2-inch thickness and a minimum for silicon above the 1 inch thickness for plates. Thus more is involved than simply a limitation on carbon content. The killed steel for plates is very desirable, especially when low hydrogen types of electrodes are used, and for service at low temperatures.

The welding specifications state specific requirements for minimum preheat and interpass temperature for manual shielded metal-arc welding for both regular and low-hydrogen type of electrodes. These requirements are shown in Table 2, Paragraph 505 - Procedures for Manual Shielded Metal-Arc Welding.

Section 7 - Concrete Design

The design of conventional reinforced concrete structures is covered in this section. The treatment is standard and no explanations are necessary. However, Article 3.7.10 - Columns has been entirely rewritten and the revised article as it will appear in the Seventh Edition is printed in the Appendix.

In general, the specification for concrete columns follows the recommendations of the 1940 Joint Committee on Standard Specifications for Concrete and Reinforced Concrete and of the ACI Building Code Requirements for Reinforced Concrete.[20] Both axially and eccentrically loaded columns are covered. The design specifications for axially loaded columns are based on an extensive series of tests sponsored by the ACI and conducted at the University of Illinois and at Lehigh University between 1930 and 1933. The design formulas derived from these tests have three significant characteristics: (1) They are based on the ultimate capacity of the column, divided by a suitable factor of safety. (2) The factor of safety is greatest for small percentages of longitudinal reinforcement and least for large percentages, in order to take account of the shrinkage and creep of concrete which act to increase the stresses in the reinforcement. (3) The ratio of modulus of elasticity of steel to that of concrete, the modular ratio n, does not appear in the design formulas.

The design specification for eccentrically loaded columns is based on the work of Richart and Olson[24] at the University of Illinois. Consider a reinforced concrete column which is symmetrical about two perpendicular planes through its axis and subject to an axial load combined with bending in one or both of the planes of symmetry. If the ratio of eccentricity to depth is no

greater than 0.5 in either plane, the combined fiber stress in compression may be computed without serious error on the basis of direct stress and bending applied to the full uncracked section of the column. The formulas presented in Article 3.7.10 for eccentrically loaded columns were developed in this manner.

Mention has already been made of the "Criteria for Prestressed Concrete Bridges,"(8) prepared by the Bureau of Public Roads. These are serving as a guide until such time as a complete American standard code has been prepared and adopted.

Section 8 - Timber Structures

The design of timber structures is covered in considerable detail in Section 8. No revisions are contemplated in the Seventh Edition.

Section 9 - Composite Beams

This section has been entirely rewritten and the new section as it will appear in the Seventh Edition is printed in the Appendix. The new section pertains only to structures composed of steel beams with concrete deck slabs joined together by shear connectors. The investigations upon which the new composite beam specification is based were conducted at the University of Illinois. In the tests made at the University of Illinois, the useful capacities of channels, welded studs and helical bars (spirals) as shear connectors were carefully studied. These are expressed by formulas in Article 3.9.5. The specifications, however, permit the use of other types of connectors which meet the requirements of Article 3.9.2.

In composite construction, the steel beams may either be shored prior to the placing of the concrete slab or they may be left unshored. In the former case, if the shoring is kept in place until the concrete has attained 75 percent of its 28-day strength, both the dead load and the live load plus impact are assumed to be carried by the composite beam. If, however, the steel beams are not shored before placing the concrete, the dead load is carried only by the steel beams. But any dead load, such as wearing surface, curbs and railings, which is placed after the concrete has attained 75 percent of its 28-day strength may be assumed to be carried by the composite section. The live load and impact are, of course, carried by the composite section. The new composite beam specification makes definite provision for all these cases.

In Article 3.9.5, the factor of safety is such that the shear connectors will not yield even when the composite beam is carrying its ultimate load. In preparing the new specification, the fact was recognized that under no circumstances should failure occur in the shear connectors themselves. The expression given for the factor of safety is general and it is applicable both to shored beams and to unshored beams. But for the most common case, in which the beam is not shored and no account is taken of any dead load carried by the composite section, the expression is considerably simplified, since $C_{mc} = 0$ and $C_v = 0$. The expression then becomes:

$$\text{F.S.} = 2.7(1.0 + C_{mi}C_s) - C_{mi}$$

The value of C_{mi} is simply the ratio of the maximum dead load moment to the maximum live load plus impact moment, while C_s is the ratio of the composite beam section modulus at the point of maximum moment to the section modulus of the steel beam at the point of maximum moment.

Embodied in the new section is a list of six references pertaining to the tests and to the development of the theory. These are not repeated in the Bibliography.

Section 10 - Structural Plate Pipe and Pipe-Arch
Section 11 - Structural Steel Arches

These sections deal with the use of structural plate for pipes, pipe-arches and for arches. An exhaustive laboratory investigation covering the testing of different types of corrugated metal sections used in these types of construction was conducted at Michigan Engineering Experiment Station.(21) Special consideration was given in the tests to such factors as the depth of corrugation, gage and radius of plate curvature on the plate performance, joint efficiency and bolt stresses.

Section 12 - Rating of Existing Bridges

This section provides for the rating of bridges for inventory purposes and for operating purposes. The inventory rating takes account of the deterioration of the material, the loss of section and the physical condition as shown by field examination. The design unit stresses as given in Section 4 are to be applied to the reduced sections and the inventory rating is the rating of the weakest member or connection in the bridge.

The operating rating is the safe load-carrying capacity of the structure. In determining the operating rating, the unit stresses are increased 50 percent, taking account of the deterioration and loss of section.

Article 3.12.11 deals with the subject of allowable stresses in the compression flanges of beams and girders subject to bending. The maximum limits of L/b are specified in Articles 3.4.2, 3.4.7 and 3.4.8. The requirements of the AASHO as to bracing are such that these limiting values should never be exceeded in structures designed under its Specifications. But in the event that a case is encountered in which the ratio of L/b is greater than the permissible value, an analysis should be made by the exact theory of lateral buckling of beams and girders to determine the critical compressive stress at which primary buckling would occur. For the inventory rating, a safety factor of 1.83 should then be applied. For the operating rating, a safety factor of 1.83/1.50 or 1.22 would be proper. The formula of Article 3.12.11 yields unsafe results for large values of L/b and it should not be used.

Section 13 - Bridges on the Interstate System

The axle loads and the arrangement of the axles of the standard H20-S16 truck are such that the standard truck does not make adequate provision for short members and for short loaded lengths when considering the effect of overloads. As a result, the stringers, floorbeams and hangers of standard H20-S16 bridges are frequently the most highly overstressed members under

the passage of a heavy vehicle. In order to overcome this deficiency in bridges on the Interstate System, the H20-S16 loading has been modified by the addition of an alternate loading consisting of two 24,000-lb. tandem axles spaced 4.0 feet apart on centers. This loading is shown in diagram on Figure 2. The alternate loading is to be applied in each traffic lane whenever it produces moments or shears greater than those produced by the standard H20-S16 truck. The standard truck and the lane loading are unchanged.

The H20-S16 loading, as modified, is to be used in the design of all main carrying members, whether of steel, concrete or timber. This includes slabs or floors of other types which are supported or reinforced longitudinally, parallel to the direction of traffic. On the other hand, slabs and other floors which are supported or reinforced in a transverse direction distribute loads to the main carrying members and as such they do not come within this category.

The use of the loading for bridges on the Interstate System involves a few changes in the AASHO Standard Specifications. The necessary changes in Articles 3.2.5 and 3.2.7 should be self-evident. In particular, the note referring to Figure 6, at the bottom of page 162 and pertaining to axle loads, is to apply only to slabs which are reinforced or supported transversely, since they are not main carrying members.

An important change occurs in cases B and C of Article 3.3.2. Under Case A, there is to be no change, since the transversely reinforced slabs are not considered to be main carrying members. The single 24,000-lb. axle or the two 16,000-lb. axles are applicable in this case. However, under Cases B and C, the slabs are reinforced longitudinally in a direction parallel to traffic and therefore they must be considered as main carrying members. They may be simply supported or continuous. The modified H20-S16 loading is applicable to Cases B and C, and either the alternate loading, the standard truck loading or the lane loading should be used, whichever produces the greatest moment or shear. The amounts of lateral distribution per wheel which are specified in Cases B and C are satisfactory. A wheel load is to be taken equal to one-half axle load and the distance between the wheels per axle, both for the alternate loading and the truck loading is 6.0 feet. When the alternate loading or the truck loading is applicable, the slabs must be designed for a single 32,000-lb. axle or for two 24,000-lb. tandem axles spaced 4.0 feet apart on centers. For spans of 11.0 feet and less, the single axle governs, and for spans greater than 11.0 feet, the tandem axles govern. In the AASHO Specifications, Article 3.3.2, Cases B and C, the "Formulas for Moment" are expressed in terms of a single wheel load. It is proper to use a single wheel load "P" of 16,000 lbs. However, when the span length "S" exceeds 11.0 feet, the values of the moment "M" must be increased 1.8(S-11.0) percent to yield moment values equal to those produced by the tandem axles. The same correction applies to "Edge Beams - Longitudinal" on page 171, and to "Cantilever Slabs" which are reinforced parallel to traffic.

In Article 3.3.3 - Distribution of Wheel Loads Through Earth Fills, the concentrated loads should be taken to be those from the modified H20-S16 loading.

The roadway width of all bridges on the Interstate System having a length of 150 feet or less between abutments or end supporting piers must equal the

full roadway width of the approaches including the usable width of shoulders. Other provisions of the AASHO regarding clearances and roadway widths will be found in "A Policy on Design Standards."(22)

BIBLIOGRAPHY

1. "Motor Truck Loading on Highway Bridges" by Manville and Gastmeyer, Engineering News, Vol. 72, No. 10, September 3, 1914, page 492.
2. "Moments in I-Beam Bridges" by Newmark and Siess, University of Illinois Bulletin No. 336, June 1942.
3. "Design of I-Beam Bridges" by Newmark, Transactions, ASCE, Vol. 114 (1949), page 997.
4. "Computation of Stresses in Bridge Slabs Due to Wheel Loads" by H. M. Westergaard, Public Roads, March 1930.
5. "Distribution of Wheel Loads and Design of Reinforced Concrete Bridge Floor Slabs" by H. R. Erps, A. L. Googins and J. L. Parker, Public Roads, October 1937.
6. University of Illinois Bulletin No. 303 by Vernon P. Jensen, Bulletin No. 304 by N. M. Newmark, Bulletin No. 315 by Vernon P. Jensen, Bulletin No. 346 by V. P. Jensen, R. W. Kluge and C. B. Williams, Jr., Bulletin No. 363 by N. M. Newmark, C. P. Siess and R. R. Penman, Bulletin No. 369 by V. P. Jensen and John W. Allen, Bulletin No. 375 by N. M. Newmark, C. P. Siess and W. M. Peckham, Bulletin No. 396 by C. P. Siess, I. M. Viest and N. M. Newmark, Bulletin No. 405 by I. M. Viest, C. P. Siess, J. H. Appleton and N. M. Newmark.
7. Bulletin, AREA, Vol. 35, No. 364, February 1934, page 1021.
8. "Criteria for Prestressed Concrete Bridges," Superintendent of Documents, U. S. Government Printing Office, Washington 25, D. C. (15¢)
9. "Rational Design of Steel Columns" by D. H. Young, Transactions, ASCE, Vol. 101 (1936), page 422.
10. "Investigation of Web Buckling in Steel Beams" by Lyse and Godfrey, Transactions, ASCE, Vol. 100 (1935, page 675. In Authors' Closure, page 701.
11. "Theory of Elastic Stability" by Timoshenko, McGraw-Hill Book Co., First Edition, 1936.
12. "Theory of Elastic Stability Applied to Structural Design" by Moisseiff and Lienhard, Transactions, ASCE, Vol. 106 (1941), page 1052.
13. "Buckling Strength of Metal Structures" by Bleich, McGraw-Hill Book Co., First Edition, 1952.
14. "How Much Combined Stress Can a Rivet Carry" by Higgins and Munse, Engineering News-Record, Vol. 149, No. 23, December 4, 1952, page 40.
15. "Shears on Column Lacing" by Shortridge Hardesty, Bulletin, AREA, Vol. 36, No. 374, February 1935, page 724.

16. National Bureau of Standards Journal of Research, R.P. 1880, Vol. 40, 1948, by A. H. Stang and Martin Greenspan.

17. "Rules for Rivet Hole Deductions in Tension Members" by Victor H. Cochrane, Engineering News-Record, Vol. 89, No. 20, November 16, 1922, page 847. For a history and elucidation of this method see "The Net Section of Riveted Tension Members" by C. H. Chapin, Proceedings, AREA, Vol. 36, 1935, page 775.

18. "Theory of Elastic Stability" by Timoshenko, McGraw-Hill Book Co., First Edition, 1936, page 417.

19. "Buckling Strength of Metal Structures" by Bleich, McGraw-Hill Book Co., First Edition, 1952, page 409.

20. "Building Code Requirements for Reinforced Concrete" (ACI 318-51), American Concrete Institute, April 1951.

21. "Load Deflection Tests on Corrugated Metal Sections," Michigan Engineering Experiment Station, East Lansing, Bulletin 109, Summer 1951.

22. "A Policy on Design Standards," Revised September 1, 1956, American Association of State Highway Officials, Washington, D. C.

23. "Distribution of Loads to Girders in Slab and Girder Bridges," by Siess and Veletsos, Research Report 14-B, Highway Research Board, 1952.

24. "The Resistance of Reinforced Concrete Columns to Eccentric Load," by F. E. Richart and T. A. Olson, Proc. ACI, Vol. 34 (1938), page 401.

APPENDIX

Major Revisions to Appear in Seventh Edition AASHO Standard Specifications for Highway Bridges

T.11(54) Insert a new article temporarily numbered, Page 95:
Art. 2.10.18-A, Connections Using High-Tensile-Strength Bolts.

(a) General.

This specification covers the assembly of structural connections using high-tensile-strength bolts and nuts with hardened washers where the initial tension in the bolt produces friction on the contact surfaces of the connected pieces sufficient in magnitude to resist shear.

High-tensile-strength bolts may be substituted for rivets as indicated by the plans or special provisions.

Except as otherwise provided in this article, construction shall conform to applicable specifications for riveted structures.

All bolts, nuts, and washers shall conform to the requirements of Division IV with regard to materials, dimensions, and identification marking.

(b) Dimensions of Bolts, Nuts, and Washers.

Bolt lengths shall be determined by adding the values given in Table I to the total thickness of connected material. The values in Table I compensate for thickness of nut, two flat washers, and bolt point. The total length shall

be adjusted to the next 1/4-inch increment up to 5-inch length and to the next longer 1/2-inch increment for lengths over 5 inches.

TABLE I—BOLT LENGTHS

Bolt Size (In.)	Add to Grip (In.)
1/2	1
5/8	1-1/8
3/4	1-1/4
7/8	1-1/2
1	1-5/8
1-1/8	1-3/4
1-1/4	1-7/8

If other than the standard thickness of circular washer as given in Table II is used, the necessary bolt length shall be adjusted accordingly. Where beveled washers of the dimensions given in Table II are used, an additional 1/8 inch shall be added for each such beveled washer.

TABLE II—WASHER DIMENSIONS

	Circular Washers			Square Beveled Washers For American Standard Beams and Channels		
Bolt Size (In.)	Inside Diameter (In.)	Outside Diameter	Thickness Gage No.	Width	Mean Thickness	Slope
1/2	9/16	1-3/8	12	1-3/4	5/16	1:6
5/8	11/16	1-3/4	10	1-3/4	5/16	1:6
3/4	13/16	2	9	1-3/4	5/16	1:6
7/8	15/16	2-1/4	8	1-3/4	5/16	1:6
1	1- 1/16	2-1/2	8	1-3/4	5/16	1:6
1-1/8	1- 1/4	2-3/4	8	2-1/4	5/16	1:6
1-1/4	1- 3/8	3	8	2-1/4	5/16	1:6

Circular washers shall be flat and smooth and their dimensions shall be not less than would conform to the current requirements of the American Standards Association (ASA Designation: B27.2) as given in Table II. Where clearance is necessary, washers may be clipped on one side to a point not closer than 7/8 of the bolt diameter from the center of the washer. Where bearing faces of the bolted parts have a slope of more than 1:20 with respect to a plane normal to the bolt axis, smooth beveled washers shall be used to compensate for lack of parallelism.

Nut dimensions shall conform to the current requirements for Heavy Semifinished Hexagon Nuts of the American Standards Association (ASA Designation: B18.2).

(c) Bolted Parts and Assembly.

Holes may be punched, subpunched and reamed or drilled, as required by the applicable specification for riveted construction, and shall be of a diameter not more than 1/16 inch in excess of the nominal bolt diameter.

Bolted parts shall fit solidly together when assembled. Contact surfaces, including those adjacent to the washers, shall be descaled or carry the normal tight mill scale. Contact surfaces shall be free of dirt, oil, loose scale, burrs, pits and other defects that would prevent solid seating of the parts.

Contact surfaces of joints shall be free of paint or lacquer unless otherwise indicated by the plans or special provisions.

Connections shall be assembled with a hardened washer under the bolt head and nut. Surfaces of bolted parts in contact with the bolt head and nut shall be parallel, except flat washers may be used on surfaces having a slope not greater than 1:20 with respect to a plane normal to the bolt axis, provided the nut is torqued against a non-sloping surface. For slopes greater than 1:20, smooth beveled washers shall be used to produce parallelism.

All nuts shall be tightened to give at least the required minimum bolt tension values given in Table III upon completion of the joint.

TABLE III—BOLT TENSION AND TORQUE VALUES

Bolt Size (In.)	Recommended* Bolt Tension For Calibrating Wrenches (Lb.)	Required** Minimum Bolt Tension (Lb.)	Approximate*** Equivalent Torque For Required Minimum Bolt Tension (Lb. Ft.)
1/2	12,500	10,850	90
5/8	20,000	17,250	180
3/4	29,000	25,600	320
7/8	37,000	32,400	470
1	49,000	42,500	710
1-1/8	58,000	50,800	960
1-1/4	74,000	64,500	1,350

*Approximately 15 percent in excess of the Required Minimum Bolt Tension.

**Equal to 90 percent of the minimum Proof Load of Bolt (ASTM A325). There is no recommended maximum bolt tension.

***Equal to 0.0167 lb. ft. per in. bolt diameter per lb. tension for non-lubricated bolts and nuts. Values given are experimental approximations. If torque rather than tension is to be measured, the torque-tension ratio shall be determined by the actual conditions of the application.

Note 1 - Wrenches should be set to induce a bolt tension in excess of the Required Minimum Bolt Tension as given in Table III. Because of the varying relation between torque and induced tension, it is suggested that wrenches be set to induce the Recommended Bolt Tension for Calibrating Wrenches as given in Table III.

Note 2 - In using a power wrench, the recommendations of the wrench manufacturer should be consulted in its operation and care should be taken that the machine is maintained in proper working condition and proper calibration.

Note 3 - In using a manual torque wrench, the required torque can be read from the wrench dial, or in other types of wrenches, the torque may be

indicated by a release of the wrench. Care should be taken that the wrench is properly calibrated. Nuts shall be in motion when torque is measured.

Note 4 - In using manual plain wrenches, a ratchet wrench of length consistent with the man-effort available should be used so that the product of the effective wrench length in feet times the man-effort in pounds exceeds the equivalent torque required.

(d) Inspection.

The inspector shall satisfy himself that all requirements of this specification are met. The inspector shall approve the procedure for calibration of wrenches and installation of bolts. The inspector shall further observe the field installation to determine that these procedures are followed. No further inspection is normally required. Where further inspection is required by the engineer, he shall specify in advance the method the inspector is to follow.

T.15(56) Art. 2.10.18A, T.11(54) (c), Connections Using High-Tensile-Strength Bolts - Bolted Parts and Assembly, Page 95.

Insert just before "(d) Inspection" the following:

As an alternate method of insuring the required minimum tension the one-turn-of-the-nut method may be used. The nuts shall be run up on the bolts to a "finger-tight" position by means of hand turning only (no spud wrench). From this commencing position, the nut shall be given one turn with the wrench. The turning can be controlled by means of a permanent mark or protrusion on the chuck of a pneumatic wrench.

In using this method it is essential to commence the tightening operation from a tightly fitted joint. Where this cannot be effected with ordinary erection bolts, the joint shall be brought to a tight fit by a preliminary tightening of a few of the high-tensile bolts which must be marked for identification and release after the other bolts have been tightened.

In order to finish the connection, the high-tensile bolts used for the preliminary tightening of the joint, if any, shall be released, set at the finger-tight position and given one turn.

When breakages occur for bolts having short grips using this method, the amount of turn may be reduced on the basis of tests with a torque wrench.

T.16(56) Art. 2.10.18, Bolts and Bolted Connections, Pg. 94. Delete entirely the present specification and substitute the following:

2.10.18—Bolts and Bolted Connections.

The specifications of this article do not pertain to the use of high-tensile-strength bolts. Bolted connections fabricated with high-tensile-strength bolts shall conform to Article 2.10.18-A.

(a) General.

Bolts shall be unfinished, turned, or an approved form of ribbed bolt. Bolted connections shall be used only as indicated by the plans or special provisions. Bolts shall have single self-locking nuts or double nuts. Beveled washers shall be used where bearing faces have a slope of more than 1:20 with respect to a plane normal to the bolt axis.

Except as otherwise provided in this article, construction shall conform to applicable specifications for riveted structures.

(b) Unfinished Bolts.

Unfinished bolts shall be standard bolts (ordinary rough or machine bolts).

(c) Turned Bolts.

Holes for turned bolts shall be carefully reamed and the bolts turned to a light driving fit with the threads entirely outside of the holes and a washer shall be used. Turned bolts shall be finished by a finishing cut. Heads and nuts shall be hexagonal.

(d) Ribbed Bolts.

Ribbed bolts shall make a driving fit with the holes. If for any reason the bolt twists before drawing tight, the hole shall be carefully reamed and an oversize bolt used as a replacement. Nuts shall be hexagonal.

T.1(56) Art. 3.2.14, Wind Loads, Page 165. Delete entire T.10(55) and substitute the following:

The following wind load forces per square foot of exposed area shall be applied to all structures (see Art. 3.4.1 for percentage of basic unit stress to be used under various combinations of loads and forces). The exposed area considered shall be the sum of the areas of all members, including floor system and railing, as seen in elevation at 90 degrees to the longitudinal axis of the structure. The forces and loads given herein are for a wind velocity of 100 miles per hour. For Group II loading, but not for Group III loading, they may be reduced or increased in the ratio of the square of the design wind velocity to the square of 100, provided the maximum probable wind velocity can be ascertained with reasonable accuracy, or there are permanent features of the terrain which make such changes safe and advisable. If change in the design wind velocity is made, the design wind velocity shall be shown on the plans.

(1) Superstructure Design.

A moving uniformly distributed wind load of the following intensity shall be applied horizontally at right angles to the longitudinal axis of the structure in the design of the superstructure:

For trusses and arches . . . . 75 pounds per square foot
For girders and beams . . . . 50 pounds per square foot

The total force shall not be less than 300 pounds per linear foot in the plane of the loaded chord and 150 pounds per linear foot in the plane of the unloaded chord on truss spans and not less than 300 pounds per linear foot on girder spans.

The above forces shall be used for Group II loading. For Group III loading there shall be added thereto a load of 100 pounds per linear foot applied at right angles to the longitudinal axis of the structure as a wind load on a moving live load. When a reinforced concrete floor slab or a steel grid deck is keyed to or attached to its supporting members, it may be assumed that the deck resists, within its plane, the shear resulting from the wind load on the moving live load.

(2) Substructure Design.

Forces transmitted to the substructure by the superstructure and forces applied directly to the substructure by wind loads shall be assumed to be as follows:

(a) Forces from superstructure.

The transverse and longitudinal forces transmitted by the superstructure to the substructure for varying angles of wind direction shall be as set forth in the following table. The skew angle is measured from the perpendicular to the longitudinal axis. The assumed wind direction shall be that which produces the maximum stress in the substructure being designed. The transverse and longitudinal forces shall be applied simultaneously at the elevation of the center of gravity of the exposed area of the superstructure.

	Trusses		Girders	
Skew Angle of Wind (Degrees)	Lateral Load Per Sq.Ft. of Area (Pounds)	Longitudinal Load Per Sq.Ft. of Area (Pounds)	Lateral Load Per Sq.Ft. of Area (Pounds)	Longitudinal Load Per Sq. Ft. of Area (Pounds)
0	75	0	50	0
15	70	12	44	6
30	65	28	41	12
45	47	41	33	16
60	25	50	17	19

The loads listed above shall be used in Group II loading as given in Art. 3.4.1.

For Group III loading, these loads may be reduced 70 percent and there shall be added thereto, as a wind on a moving live load, a load per linear foot of structure as given in the following table:

Skew Angle of Wind (Degrees)	Lateral Load Per Lin. Ft. (Pounds)	Longitudinal Load Per Lin. Ft. (Pounds)
0	100	0
15	88	12
30	82	24
45	66	32
60	34	38

This load shall be applied at a point 6 feet above the deck.

For the usual girder and slab bridges having maximum span lengths of 125 feet, the following wind loading may be used in lieu of the more precise loading specified above:

W (wind load on structure)
50 pounds per square foot, traverse;
12 pounds per square foot, longitudinal
Both forces shall be applied simultaneously.
WL (Wind load on live load)

100 pounds per linear foot, transverse;
40 pounds per linear foot, longitudinal
Both forces shall be applied simultaneously.

(b) Forces applied directly to the substructure.

The transverse and longitudinal forces to be applied directly to the substructure for a 100-mile-per-hour wind shall be calculated from an assumed wind force of 40 pounds per square foot. For wind directions assumed skewed to the substructure this force shall be resolved into components perpendicular to the end and front elevations of the substructure according to the functions of the skew angle. The component perpendicular-to-the-end elevation shall act on the exposed substructure area as seen in end elevation and the component perpendicular to the front elevation shall act on the exposed substructure area as seen in front elevation. These loads shall be assumed to act on horizontal lines at the centers of gravity of the exposed areas and shall be applied simultaneously with the wind loads from the superstructure. The above loads are for Group II loading and may be reduced 70 percent for Group III loading.

(3) Overturning Forces.

The effect of forces tending to overturn structures shall be calculated under Group II and Group III of Article 3.4.1, and there shall be added an upward force applied at the windward quarter point of the transverse superstructure width. This force shall be 20 pounds per square foot of deck and sidewalk plan area for Group II combination and 6 pounds per square foot for Group III combination. The wind direction shall be at right angles to the longitudinal axis of the structure.

T.4(54) Art. 3.5.1(h), Concrete Piles (Cast-in-Place), Page 203.

Delete third paragraph and substitute the following:

"Cast-in-place piling shall be reinforced when specified or shown on the plans. Cast-in-place foundation piling, carrying axial loads only and where the possibility of lateral forces being applied to the piles is insignificant, need not be reinforced when the soil provides adequate lateral support. Those portions of cast-in-place piling which are not supported laterally shall be designed as reinforced concrete columns in accordance with Art. 3.7.10, and the reinforcing steel shall extend ten feet below the plane where the soil provides adequate lateral restraint. Where the shell is more than 0.12 inch in thickness, it may be considered as reinforcement."

T.12(55) Art. 3.5.2(d), Spread Footings, Page 205.

Delete the last paragraph, which reads, "Except in small structures, no footing shall have a thickness at the edge of less than 2 feet. When piles are used, the footing shall have an edge thickness of not less than 18 inches above the tops of the piles."

T.13(55) Art. 3.6.20, Pitch of Rivets, Page 214.

Change the title of the article to "Spacing of Rivets" and add a new first paragraph as follows:

"The pitch of rivets is the distance along the line of principal stress, in inches, between centers of adjacent rivets, measured along one or more rivet lines. The gage of rivets is the distance in inches between adjacent lines of rivets or the distance from the back of an angle or other shape to the first line of rivets. The pitch of rivets shall be governed by the requirements for sealing of stitch, whichever is the minimum."

T.14(55) Art. 3.6.22, Maximum Pitch, Page 215.

Change title of the article to "Maximum Pitch of Sealing Rivets" and revise the entire article to read:

"For sealing, the pitch on a single line adjacent to a free edge of an outside plate or shape shall not exceed 4 inches + 4t or 7 inches. If there is a second line of rivets uniformly staggered with those in the line adjacent to the free edge, at a gage "g," less than 1-1/2 inches + 4t therefrom, the staggered pitch in two such lines, considered together, shall not exceed 4 inches + 4t - 3/4g or 7 inches - 3/4g but need not be less than one-half the requirement for a single line. t = the thickness in inches of the thinner outside plate or shape."

T.15(55) Art. 3.6.23, Stitch Rivets, Page 215.

Change the title of the article to "Maximum Pitch of Stitch Rivets" and revise the entire article to read:

"In built-up members where two or more plates or shapes are in contact, stitch rivets shall be used to insure uniform action and, in compression members, to prevent buckling. In compression members the pitch of stitch rivets on any single line in the direction of stress shall not exceed 12 t, except that, if the rivets on adjacent lines are staggered and the gage "g" between the line under consideration and the farther adjacent line (if there are more than two lines) is less than 24 t, the staggered pitch in the two lines, considered together, shall not exceed 12 t or 15 t - 3/8 g. The gage between adjacent lines of rivets shall not exceed 24 t. t = thickness, in inches, of the thinner outside plate or shape. In tension members the pitch shall not exceed twice that specified for compression members and the gage shall not exceed that specified for compression members."

T.4(56) Art. 3.5.2(e), T.21(55), Footings, Internal Stresses in Spread Footings, Page 206.

Delete entire tentative specification T.21(55) and revise the seventh paragraph of Article 3.5.2(e) to read:

The critical section for diagonal tension in footings supported on piles shall be considered as the concentric vertical section through the footing at a distance, d/2, from each face of the column, pedestal or wall, and any piles whose centers are at or outside this section shall be considered in computing the diagonal tension.

T.8(56) Art. 3.7.10, Columns, Page 242. Delete the entire article and substitute the following:

3.7.10—Columns.*

(a) General.

The provisions of Section 7, Concrete Design, shall apply in the design of columns unless specifically modified by this article.

In the design of columns the unsupported length shall be defined as the clear distance between struts, cross beams, footings or other types of adequate restraint to lateral movement. Where bracing members have haunches at their junction to columns, the unsupported column length shall be measured from the junction of the haunch with the column, provided that the face of the haunch makes an angle with the face of the column of at least 45 degrees. Struts or cross beams joining columns at angles greater than 30 degrees from the plane of symmetry of the column shall not be considered as adequate support. The least lateral dimension of a column shall be taken as: (1) for rectangular columns, the overall thickness along the principal axes; (2) for spirally reinforced columns, the overall diameter including the encasement of the spirals; (3) for "T"-shaped columns, the width or depth of the T.

In a column which, for architectural or other reasons, has a larger cross section than required by the load carried, the minimum amount of longitudinal steel hereinafter specified may be reduced provided that in no case shall less longitudinal steel be used than that required by the minimum column designed with one percent of longitudinal steel.

The notations used in this section are as follows:

A_g = overall or gross cross-sectional area of a spirally reinforced or tied pier, pedestal or column in square inches

A_c = cross-sectional area of core of spirally reinforced columns measured to the outside diameter of the spiral, square inches

A_s = cross-sectional area of longitudinal steel

C = $\dfrac{f_a}{0.40\,f'_c}$, a factor used in the design of members subjected to combined axial and bending stresses

d = least lateral dimension of column, inches

e = eccentricity of resultant load on a column, measured from a gravity axis

f_a = $\dfrac{0.225\,f'_c + f_s\,p}{1 + (n - 1)\,p}$

f'_c = crushing strength of 6" x 12" concrete test cylinders at age of 28 days, psi.

f_e = maximum allowable compressive stress in members subjected to combined axial and bending stress, psi.

f_s = nominal working stress in longitudinal reinforcing steel (see Article 3.4.12), psi.

f'_s = yield stress of spiral reinforcement (for steel grades not having a definite yield point, the stress causing a 0.2 percent plastic set), psi.

*This section, covering the design of reinforced concrete columns, follows in general the recommendations of the Joint Committee on Standard Specifications for Concrete and Reinforced Concrete and of the 1951 ACI Building Code Requirements for Reinforced Concrete.

$K \quad = \dfrac{t^2}{2\,r^2}$, a factor used in the design of members subjected to combined axial and bending stresses
$L \quad$ = unsupported length of column, inches
$n \quad$ = ratio of modulus of elasticity of steel to that of concrete
$P_e \quad$ = a load eccentrically applied
$P_p \quad$ = total load on pier or pedestal, pounds
$P_s \quad$ = total load on spirally reinforced column, pounds
$P_{sl} \quad$ = total load on spirally reinforced long column, pounds
$P_t \quad$ = total load on tied column, pounds
$P_{tl} \quad$ = total load on tied long column, pounds
$p \quad$ = ratio of longitudinal steel area to gross column area
$p' \quad$ = ratio area of spiral reinforcement to CORE area
$r \quad$ = least radius of gyration of section (transformed section)
$t \quad$ = overall depth of column in the direction of eccentricity or bending

(b) Piers and Pedestals.

The ratio of the unsupported length of unreinforced concrete piers or pedestals to their least dimension shall not exceed 3. The total load on any unreinforced concrete pier or pedestal shall not exceed that given by the following formula:

$$P_p = 0.25\,A_g f'_c \tag{1}$$

(c) Spirally Reinforced Columns.

(1) Longitudinal Reinforcement.

Longitudinal reinforcement shall be placed within the area contained by the spiral reinforcement. The ratio between the area of longitudinal reinforcement and the gross area of the column, including the encasement outside the spiral reinforcement, shall be not less than 0.01 nor more than 0.08. There shall be a minimum of six longitudinal bars evenly spaced around the periphery of the column core. The clear spacing between the individual bars or pairs of bars at lapped splices shall be not less than 1-1/2 inches or 1-1/2 times the maximum size of the coarse aggregate used, subject to the further requirement that the center-to-center spacing shall be not less than 2-1/2 times the diameter of round bars or three times the side dimension of square bars. The diameter of bars shall be not less than five-eighths inch. For columns with a circular spirally reinforced core having excessive size or other outside shapes, the gross area to be used in determining percentage of reinforcement shall be a circle with a diameter equal to the minimum core required for structural design plus the specified outside cover.

(2) Spiral Reinforcement.

Spiral reinforcement shall consist of uniform spirals held firmly in position by attachment to the longitudinal reinforcement. For columns with 18-inch-or-less core diameter, the diameter of the spiral bars shall be not less than one-fourth inch. For columns with core diameters greater than 18 inches, the diameter of the spiral bars shall be not less than three-eighths

inch. Splices in spiral bars should be avoided if practicable and, if necessary, shall be made by welding or by a lap of 1-1/2 turns. The pitch of spirals shall not exceed one-sixth the core diameter. The clear distance between individual turns of the spiral shall not exceed 3 inches nor be less than 1-3/8 inches or 1-1/2 times the maximum size of the coarse aggregate used. Spiral reinforcement shall extend from the footing or other support to the level of the lowest horizontal reinforcement of members supported by the column.

The ratio of the volume of the spiral reinforcement to the volume of core of the column, out to out of spirals, shall be not less than

$$p' = 0.45\left[\frac{A_g}{A_c} - 1\right]\frac{f'_c}{f'_s} \tag{2}$$

(3) Allowable Load - Short Columns.

The provisions of this subarticle shall apply only to columns having ratios of unsupported height to least lateral dimension of not more than 10. The total axial load on a column shall not exceed that given by the following formula:

$$P_s = 0.225\ f'_c\ A_g + A_s f_s \tag{3}$$

(4) Long Columns.

The total axial load on a column having a ratio of unsupported height to least lateral dimension greater than 10, but not greater than 20, shall be not greater than given by the following formula:

$$P_{s1} = P_s\ (1.3 - 0.03\ \frac{L}{d}) \tag{4}$$

If the L/d ratio of columns exceeds 20, the column shall be investigated for elastic stability.

(d) Tied Columns.

(1) Longitudinal Reinforcement.

The longitudinal reinforcement shall consist of at least four bars, and, when only four bars are used, they shall be placed at the corners of the section. Bars shall be placed at each intersection of column faces. The bars shall not be less than five-eighths inch in diameter. The ratio of the total cross-sectional area of the bars to the total cross-sectional area of the column shall be not less than 0.01 nor more than 0.04.

(2) Hoops and Lateral Ties.

Hoops shall surround the longitudinal reinforcement. They shall be not less than one-fourth inch in diameter and shall be spaced not more than 12 inches apart except that this spacing may be increased in the case of pier shafts or columns having a larger cross section than required by conditions of loading. Adequate auxiliary ties shall be provided to support intermediate longitudinal bars whose distance from any tied bar exceeds 2 feet.

(3) Allowable Load - Short Columns.

The provisions of this subarticle shall apply only to columns having ratios of unsupported height to least lateral dimension of not more than 10. The total axial load on a column shall be not greater than 0.8 of that given by equation 3, which results in

$$P_t = 0.8\ (0.225\ f'_c\ A_g + A_s f_s) \tag{5}$$

The total axial load on a column having a ratio of unsupported height to least lateral dimension greater than 10 but not greater than 20 shall be not greater than given by the following formula:

$$P_{t1} = P_t\ (1.3 - 0.03\ \frac{L}{d}) \tag{6}$$

If the L/d ratio exceeds 20, the column shall be investigated for elastic stability.

(e) Bending Moments in Columns.

When beams or slabs are connected to columns, the moments induced in the columns by such beams or slabs shall be provided for in the column design.

(f) Combined Axial and Bending Stress.

A reinforced concrete column which is symmetrical about two mutually perpendicular planes through its axis and which is subject to an axial load P_e combined with bending in one or both of the planes of symmetry may be designed on the basis of uncracked sections provided the ratio of eccentricity to depth, e/t, is not greater than 0.5 in either plane. The combined fiber stress in compression is given by the following formula:

$$f_c = \frac{P_e}{A_g}\left[\frac{1 + \frac{Ke}{t}}{1 + (n - 1)\ p}\right] \tag{7)*}$$

The column may be designed for an equivalent axial load P_s or P_t as given by the following formula:

$$P = P_e\ (1 + \frac{CKe}{t}) \tag{8}$$

When bending exists in both planes of symmetry, $\frac{Ke}{t}$ is the sum of the $\frac{Ke}{t}$ values in both planes.*

Reinforced concrete columns in which the eccentricity, e, is greater than 0.5 t shall be designed on the basis of cracked sections with no tension resisted by the concrete, and tensile stresses in the reinforcing steel shall be investigated. In such cases the value of the compressive reinforcement may

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*For approximate or trial design, K may be taken as 8 for a circular spiral column and as 5 for a rectangular, tied or spiral, column. The assumed value of K shall be checked by the adopted section.

be taken as twice the value given by a straight line relationship between stress and strain and the modular ratio, n, given in Article 3.4.11(a). (See appendix D for procedure in determining position and direction of neutral axis.)

The maximum allowable compressive stress in concrete, f_e, in spiral and tied columns, eccentrically loaded or otherwise, when subjected to combined and bending stress shall not exceed that given by the following formula:

$$f_e = f_a \left[\frac{t + Ke}{t + CKe} \right] \qquad (9)^*$$

where $f_a = \dfrac{0.225 f'_c + f_s p}{1 + (n - 1) p}$ for spiral columns and 0.8 that amount for tied columns. The limiting steel ratio of 0.04 provided in (d) may be increased to 0.08 for tied columns designed to withstand combined axial and bending stresses provided that the amount of steel spliced by lapping in any 3-foot length of column shall not exceed a steel ratio of 0.04. The size of the column shall be not less than that required by axial load alone.

T.14(56) Div. III, Sect. 9 - Composite Beams, Page 250. Revise entire section as follows:

3.9.1—General.

This section pertains to structures composed of steel beams with concrete deck slabs connected by shear connectors.

General specifications pertaining to the design of concrete and steel structures shall apply to structures utilizing composite beams where such specifications are applicable. Composite beams and slabs shall be designed and the stresses computed by the composite moment of inertia method and shall be consistent with the predetermined properties of the various materials used.

The modulus of elasticity of all grades of structural steel shall be taken to be 29,000,000 pounds per square inch. The ratio of the moduli of elasticity of steel to those of concrete of various design strengths shall be in accordance with the provisions of Article 3.4.11.

The effect of creep shall be considered in the design of composite beams which have dead loads acting on the composite section. In such structures, stresses and horizontal shears produced by dead loads acting on the composite section shall be computed for "n" as given in Article 3.4.11 or for this value multiplied by 3, whichever gives the higher stresses and shears.

If concrete with expansive characteristics must be used, composite design should be used with caution and provision must be made in the design to accommodate the expansion.

Composite sections should preferably be proportioned so that the neutral axis lies below the top surface of the steel beam. If concrete is on the tension side of the neutral axis, it shall not be considered in computing moments of inertia or resisting moments except for deflection calculations. Mechanical anchorages shall be provided to tie the sections together and to develop stresses on the plane joining the concrete and the steel.

The steel beams, especially if not supported by intermediate falsework, shall be investigated for stability during the time the concrete is in place and before it has hardened.

*For approximate or trial design, K may be taken as 8 for a circular spiral column and as 5 for a rectangular, tied or spiral column. The assumed value of K shall be checked by the adopted section.

3.9.2—Shear Connectors.

The mechanical means which are used at the junction of the beam and slab for the purpose of developing the shear resistance necessary to produce composite action shall conform to the specifications of the respective materials as provided in Division IV. The shear connectors shall be of types which permit a thorough compaction of the concrete in order to insure that their entire surfaces are in contact with the concrete. They shall be capable of resisting both horizontal and vertical movement between the concrete and the steel.

The shear connectors shall be riveted or welded to the beams. The capacity of the rivets or welds at permissible working stresses shall equal or exceed the resistance value "Q" of the shear connector.

The clear depth of concrete cover over the tops of the shear connectors shall be not less than one inch.

The clear distance between the edge of a beam flange and the edge of the shear connectors shall be not less than one inch.

Welded studs for shear connectors shall be of weldable steel and shall be end-welded.

3.9.3—Effective Flange Width.

In composite beam construction the assumed effective width of the slab as a T-beam flange shall not exceed the following:

(1) One-fourth of the span length of the beam.
(2) The distance center to center of beams.
(3) Twelve times the least thickness of the slab.

For beams having a flange on one side only, the effective flange width shall not exceed one-twelfth of the span length of the beam, nor six times the thickness of the slab, nor one-half the distance center to center of the next beam.

Composite beam-type construction shall not be used for isolated beams.

3.9.4—Stresses.

Maximum compressive and tensile stresses in beams which are not provided with temporary supports during the placing of the permanent dead load, shall be the sum of the stresses produced by the dead loads acting on the steel beams or girders alone and the stresses produced by the superimposed loads acting on the composite beam. When beams are provided with effective intermediate supports which are kept in place until the concrete has attained 75 percent of its required 28-day strength, and the dead and live load stresses shall be computed on the basis of the composite section.

In continuous spans, the positive moment portions may be designed with composite sections as in simple spans. The negative moment portions shall be designed on the assumption that concrete on the tension side of the neutral axis is not effective except as a device to develop the reinforcement steel embedded in it. In case reinforcement steel embedded in the concrete is not used in computing the section, shear connectors need not be provided in these portions of the spans.

3.9.5—Shear.

Resistance to horizontal shear shall be provided by mechanical shear connectors at the junction of the concrete slab and the steel beam or girder. The

shear connectors may be continuous helical- or serpentine-shaped bars adequately attached to the beam at regular or variable intervals, or other mechanical devices placed transversely across the flange of the beam, spaced at regular or variable intervals.

The horizontal shear to be transferred by the shear connectors shall be computed by the formula

$$S = \frac{Vm}{I}, \text{ in which}$$

S = the horizontal shear per linear inch at the junction of the slab and beam at the point in the span under consideration

V = the total external shear due to the superimposed loads applied after the concrete has attained 75 percent of its required 28-day strength

m = the statical moment of the transformed compressive concrete area about the neutral axis of the composite section or the statical moment of the area of reinforcement embedded in the concrete for negative moment

I = the moment of inertia of the transformed composite beam

In the above, the compressive concrete area is transformed into an equivalent area of steel by dividing the effective concrete flange width by the modular ratio "n."

The resistance value "Q" at working load of an individual shear connector, or of one pitch of continuous bar, is its critical load capacity divided by the factor of safety. The required pitch of the shear connectors is determined by dividing the resistance "Q" of all connectors at one transverse beam cross section by the shear "S" per linear inch with a maximum pitch of 24 inches.

The useful capacities of the following types of shear connectors, as determined by tests, are as follows:

Channels — $Quc = 182 (h + 1/2\, t)\, w \sqrt{f'_c}$

Welded studs (Diameters "d" less than 1 inch) —

$$^*Quc = 332\, d^2 \sqrt{f'_c}$$

Welded studs (Diameters "d" equal to or greater than 1 inch) —

$$^*Quc = 316\, d \sqrt{f'_c}$$

Helical bars (spirals) — $Quc = 3840\, d \sqrt[4]{f'_c}$

In the above the following notations apply:

Quc = the critical load capacity of one shear connector

h = the maximum thickness of a channel flange, in inches, measured at the face of the web

t = the thickness of the web of a channel shear connector, in inches

w = the length of a channel shear connector, in inches, measured in a transverse direction on the flange of a beam

f'_c = the required compressive strength of concrete at 28 days, as determined by tests of 6" by 12" cylinders, in pounds per square inch

*These formulas apply to studs 4 inches high and higher; for three-inch height the values should be reduced 15 percent.

d = diameter of studs or of the round bars used in helical connectors (spirals), in inches

The factors of safety to be used shall be determined as follows:

$$*F.S. = \frac{2.7\,(1 + C_{mc} + C_{mi}\,C_s) - (C_{mc} + C_{mi}) + C_v}{(1 + C_v)}$$

where

$$C_{mc} = \frac{\text{Max. moment caused by dead loads acting on the composite section}}{\text{Max. moment caused by live load}}$$

$$C_{mi} = \frac{\text{Max. moment caused by dead loads acting on the steel beam alone}}{\text{Max. moment caused by live load}}$$

$$C_s = \frac{\dfrac{\text{Moment of inertia of composite beam at point of max. moment}}{\text{Distance from neutral axis to extreme tensile fiber}}}{\dfrac{\text{Moment of inertia of steel beam at point of max. moment}}{\text{Distance from neutral axis to extreme tensile fiber}}}$$

$$C_v = \frac{\text{Vertical shear at section considered caused by the dead load acting on the composite section}}{\text{Vertical shear caused by live load}}$$

The intensity of unit shearing stress in a composite beam may be determined on the basis that the web of the steel beam carries the total external shear, neglecting the effects of the steel flanges and of the concrete slab. The shear may be assumed to be uniformly distributed throughout the gross area of the web.

3.9.6—Deflection.

The provisions of Article 3.6.10 in regard to deflections from live load plus impact shall be applicable also to composite beams and girders. Preferably the ratio of the length of span to the overall depth of beam (concrete slab plus steel beam or girder) shall not be greater than 25 and the ratio of the length of span to the depth of the steel beam alone shall be not greater than 30. For continuous spans the span length shall be considered as the distance between dead load points of contraflexure.

If depths less than these are used, the sections shall be so increased that the maximum deflection will be not greater than if these ratios had not been exceeded.

When the beams are not provided with falsework or other effective intermediate support during the placing of the concrete slab, the deflection due to the weight of the slab and other permanent dead loads added before the concrete has attained 75 percent of its required 28-day strength shall be computed on the basis of no composite action.

*A factor of safety of 4 may be used in lieu of calculating the factor of safety by these formulas.

REFERENCES

1. "Composite Construction for I-Beam Bridges," Viest and Siess, Proceedings of the Highway Research Board, Vol. 32, 1953, p. 161.

2. "Design of Channel Shear Connectors for Composite I-Beam Bridges," Viest and Siess, Public Roads, Vol. 28, No. 1, April 1954, p. 9.

3. "Investigation of Stud Shear Connectors for Composite Concrete and Steel T-Beams," Viest, Journal, American Concrete Institute, April 1956, Proceedings, Vol. 52, pp. 875-891.

4. "Alpha Composite Construction Engineering Handbook," Porete Manufacturing Company, Second Edition, 1949.

5. "Tests of Spiral Shear Connectors," Nelson Stud Welding Division, Lorain, Ohio.

6. "Strength of Alpha Composite Sections Under Static and Dynamic Stresses," Voellmy, Porete Manufacturing Co., North Arlington, New Jersey.

For beams erected without temporary shoring, the total external shear "V" as used in Article 3.9.5 is the total external shear from live load and impact plus any shear from dead load added after the concrete has attained the 75 percent of its required 28-day strength. For beams provided with properly designed temporary shoring during construction, "V" is the external shear from dead, live, impact and shoring-removal loads.

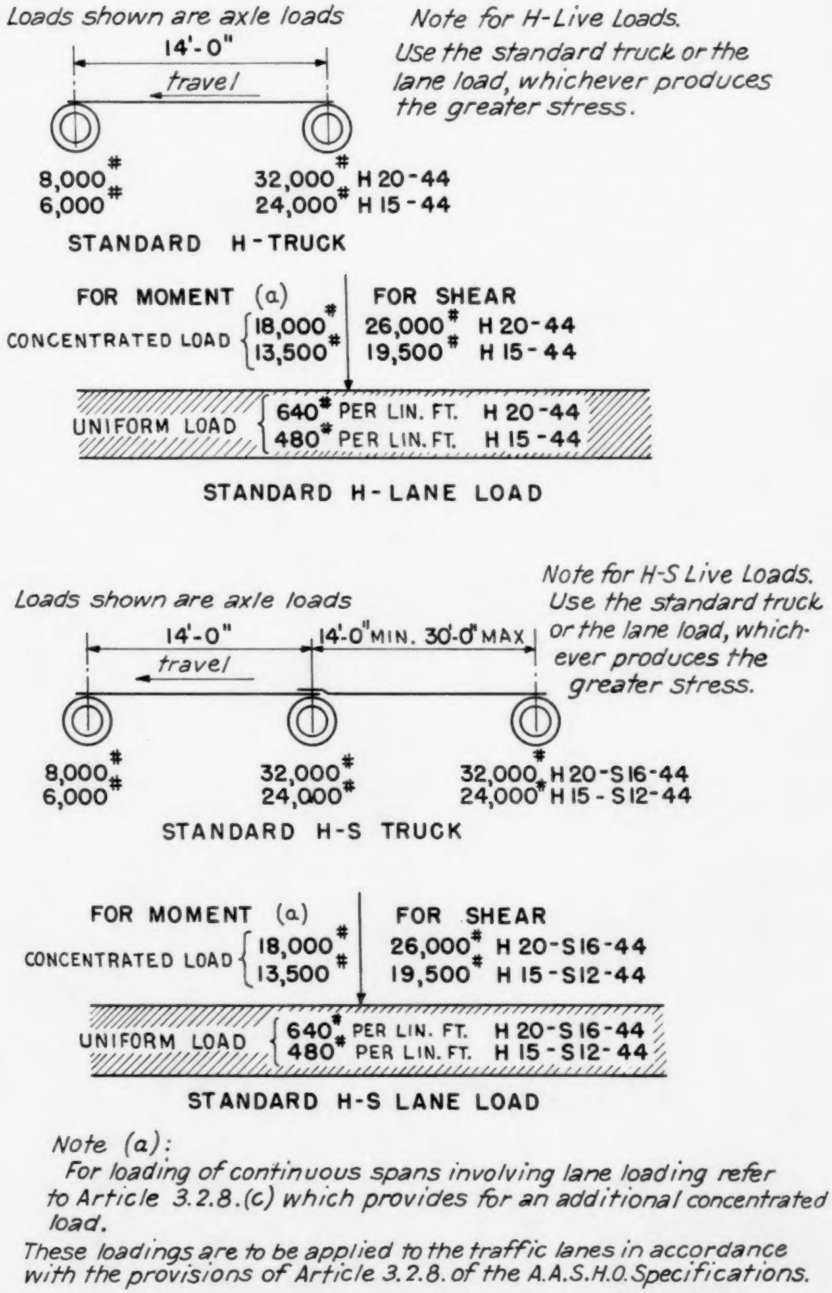
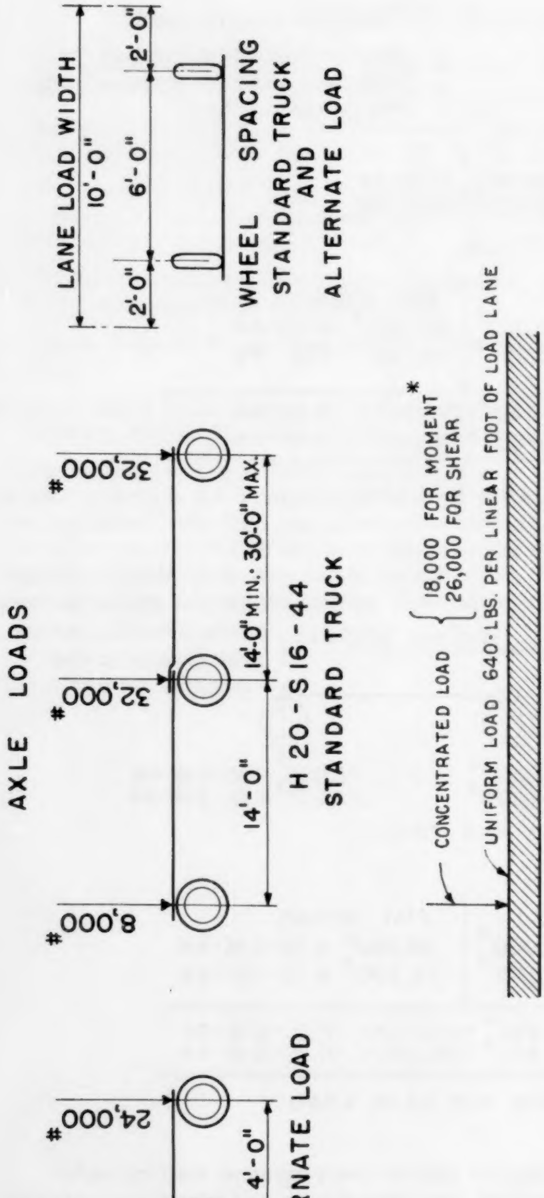


Fig. 1 STANDARD LIVE LOADS FOR BRIDGES



* *Note:*
For loading of continuous spans involving lane loading refer to Article 3.2.8.(c) which provides for an additional concentrated load.

Use the Standard Truck or the Alternate Loading or the Lane Loading whichever produces the greatest stress.

These loadings are to be applied to the traffic lanes in accordance with the provisions of Article 3.2.8. of the A.A.S.H.O. Specifications.

Fig. 2 LIVE LOAD FOR BRIDGES ON INTERSTATE SYSTEM

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BACKFILL GUIDE*

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(Proc. Paper 1321)

ABSTRACT

Recent field investigations have shown that improperly placed and compacted backfill is commonly associated with pavement settlement and the movement of retaining structures. It is the purpose of the "Backfill Guide" to demonstrate the importance of the backfill, to point out the reasons for specified backfilling procedures, and to show various construction practices that should be avoided.

FOREWORD

Even the casual observer cannot help but notice that the settlement of pavement placed on backfill at structures is one of the most frequent defects to occur in highway construction. Less obvious, but no less important, is the excessive movement of backfilled retaining walls, abutments, and wingwalls that also takes place.

Recent field investigations that have been made during the course of a research study sponsored by the Illinois Division of Highways and the U.S. Department of Commerce, Bureau of Public Roads and conducted by the University of Illinois have shown that it is not uncommon to find improperly placed and compacted backfill associated with pavement settlement and the movement of structures.

It is unfortunate that the problems of working in a confined space should occur where the need for a good fill is the greatest. The cramped working space, the relatively small volume of fill involved, and the backfill material locally available are undoubtedly responsible for much lax enforcement of

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specifications. However, the knowledge that prominent defects are almost sure to develop if the specifications are not enforced should serve as an incentive for better enforcement of backfill specifications. It is the purpose of this guide to demonstrate the importance of the backfill, to point out the reasons for specified backfilling procedures, and to show some construction practices that should be avoided.

Why Build a Retaining Wall?

A retaining wall is built to make possible a difference in ground level at points located so close to each other that a natural slope would occupy too much space. The wall must support the ground that extends to the upper level plus any additional burden placed on this ground (Fig. 1). If the wall also supports the end of a bridge it is known as an abutment, but its purpose is essentially the same. That is, it must support and hold back the soil upon which the pavement or railroad track rests so that there can be a clear opening beneath the bridge (Fig. 2).

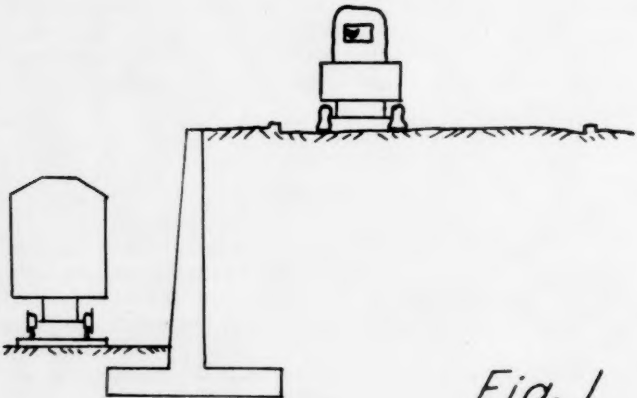

Fig. 1

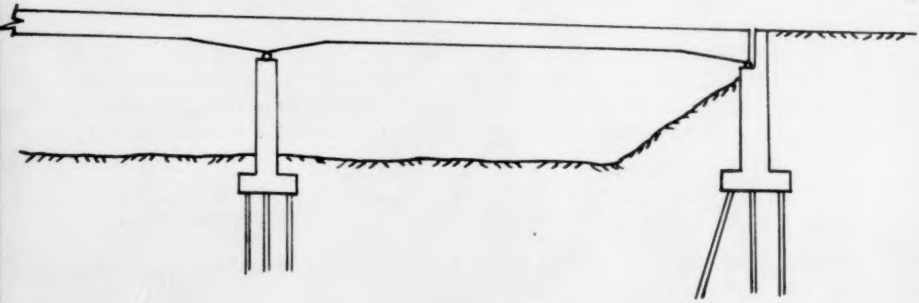

Fig. 2

Why Backfill a Wall?

Backfilling is necessary simply because a wall or an abutment has to be constructed, and construction takes room. If the upper ground will stand temporarily on a slope it is sometimes cut away and the retaining wall constructed in the cut (Fig. 3). If the upper ground will not stand alone it may be necessary to drive sheet piles for protection and to provide the working space (Fig. 4). Sometimes, especially if a bridge abutment is being constructed for an overpass, (Fig. 2), an abutment can be built completely in the open. Sooner or later, the space behind the wall must be filled to restore

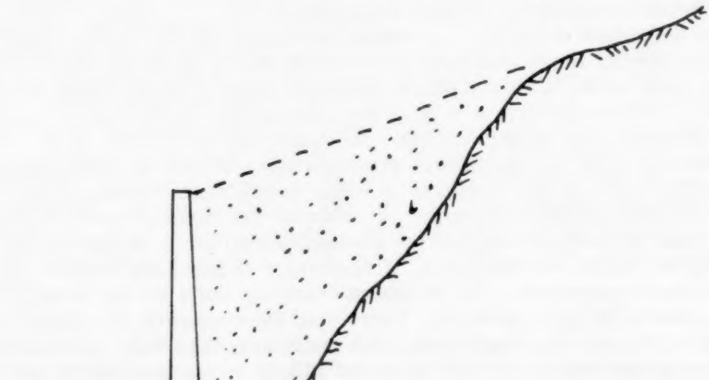

Fig. 3

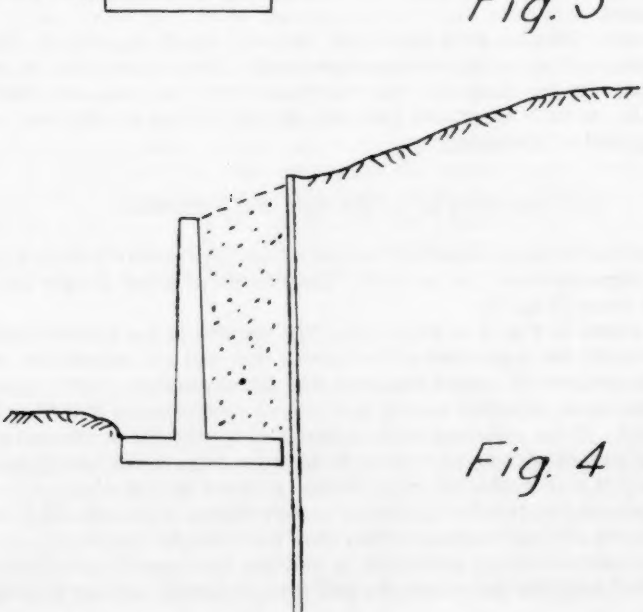

Fig. 4

the ground surface to its original elevation, or to provide the support for the roadway behind the abutment. Hence, backfill is a necessity for every retaining wall or abutment and since the backfill is the portion of the ground closest to the wall, it has an important effect on the forces that the wall must resist.

What Forces Must the Wall Stand?

The weight of the backfill behind the wall tends to make this material settle and push outward against the back of the wall. This push is known as the earth pressure. If the retaining wall or abutment is to remain in place it must be able to withstand this earth pressure.

The magnitude of the earth pressure increases rapidly as the height of the wall increases. It also depends somewhat on the weight of the material used for backfill. Most of all it depends on the strength of the material used as backfill. If this material is strong, the tendency for the backfill to push outward under its own weight is much less than if the material is weak.

Retaining walls and abutments are expensive. If they must be designed for the large pressures that result from weak backfills, their bases must be very wide, and the structures must be very strong. On the other hand, if retaining walls need be designed only for the pressures exerted by strong backfills, they can be much less expensive. It is cheaper to find good backfill material and to place it correctly, than to design retaining walls for the large pressures exerted by poor backfills. Therefore, the engineers who design retaining walls, and who must think about their cost, base their calculations on the assumption that the backfill material will be strong and will be properly placed. This is only good sense, because it leads to economy. However, the best efforts of the designer will be of no value if the backfill is not strong and well placed.

The strength of the backfill, therefore, is particularly important. The designer has assumed a certain minimum strength. This assumption is as important as those he has made for the strengths of the concrete and steel in the wall itself. In fact, we should consider the backfill as an inherent part of the retaining wall or abutment.

How Important Is the Strength of the Backfill?

We do not need to understand the details of earth pressure theory to appreciate the importance of the backfill. The results of a few simple calculations tell the story (Fig. 5).

The wall shown in Fig. 5 is 15 ft high. The lengths of the arrows behind the wall represent the magnitude of the forces that will act against the wall if the backfill consists of a good material like dense sand or gravel, if it consists of the same material placed in a looser condition, or if it is soft wet clay or silt. If the retaining wall is designed for the force exerted by dense sand or gravel with a reasonable factor of safety, it can hardly be expected to stand if it is subjected to the forces exerted by soft clay.

In Fig. 6 we see the results of similar computations for a wall 30 ft high. All the pressures are much greater than they were for the low wall.

These diagrams should be sufficient to indicate the importance of using the right kind of backfill, for which the wall was designed, and for placing it correctly.

What About a Soft Backfill?

Not only should a backfill be strong, in order to produce smaller pressures against the wall, but it should also be reasonably rigid. Otherwise it will settle after it is placed. Out in the open country, if nothing of importance is to be constructed above the retaining wall, perhaps the settlement will not be important. But if the backfill is to support a railroad track, a highway, or a building, the settlement will be most undesirable. We have all felt the bump in a concrete highway just as the pavement reaches a bridge. The bump develops, not because the bridge is hard, but because the pavement has settled. It settled because the backfill was soft and compressible.

Compressible backfill behind bridge abutments has become so common that designers of highway bridges now include an approach slab, a heavily reinforced slab that rests partly on the abutment and partly on the approach fill. The slab is supposed to be strong enough to bridge over the space left under the slab by settlement of the backfill. (Fig. 7). Many times the soil under the entire approach slab has settled and the bump is still there, although it is spread out over a greater length. If this space beneath the approach slab extends far enough, the slab will fail and repairs will be required.

It is seen then, that an incompressible backfill is also important and is often as important as a strong backfill. Fortunately strength and incompressibility usually go hand in hand and if one is obtained the other is also obtained.

What About Drainage?

A retaining wall or an abutment is not designed as a dam. If it were, it would have been much stronger and more expensive. The earth pressures for which most retaining walls are designed are not more than half the pressures that would be exerted by water of the same depth as the backfill.

Again it would be poor economy to design a retaining wall to serve also as a dam. However, if it is not so designed and is subjected to the combined action of earth and water, it is likely to fail.

To avoid the excessive pressures caused by a saturated backfill, designers provide for drainage of the material behind the retaining wall, and specify that the backfill should be material that can be drained. This is no mere whim of the designer. It is far cheaper to find a backfill that can be drained, and to see that the drains are properly installed, than to design the retaining wall or abutment to serve also as a dam.

What Is the Answer?

The field forces have two major responsibilities related to the backfill which makes the difference between a successful and an unsuccessful job. These are the selection of good materials and their proper placement.

The best materials are broken stone, gravel, and sand. These materials are good because they are strong; that is, they have high shearing resistances, even if placed in a fairly loose condition. They are practically incompressible if properly placed, and they are drainable without any special effort. Dirty or silty sandy and silts are not such good materials because they drain quite slowly, they have a tendency to resist compaction, and they

are inherently not as strong as the more granular materials. If they are used, they must be placed with extreme care or they will eventually exert excessive pressures against the wall.

Clays are poor backfill materials. They cannot be satisfactorily drained, they shrink as they dry out and swell as they become wet. They are also likely to be quite compressible and lead to undesirable settlement. Therefore, they should be avoided whenever possible.

The selection of the best available material is important. A glance at Figs. 5 and 6 should be sufficient to demonstrate this. The designer depends on the cooperation of the engineers in the field in getting material for backfill that will behave as he has assumed in his calculations, on which he had expended much thought and in which he has sensibly tried to be both conservative and economical.

How to Do the Job Wrong

It is easy to get a poor backfill job. Many ordinary construction operations, sometimes considered acceptable because of their commonness, yield poor results. If we were to try deliberately to create unsatisfactory conditions, we could hardly do better than to follow some of the most common practices.

For example, specifications often call for compaction in layers perhaps 4 or 6 inches thick. But if the material is brought in by a bulldozer, a truck, or a clamshell, it is usually heaped or dumped in a pile several feet thick. The pile is spread out over the area of the backfill into a layer, but how often is the pile distributed to a thickness of only 4 or 6 inches at its deepest part? It is only natural to spread as little dirt as possible, but if we do this there remains at the location of the pile a loose accumulation, perhaps 1 or 2 ft thick, that can be compacted at the top but not in the lower part. This loose lens is forever a source of weakness in the backfill, and is also likely to be the source of settlement.

The loose material taken from the excavation may be stockpiled above the wall and pushed by a bulldozer into the hole when backfilling is started. Perhaps there are men in the bottom of the hole with pneumatic tampers to compact the material in layers, but if the material comes too fast, the men cannot compact it because they must work on the top of the material. If the soil is added faster than effective compaction can be carried out, the backfill never gets compacted. This is perhaps the most common cause of a poor job. It is even possible that at opportune moments a whole load of material may be dumped behind the wall so rapidly that it receives no compaction at all.

Of course the upper part of the fill is likely to be carefully compacted, because this material is visible to all. However, if at the bottom of the backfill, even in a narrow wedge, there is an accumulation of loose material, large earth pressures and a tendency toward settlement are built into the backfill and no amount of surface compaction can be helpful.

If a bridge abutment is to be constructed near a stream crossing, the excavation for the embankment is very likely to be below ground-water level and the bottom of the hole will be wet. Possibly the construction will be caught in a temporary flood and a deposit of soft mucky material will settle down on top of the footing, (Fig. 8a). We may backfill without removing the water, and we are very likely to backfill without removing the soft muck that

has accumulated in the bottom of the hole. But if we do so, no matter how much the overlying material is compacted, the built-in zone of softness and weakness is there forever. Extremely great pressures can be exerted against a wall as a result of this procedure. It is as if we had a rubber balloon filled with water beneath the backfill, because the vertical load transmitted to the soft material is likely to be redirected against the abutment (Fig. 8b). Of course it costs money to pump the water out and clean out the excavation, but these operations are part of the job and must be carried out if a satisfactory backfill job is to be obtained. The designer has a right to expect satisfactory backfill.

We may select good sand, perhaps a little on the fine side, and dump it in large loads behind the retaining wall. We may then flood the sand with a fire hose for compaction. This process does not really produce a dense sand. It breaks down large voids that may exist, and to that extent is helpful, but it is not as satisfactory as compaction in layers with the accompaniment of vibration. If the backfill is silty or consists of silt, the water used for flooding never drains out of the material and it remains a soft soggy mass, very weak, and capable of exerting large pressures against the wall. If the backfill is clay, the flooding process is even more undesirable because the clay is softened and rendered far more compressible than it would otherwise have been, and is unlikely ever to drain. If it should dry out it will certainly settle and develop large shrinkage cracks. Hence, flooding can certainly not be considered good practice, unless the backfill material is clean and coarse.

In freezing weather, backfill material is often frozen and may be intermixed with snow or pieces of ice. When this mixture is placed, it may be compacted and become very hard. In this state it gives the impression that it will be very satisfactory. However, as the ice and snow melt and the frozen soil thaws, the backfill loses strength, and becomes a quagmire. In this condition, the pressures exerted on the retaining wall are comparable to those demonstrated by Fig. 8 and have been known to cause complete failures.

How to Do the Job Right

The best way to do the job properly is to read the specifications and enforce them. What has been written in this paper should be enough to indicate that the provisions in the specifications are important. If they are not followed, the wall will be acted on by forces for which it is not, and often cannot be, designed.

Contractors have a natural interest in getting the job done, in getting paid for it, and getting off the job. The cost of future maintenance is not their concern. But maintenance is of vital importance to the owner of the structure. The field engineer as representative of the owner has an obligation to see that the conditions assumed by the designer and written in the specifications are actually obtained. Otherwise his job is likely to become a source of continued annoyance and expense for many years in the future. These interests are somewhat different from those of the contractor.

Therefore, vigilance is really the answer. To be present when the backfilling operations are carried out, to require compliance with the specifications and to accept no unsatisfactory substitute, are the essential contributions of the field engineer. Backfilling is often done, during the brief absence

of the inspector. The reasons are obvious, but the consequences are also obvious. Of all practices in the field, this should be the one least condoned by the resident engineer who wishes to do a good job.

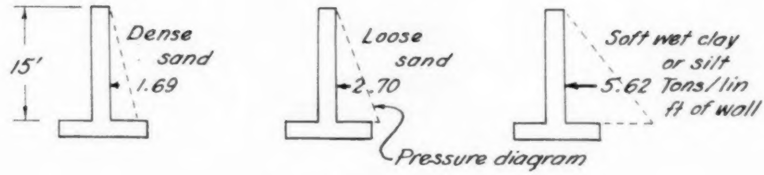
15'
Dense sand
1.69
Loose sand
2.70
Soft wet clay or silt
5.62 Tons/lin ft of wall
Pressure diagram

Fig. 5

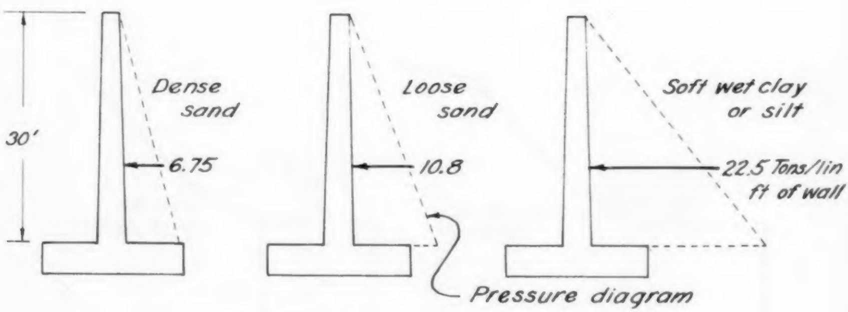
30'
Dense sand
6.75
Loose sand
10.8
Soft wet clay or silt
22.5 Tons/lin ft of wall
Pressure diagram

Fig. 6

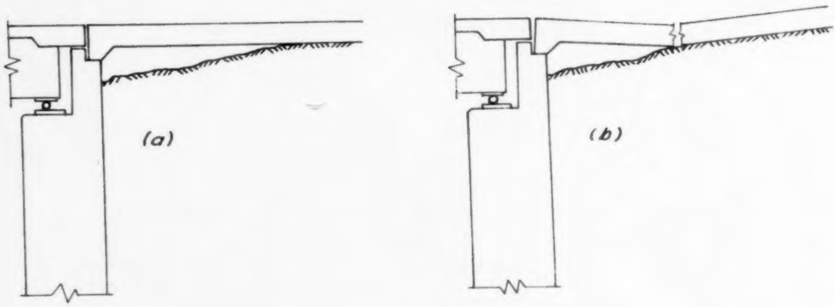

Fig. 7

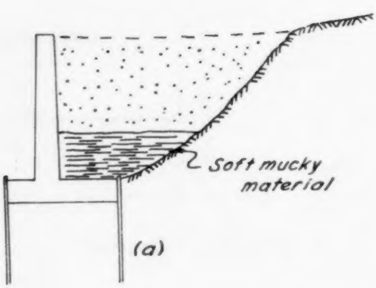

Fig. 8

Journal of the
STRUCTURAL DIVISION
Proceedings of the American Society of Civil Engineers

CONTENTS

DISCUSSION
(Proc. Paper 1329)

Discussion of
"APPLICATION OF AASHO SPECIFICATIONS TO BRIDGE DESIGN"

by Eric L. Erickson and Neil Van Eenam
(Proc. Paper 1320)

RAYMOND ARCHIBALD,[1] M. ASCE.—In the preparation of specifications for the use of the engineering profession it is desirable and essential that references and historical data be included in the specifications especially those involving the design of structures. The authors are to be complimented on their detailed references for the provisions of the design specifications for highway bridges adopted by the A.A.S.H.O. Bridge Committee.

While the writer was chairman of the A.A.S.H.O. Bridge Committee many of the empirical formulas and provisions in the specifications which were based on valid data, experiments or practical limitations gained by experience were questioned. Some of these references were lost or never recorded, and it became quite a task to verify or substantiate them.

The authors have referred to the distribution of loads in concrete slabs as covered by Article 3.3.2 and have stated that legal axle loads are considered. The historical background of the adoption of this provision should be of interest to those using the specifications.

The H and H-S standard truck design load is only a hypothetical vehicle which produces stresses in stringers, girders and truss members approximately equivalent to the stresses produced by the legal heavy trucks using the highways. However, the axle load of 32,000 pounds for the rear axle of the H and H-S trucks is heavier than any legal axle load permitted on the highways. Because of this the Committee deemed it inadvisable and unnecessary to design the floor slab supported by stringers or girders for an axle load that exceeded the legal load to any great extent. Consequently the 24,000 pound axle load or the dual axles of 16,000 pounds each was adopted for the design of slabs supported by stringers or girders for both the H 15, or the H 20 series.

The floor slab, even though it might fail would cause very little serious damage to the bridge or its use. About the worst that could happen would be a hole punched through the floor. Also the records of the States indicated that very few if any slab failures due to overload, have been recorded. Considerable economy could be gained by the adoption of these provisions since the floor slab of any bridge constitutes a rather large proportion of the total dead load of the span.

Reference has been made to Article 3.6.5 relating to reversals of stresses and suggests that the provision is ambiguous, recommending that the first sentence of the last paragraph should be deleted. This sentence reads—"If the live load increased by 50% produces live load stresses greater than and of opposite sign to the dead load stresses, counters of compression members should be used." The purpose of this provision may have been overlooked by the authors. It is merely a safety provision in the design of truss members

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wherein an overload may reverse the stress in a tension member and require it to carry compression. Should this happen the tension member would probably buckle under the compressive load and therefore should be designed as a compression member or a counter provided.

The authors have questioned the wisdom of the provision in Article 3.6.9 which concerns the area of any portion of a compression member may be considered in determining the radius of gyration and the effective area. This has been the subject of many discussions in the Bridge Committee, but the provisions seem to have stood the test of time and thinking of bridge engineers. It appears to be on the safe side and for that reason has been considered favorably by the Committee.

A reference of early vintage is given for computing the net sections of riveted tension members as specified in Article 3.6.38. It should be pointed out that the University of Illinois has been conducting a research project over the past few years to determine the effective net sections of tension members and should have a new formula for consideration of the Bridge Committee in the very near future.

In the discussion of welded stiffeners, it might be well to point out that the University of Illinois is conducting tests on beams with and without welded stiffeners in which the preliminary tests indicate that the whole problem of welded stiffeners needs a thorough investigation.

PROCEEDINGS PAPERS

The technical papers published in the past year are identified by number below. Technical-division sponsorship is indicated by an abbreviation at the end of each Paper Number, the symbols referring to: Air Transport (AT), City Planning (CP), Construction (CO), Engineering Mechanics (EM), Highway (HW), Hydraulics (HY), Irrigation and Drainage (IR), Pipeline (PL), Power (PO), Sanitary Engineering (SA), Soil Mechanics and Foundations (SM), Structural (ST), Surveying and Mapping (SU), and Waterways and Harbors (WW), divisions. Papers sponsored by the Board of Direction are identified by the symbols (BD). For titles and order coupons, refer to the appropriate issue of "Civil Engineering." Beginning with Volume 82 (January 1956) papers were published in Journals of the various Technical Divisions. To locate papers in the Journals, the symbols after the paper numbers are followed by a numeral designating the issue of a particular Journal in which the paper appeared. For example, Paper 1113 is identified as 1113 (HY6) which indicates that the paper is contained in the sixth issue of the Journal of the Hydraulics Division during 1956.

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JULY: 1019(ST4), 1020(ST4), 1021(ST4), 1022(ST4), 1023(ST4), 1024(ST4)^c, 1025(SM3), 1026 (SM3), 1027(SM3), 1028(SM3)^c, 1029(EM3), 1030(EM3), 1031(EM3), 1032(EM3), 1033(EM3)^c.

AUGUST: 1034(HY4), 1035(HY4), 1036(HY4), 1037(HY4), 1038(HY4), 1039(HY4), 1040(HY4), 1041(HY4)^c, 1042(PO4), 1043(PO4), 1044(PO4), 1045(PO4), 1046(PO4)^c, 1047(SA4), 1048 (SA4)^c, 1049(SA4), 1050(SA4), 1051(SA4), 1052(HY4), 1053(SA4).

SEPTEMBER: 1054(ST5), 1055(ST5), 1056(ST5), 1057(ST5), 1058(ST5), 1059(WW4), 1060(WW4), 1061(WW4), 1062(WW4), 1063(WW4), 1064(SU2), 1065(SU2), 1066(SU2)^c, 1067(ST5)^c, 1068 (WW4)^c, 1069(WW4).

OCTOBER: 1070(EM4), 1071(EM4), 1072(EM4), 1073(EM4), 1074(HW3), 1075(HW3), 1076(HW3), 1077(HY5), 1078(SA5), 1079(SM4), 1080(SM4), 1081(SM4), 1082(HY5), 1083(SA5), 1084(SA5), 1085(SA5), 1086(PO5), 1087(SA5), 1088(SA5), 1089(SA5), 1090(HW3), 1091(EM4)^c, 1092 (HY5)^c, 1093(HW3)^c, 1094(PO5)^c, 1095(SM4)^c.

NOVEMBER: 1096(ST6), 1097(ST6), 1098(ST6), 1099(ST6), 1100(ST6), 1101(ST6), 1102(IR3), 1103 (IR3), 1104(IR3), 1105(IR3), 1106(ST6), 1107(ST6), 1108(ST6), 1109(AT3), 1110(AT3)^c, 1111(IR3)^c; 1112(ST6)^c.

DECEMBER: 1113(HY6), 1114(HY6), 1115(SA6), 1116(SA6), 1117(SU3), 1118(SU3), 1119(WW5), 1120(WW5), 1121(WW5), 1122(WW5), 1123(WW5), 1124(WW5)^c, 1125(BD1)^c, 1126(SA6), 1127 (SA6), 1128(WW5), 1129(SA6)^c, 1130(PO6)^c, 1131(HY6)^c, 1132(PO6), 1133(PO6), 1134(PO6), 1135(BD1).

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JANUARY: 1136(CP1), 1137(CP1), 1138(EM1), 1139(EM1), 1140(EM1), 1141(EM1), 1142(SM1), 1143(SM1), 1144(SM1), 1145(SM1), 1146(ST1), 1147(ST1), 1148(ST1), 1149(ST1), 1150(ST1), 1151(ST1), 1152(CP1)^c, 1153(HW1), 1154(EM1)^c, 1155(SM1)^c, 1156(ST1)^c, 1157(EM1), 1158 (EM1), 1159(SM1), 1160(SM1), 1161(SM1).

FEBRUARY: 1162(HY1), 1163(HY1), 1164(HY1), 1165(HY1), 1166(HY1), 1167(HY1), 1168(SA1), 1169(SA1), 1170(SA1), 1171(SA1), 1172(SA1), 1173(SA1), 1174(SA1), 1175(SA1), 1176(SA1), 1177(HY1)^c, 1178(SA1), 1179(SA1), 1180(SA1), 1181(SA1), 1182(PO1), 1183(PO1), 1184(PO1), 1185(PO1)^c.

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c. Discussion of several papers, grouped by Divisions.

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